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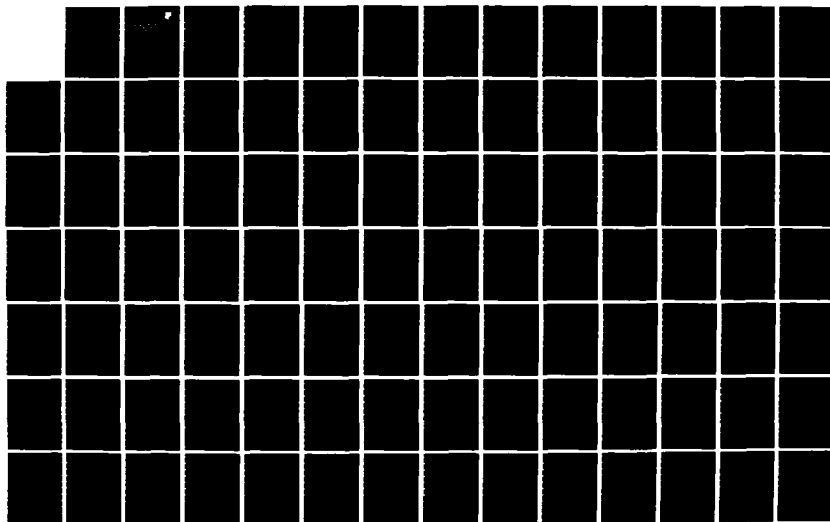
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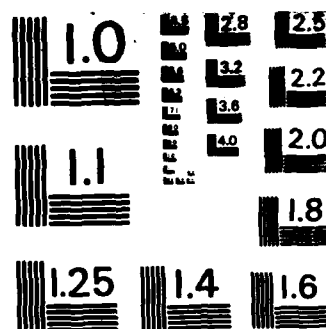
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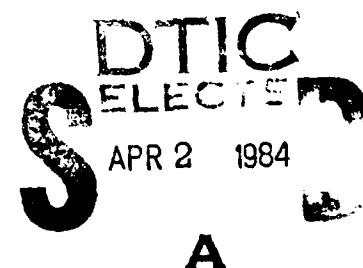


# **MATHEMATICAL MODELLING OF WAVEGUIDING TECHNIQUES AND ELECTRON TRANSPORT**

**Arcon Corporation**

**Peter P. Wintersteiner  
Stanley Woolf**

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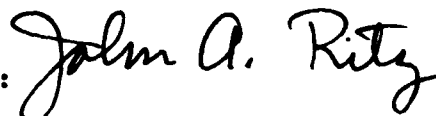
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) In this report we will describe the techniques and computer programs developed to analyze various problems. The scope of our work extended into three technical areas: 1) the study of dielectric waveguide modes and arrays of dielectric waveguides. 2) the study of a mechanism for producing diffraction gratings in crystals exhibiting photorefractivity. 3) the problem of electron scattering and transport in solids.		

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Block 19 Cont'd (Key Words)

Method of Discrete Ordinates

Multigroup Transport

Monte Carlo Method

Multiple Scattering

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- TRUNCN -

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PROGRAM TRUNCN(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)

PREPARED BY PETER P. WINTERSTEINER, ARCON CORP: COMPLETION DATE 11/30/78  
REVISION COMPLETE 6/ 5/79  
REVISION COMPLETE 1/ 7/80  
REVISION COMPLETE 5/28/81  
REVISION COMPLETE 11/ 5/81

PROGRAM TRUNCN FINDS THE MODES OF A DIELECTRIC WAVEGUIDE IN THE  
WEAKLY-GUIDING APPROXIMATION BY SATISFYING THE CONDITIONS  $\text{DET}(R) = 0$   
OR  $\text{DET}(I) = 0$ . R AND I ARE MATRICES OBTAINED BY TRUNCATING (AT  
NOT TERMS) THE INFINITE SET OF INFINITE LINEAR EQUATIONS FOR THE FIELD  
EXPANSION COEFFICIENTS (C OR D). R AND I ARE ACTUALLY LINE  
INTEGRALS WHICH ARE EVALUATED BY CHEBYSHEV QUADRATURE. UP TO 20 SEPARATE  
CASES CAN BE HANDLED IN A SINGLE JOB, SO LONG AS NOT IS KEPT FIXED.  
NOT MAY BE BETWEEN 1 AND 7.

INTEGER S,SC

REAL N,NL

DIMENSION PR(10),TABLE(51,2),INDEX(20)

DIMENSION STORE(200,16),INDX(7),GO(7,7),C(7)

DIMENSION ROOT(20,13),ROOT(20,3),CFF(20,8,7)

COMMON/A,B,P2,PI,NOT

COMMON/B/STORE,INDX,GO,C

DATA (PR(1),1-1.31/10M CIRCLE,10M ELLIPSE,10M SQUARE, /

DATA (PR(1),1-4.7/10M S' ELLIPSE,10M RECTANGLE,6M COSINE,6M SINE /

DATA (PR(1),1-8.9/10M PLIPED,10M CUSPED /

LIST OF VARIABLES WITH SINGLE ASSIGNED PURPOSES

ACC ACCURACY TO WITHIN WHICH THE ROOT LOCATION PROCEDURE FINDS P2  
BP NORMALIZED FREQUENCY (READ IN)  
B VALUE OF B IS SZ.NE.1 (SEE COMMENTS)  
C VECTOR OF COEFFICIENTS IN THE FIELD EXPANSION (DESIRED RESULT)  
CFF ARRAY FOR STORING VALUES OF C FOR DIFFERENT CASES  
DET VALUE OF THE DETERMINANT OF THE MATRIX GO  
DIRHO DIRHO/D(PHI) ON THE PERIMETER OF THE GUIDE  
GD MATRIX OF COEFFICIENTS (RLS OR TLS)  
ID # OF FUNCTION EVALUATIONS DURING ROOT-LOCATION PROCEDURE  
IDIA DIAGNOSTIC CODE (SEE COMMENTS) (READ IN)  
INDEX ARRAY FOR STORING TABLE INDICES NEAR WHICH ROOTS OCCUR  
INDX ARRAY FOR STORING VALUES OF MATRIX INDICES USED  
INDOT ARRAY FOR STORING INFORMATION USED IN PRINTING RESULTS AT THE END  
JDEXT COUNTER TO KEEP TRACK OF THE NUMBER OF SEPARATE CASES COMPLETED  
L INDEX OF FIRST TERM IN FIELD EXPANSION (READ IN)  
LLL VALUE OF L FROM PREVIOUS CASE (HELPS AVOID REPEATED CALCULATIONS)  
LLS VALUE OF S FROM PREVIOUS CASE (HELPS AVOID REPEATED CALCULATIONS)  
N SUPERELLI-SE INDEX (REAL) (READ IN)  
NL VALUE OF N FROM PREVIOUS CASE (HELPS AVOID REPEATED CALCULATIONS)  
NDP # OF INTEGRATION POINTS (READ IN)  
NDT # OF TERMS IN EXPANSIONS, SIZE OF DETERMINANT (READ IN)  
NP2 # OF VALUES OF P2 TAKEN TO CONSTRUCT TABLE (READ IN)  
NR # OF ROOTS FOUND  
PHI ANGLE IN RHO, PHI) COORDINATE SYSTEM  
PI  
P1 LARGEST VALUE OF P2 USED IN TABLE (READ IN)  
P2 SMALLEST VALUE OF P2 USED IN TABLE (READ IN)  
PR ARRAY FOR STORING ALPHANUMERIC PROGRAM INFORMATION  
P2 PROPAGATION CONSTANT (DESIRED RESULT)  
R ASPECT RATIO (READ IN)

[illegible]

```

NOT = NUMBER OF TERMS TAKEN BEFORE TRUNCATION
AT TACY OF HALF-INTERVAL SEARCH IS 10.0*(-S)
NP1 = # OF POINTS IN THE INITIAL TABLE
NIP = # OF INTEGRATION POINTS IN THE RANGE (0,PI/2)
IDAG = DIAGNOSIS CODE 0 = NORMAL CONDITION (DEFAULT)
1 = TABLE, MATRIX, COEFF SUMMARY PRINTED
2 = SUMMARY PRINTED WITH MORE SIG FIGURES
3 = DETAILS OF ROOT-LOCATION PRINTED

* IF THE CROSS-SECTION IS CIRCULAR, THE PHI-INTEGRATIONS CAN BE DONE IN
* CLOSED FORM. TO SAVE TIME ONE CAN SET NOP = 1, LEAVING ONLY ONE "INTE-
* GRATION POINT". IN THIS CASE, ADJUSTMENTS IN THE VALUES OF THE TRIG
* FUNCTIONS ARE MADE AT STATEMENT 87 SO THAT THE INTEGRAL WILL COME OUT
* RIGHT. NOTE THAT IF NOP = 1, NOT IS MUST ALSO EQUAL 1 OR ELSE THE OFF-
* DIAGONAL MATRIX ELEMENTS MAY BE NONZERO AND THEREFORE INCORRECT.

READ IN INFORMATION WHICH REMAINS FIXED FOR MANY CASES.
RETURN TO STATEMENT 2 ONLY AFTER DUMPING RESULTS FOR THESE CASES.
A BLANK CARD HERE TERMINATES PROGRAM EXECUTION.

READ(5,100)NOT,S,NP2,NOP,IDIAG
IF(NOT.EQ.0)STOP
WRITE(6,130)
WRITE(6,123)
WRITE(6,101)PN,W
IF(NOT.GT.7)NOT = 7
IF(S.LE.0)S = 3
IF(S.GT.12)S = 12
ACC = 10.0*(-S)
IF(NP2.LT.1)NP2 = 10
IF(NP2.GT.50)NP2 = 50
IF(NOP.GT.200)NOP = 200
IF(NOP.EQ.0)NOP = 50
IF(NOP.EQ.1)NOT = 1

PRINT THE FIXED INFORMATION READ FROM THE FIRST DATA CARD

WRITE(6,102)NOT
WRITE(6,104)S
WRITE(6,105)NP2
WRITE(6,108)NOP
WRITE(6,103)IDIAG
*****

SC = 1 IF COSINE TERMS ARE TO BE USED; 2 IF SINE TERMS ARE DESIRED
L = INDEX OF FIRST TERM IN FIELD EXPANSION
N = INDEX OF OVOID (1 = ELLIPSE, >30 = SQUARE)
R = ASPECT RATIO
B = ASSUMED VALUE OF THE BINDING CONSTANT
PMAX, PMIN = RANGE OF VALUES OF P2 TO BE SEARCHED
SZ = SEMIMINOR AXIS OF GUIDE

DEF: 1
DEF: 1
DEF: 1, 0
DEF: 1

NOTE: NORMAL PROCEDURE IS TO LEAVE SZ BLANK, IN WHICH CASE SZ IS IGNORED.
IN THE PROGRAM B IS ALWAYS DEFINED IN TERMS OF AN AXIS OF UNITY. FOR A
GUIDE OF SIZE SZ, A NORMALIZED FREQUENCY B IS EQUIVALENT TO A NORMALIZED
FREQUENCY B*SZ IN A GUIDE OF SIZE 1.

ONE DATA CARD CORRESPONDS TO ONE INDIVIDUAL CASE

RETURN TO STATEMENT 1 IMMEDIATELY AFTER EACH CASE IS COMPLETE. A BLANK
CARD READ AT STATEMENT 1 CAUSES RESULTS FOR ALL CASES TO BE DUMPED
(PRINTED) AND CONTROL WILL THEN GO TO STATEMENT 2. A BLANK CARD THERE
TERMINATES THE JOB EXECUTION, SO TWO BLANK CARDS SHOULD FOLLOW THE LAST
DATA CARD.

```

```

C      J = 0
      1 READ(5,121)SC,L,N,R,BP,PMAX,PMIN,SZ
      IF(R.LE.0.0)GO TO 999
C
      IF(JDEX.GE.20)GO TO 999
      JDEX = JDEX + 1
      CALL SECOND(TO)
      B = BP
      IF(SZ.LE.0.0)GO TO 60
      B = BP*SZ
      60 IF(PMAX.EQ.0.0)PMAX = 1.0
      IF(N.LE.0.0)N = 1.0
      IF(B.LE.0.0)B = 1.0
      IF(SC.LE.0)SC = 1
      IF(SC.GT.2)SC = 2
C
C .....
C      DETERMINE S, THE INDEX OF THE LAST TERM IN THE FIELD EXPANSION
C .....
      S = L + 2*(NOT-1)
      IF(R.NE.1.0)GO TO 65
      I = L/2
      I = 2*I
      IF(I.NE.L)GO TO 65
      S = L + 4*(NOT-1)
C .....
C      PRINT INPUT DATA
C .....
      65 WRITE(6,111)
      WRITE(6,123)
      WRITE(6,111)
      WRITE(6,109).DEX,L,S,N,R,B
      IF(SZ.GT.0.0)WRITE(6,129)SZ,BP
      IF(PMAX.NE.1.0.OR.PMIN.NE.0.0)WRITE(6,110)PMAX,PMIN
C .....
C      INITIALIZE STORAGE ARRAYS FOR THIS CASE
C .....
      DO 70 I = 6,13
      DO 69 K = 1,NOT
      69 CFF(JDEX,I-S,K) = 0.0
      70 ROOT(JDEX,I) = 0.0
      ROOT(JDEX,3) = N
      ROOT(JDEX,4) = R
      ROOT(JDEX,5) = BP
      ROOT(JDEX,1) = 0
      ROOT(JDEX,2) = L
      ROOT(JDEX,3) = 5
C .....
C      IDENTIFY SHAPE OF CROSS-SECTION
C .....
      J = SC + 5
      I = 4
      IF(N.GT.1.0)GO TO 90
      IF(N-5)94,95,96
      94 I = 9
      GO TO 92
      95 I = 8
      GO TO 92

```

```

96 IF(NE.1.0)GO TO 92
1(
IF(LEQ.1.0)I = 1
90 IF(LE.30.)GO TO 92
I = 5
IF(LEQ.1.0)I = 3
92 CONTINUE
WRITE(6,112)PR(J),PR(I)
ROOT(JDEX,2) = PR(J)
ROOT(JDEX,1) = PR(I)
IF(NOP.EQ.1.AND.I.NE.1)WRITE(6,131)
.....
C
C
PN = NOP
W = PI/(PN*2.0)
IF(R.EQ.RL.AND.N.EQ.NL)GO TO 79
RL = R
NL = N
C
C
C CALCULATE AND STORE RHO(PHI) AND D(RHO)/D(PHI)
DO 75 I = 1,NOP
PN = 2*(NOP - I) + 1
PHI = W*PN/2.0
CALL PERIM(N,R,PHI,RHO,DRDP)
STORE(I,1) = RHO
75 STORE(I,2) = DRDP
C
C
C CALCULATE AND STORE COS(M*PHI) AND SIN(M*PHI) WHERE M CAN BE L.....S.
VALUES OF M ARE STORED AS INDX.
79 CONTINUE
IF(L.EQ.LLL.AND.S.EQ.LLS)GO TO 89
LLL = L
LLS = S
INDX(1) = L
IF(NOT.EQ.1)GO TO 86
LST = (S - L)/(NOT - 1)
DO 85 K = 2,NOT
INDX(K) = L + (K - 1)*LST
85 LST = 2*NOT + 2
IF(NOP.EQ.1)GO TO 87
DO 80 I = 1,NOP
PN = 2*(NOP - I) + 1
PHI = W*PN/2.0
DO 80 M = 3,LST,2
K = M/2
PN = INDX(K)
PN = PN*PHI
STORE(I,M) = COS(PN)
80 STORE(I,M+1) = SIN(PH)
GO TO 89
C
C
C IF NOP = 1, ASSUME A CIRCLE. SET TRIG FUNCTIONS EQUAL TO CONSTANTS
C CHOSEN SO THAT THE ANALYTICAL RESULTS FOR THE INTEGRAL ARE OBTAINED.
C
87 STORE(1,3) = SORT(.5)
STORE(1,4) = SORT(.5)
IF(INDX(1).NE.0)GO TO 89
STORE(1,3) = 1.0
.....
C
C
C CONSTRUCT THE TABLE OF DETERMINANT VALUES FOR PMIN < P2 < PMAX.
C FOR EACH VALUE OF P2, SUBROUTINE CHEBY CALCULATES THE MATRIX
C

```

```

C      GO, WHOSE DETERMINANT IS SOUGHT. SUBROUTINE MATINV CALCULATES
C      THE DETERMINANT.
C
      89 LST = NP2 + 1
      N = 3*HRLS
      IF(SC.EQ.2)W = 3*HLS
      IF(IDIAG.GT.0)WRITE(6,106)W
      PN = NP2
      PN = (PMAX - PMIN)/PN
      WW = PN/100.
      P2 = PMAX
      IF(PMAX.EQ.1.0)P2 = 1.0 - WW
      DO 200 M = 1,LST
      CALL CHEBY(NOP,SC)
      CALL MATINV(GD,NOT,C,DET,0)
      TABLE(M,1) = P2
      TABLE(M,2) = DET
      IF(IDIAG.GT.0)WRITE(6,107)M,P2,DET
      P2 = PMAX - M*PN
      200 IF(P2.LT.WW)P2 = WW
      IF(IDIAG.GT.0)WRITE(6,114)
C
      CALL SECOND(T1)
      T1 = T1 - TO
      WRITE(6,115)T1
C
C .....
C      MAKE TENTATIVE IDENTIFICATION OF ROOTS OF EQUATION DET(RLS) = 0 (OR T1S)
C
      NR = 0
      IF(TABLE(1,2).NE.0.0)GO TO 210
      NR = 1
      INDEX(NR) = 1001
      210 DO 220 M = 2,LST
      PN = ABS(TABLE(M,2))
      IF(PN.GT.0.0)GO TO 215
      NR = NR + 1
      INDEX(NR) = M + 1000
      GO TO 220
      215 PN = (TABLE(M,2)/PN)*TABLE(M-1,2)
      IF(PN.GE.0.0)GO TO 220
      NR = NR + 1
      INDEX(NR) = M - 1
      220 IF(NR.GE.20)GO TO 221
      221 WRITE(6,124)NR,JDEX
      IF(NR.EQ.0)GO TO 1
C
C .....
C      LOCATE THE ROOTS OF DET(RLS) PRECISELY.
C      LOOP DO 250 IS COMPLETED ONCE FOR EACH ROOT (I) IDENTIFIED ABOVE
C
      IF THERE ARE MORE THAN 9 ROOTS, MAKE NOTE OF IT, ONLY THE FIRST 9
      ROOTS WILL BE LISTED IN THE SUMMARY AT THE END. (IF THERE ARE MORE
      THAN 20 ROOTS, THE LAST ONES WILL HAVE BEEN MISSED COMPLETELY.)
C
      K = NR
      IF(NR.LE.8)GO TO 251
      K = K - 8
      WRITE(6,128)K
      K = 8
      251 IROOT(JDEX,1) = K
C
      DO 250 I = 1,NR

```

```

C      N = INDEX(I)
C      C( RTT(TABLE,ACC,PN,NOP,SC,M,IO,IOIAG)
C      RTT RETURNS A FINAL MATRIX, GD, A SET OF COEFFICIENTS, C,
C      AND THE ROOT, P2. SEE THE SUBROUTINE FOR A DESCRIPTION OF PARAMETERS.
C      PRINT THE FINAL MATRIX, IF DESIRED: STORE ROOT: CALCULATE A FINAL VALUE
C      FOR THE DETERMINANT. ALSO PRINT THE COEFFICIENTS.
C      IF(I.LE.8)ROOT(JDEX,I+5) = P2
C      IF(IOIAG.LT.1)GO TO 253
C      WRITE(6,114)
C      DO 252 J = 1,NOT
C      252 WRITE(6,116)W,(GD(J,K),K=1,NOT)
C      253 WRITE(6,114)
C      CALL MATINV(GD,NOT,C,DET,O)
C      WRITE(6,125)DEX,I,P2,PN,IO,DET
C      PN = IH
C      IF(NOT.GT.1)WRITE(6,122)((PN,INDEX(K),C(K)),K=1,NOT)
C      WRITE(6,114)
C      IF(DET.EQ.0.0)WRITE(6,113)
C      IF(I.GT.8)GO TO 250
C      DO 249 J = 1,NOT
C      249 CFF(JDEX,I,J) = C(J)
C      250 CONTINUE
C      CALL SECOND(T1)
C      T1 = T1 - TQ
C      WRITE(6,126)T1
C      GO TO 1
C      .....
C      AFTER ALL CASES (20 OR LESS) HAVE BEEN COMPLETED, PRINT A TABLE
C      OF RESULTS GIVING THE ROOTS AND THE COEFFICIENT RATIOS FOR EACH CASE.
C      999 WRITE(6,123)
C      WRITE(6,123)
C      WRITE(6,123)
C      WRITE(6,116)
C      DO 900 I = 1,JDEX
C      K = IROOT(I,1) + 5
C      IF(IOIAG.LE.1)WRITE(6,117)I,ROOT(I,1),(IROOT(I,M),M=2,3),
C      1(ROOT(I,J),J=2,K)
C      900 IF(IOIAG.GE.2)WRITE(6,127)I,ROOT(I,1),(IROOT(I,M),M=2,3),
C      1(ROOT(I,J),J=2,K)
C      IF(NOT.EQ.1)GO TO 2
C      IF(IOIAG.EQ.0)GO TO 2
C      WRITE(6,111)
C      WRITE(6,119)
C      DO 910 I = 1,JDEX
C      K = IROOT(I,1)
C      IF(K.EQ.0)GO TO 905
C      WRITE(6,117)I,ROOT(I,1),(IROOT(I,M),M=2,3),(ROOT(I,J),J=2,5),
C      1(CFF(I,IO,1),IO=1,K)
C      DO 902 J = 2,NOT
C      902 WRITE(6,120)(CFF(I,IO,J),IO=1,K)
C      GO TO 910
C      905 WRITE(6,117)I,ROOT(I,1),(IROOT(I,M),M=2,3),(ROOT(I,J),J=2,5)
C      910 CONTINUE
C      GO TO 2
C      END
C      SUBROUTINE RTT(TABLE,ACC,PN,NOP,SC,M,IO,IOIAG)

```

```

C SUBROUTINE RTT1 PRECISELY LOCATES THE ROOT SPECIFIED BY M, ID, AND THEN
C DETERMINES THE FIELD EXPANSION COEFFICIENTS C(J) CORRESPONDING TO
C THAT ROOT. ACC IS THE DESIRED ACCURACY OF THE ROOT, AND PN IS THE
C DIFFERENCE IN THE ACTUAL BOUNDS OBTAINED. ID IS THE NUMBER OF FUN-
C CTION EVALUATIONS NECESSARY. SC, IDIAG, AND NOP ARE AS IN MAIN. GO, THE
C FINAL MATRIX AT THE ROOT P2, IS ALSO RETURNED. (C, GD, AND P2 ARE ALL
C IN COMMON BLOCKS.)
C
C INTEGER SC
C DIMENSION STORE(200,16),INDX(7),GD(7,7),C(7),SINV(7,7)
C DIMENSION TABLE(51,2)
C COMMON/A/B,P2,P1,NOT
C COMMON/B/STORE,INDX,GD,C
C
C 700 FORMAT(/,'* SEARCH ID*,7X,*X0*,11X,*X1*,11X,*X2*,10X,*F0*,9X,*F1*,
C 19X,*F2*,11X,*X*,10X,*F*,10X,*DX*')
C 701 FORMAT(/,'* $$$$ ERROR IN HINTS OR MULLER, IER =*,13,* $$$$$$*')
C
C IF(IDIAG.GE.3)WRITE(6,700)
C K = 0
C ID = 0
C IF(M.LT.1000)GO TO 230
C P2 = TABLE(M-1000,1)
C CALL CHEBY(NOP,SC)
C ID = 1
C GO TO 248
C
C 230 G1 = TABLE(M+1,2)
C G2 = TABLE(M,2)
C X1 = TABLE(M+1,1)
C X2 = TABLE(M,1)
C DN = X2 - X1
C
C IF G1 AND G2 ARE TWO OR MORE ORDERS OF MAGNITUDE DIFFERENT, USE
C A HALF-INTERVAL SEARCH TO NARROW THE RANGE X1 TO X2
C
C 235 MW = -G1/G2
C IF(MW.LT.100..AND.MW.GT..01)GO TO 240
C IF(DN.LT.ACC)GO TO 240
C DN = DN/3.0
C
C CALL HINTS(X1,X2,G1,G2,DN,PN,NOP,SC,ID,IDIAG,IER)
C IF(IER.GT.0)WRITE(6,701)IER
C
C K = K + ID
C IF(PN.EQ.0.0)GO TO 248
C GO TO 235
C
C MULLER'S METHOD TO GET THE ROOT PRECISELY
C
C 240 CALL MULLER(X1,X2,G1,G2,ACC,PN,NOP,SC,ID,IDIAG,IER)
C IF(IER.GT.0)WRITE(6,701)IER
C
C 248 ID = ID + K
C M = 1
C IF(NOT.EQ.1)GO TO 280
C
C CALCULATE THE COEFFICIENTS: SUBROUTINE LOAD COPIES (NOT-1) OF THE
C NOT ROWS OF GD INTO ARRAY SINV AND VECTOR C. SUBROUTINE MATINV SOLVES
C THE (NOT-1)X(NOT-1) SYSTEM FOR THE COEFFICIENTS AND RETURNS THEM IN C.
C
C CALL LOAD(GD,SINV,C,NOT,M)
C LST = NOT - 1

```

```

C      MATINV(SINV,LST,C,DN,1)
C      STACK THE COEFFICIENTS IN THE PROPER ORDER IN VECTOR C, INCLUDING 1.000
C
      IF(M.EQ.NOT)GO TO 280
      DO 270 J = M,LST
      K = NOT + M - J
      270 C(K) = C(K-1)
      280 C(M) = 1.0
      RETURN
      END
      SUBROUTINE LOAD(SS,SINV,C,NOT,ID)
C      SUBROUTINE LOAD LOADS SS (NEE GO) INTO ARRAY SINV, ELIMINATING
C      THE ROW AND COLUMN CONTAINING THE DIAGONAL ELEMENT WHICH IS SMALLEST
C      IN MAGNITUDE. THE NEGATIVE OF THE COLUMN OF SS WHICH IS LEFT OUT IS
C      LOADED INTO ARRAY C, EXCEPT FOR THE ELEMENT IN THE ROW WHICH IS ELIM-
C      INATED. THUS, SINV IS (NOT-1) BY (NOT-1) IN DIMENSION, AND C IS A VEC-
C      TOR (NOT-1) IN LENGTH.
C      NOTE THAT ONLY THE FIRST THREE DIAGONAL ELEMENTS ARE SEARCHED FOR
C      THE SMALLEST VALUE.
C
      DIMENSION SS(7,7),SINV(7,7),C(7)
      G = 1.0E+321
      ID = 1
      K = NOT
      IF(K.GT.3)K = 3
      DO 200 J = 1,K
      H = ABS(SS(J,J))
      IF(H.GT.G)GO TO 200
      G = H
      ID = J
      200 CONTINUE
C
      JJ = 0
      DO 300 J = 1,NOT
      IF(J.EQ.ID)GO TO 300
      JJ = JJ + 1
      C(JJ) = -SS(J,ID)
      KK = 0
      DO 299 K = 1,NOT
      IF(K.EQ.ID)GO TO 299
      SINV(JJ,KK) = SS(J,K)
      299 CONTINUE
      300 CONTINUE
      RETURN
      END
      SUBROUTINE MULLER(X0,X2,F0,F2,ACC,PH,NOP,SC,ID,DTAG,IER)
C      MULLER'S ALGORITHM FOR THE SOLUTION OF DET(RLS) = 0
C
      INVERSE PARABOLIC INTERPOLATION USING POINTS
      (X0,F0), (X1,F1), AND (X2,F2)
      TO FIND THE NEW POINT (X,F), WHERE X0 < X1 < X2 AND F0*F2 < 0
      C ALWAYS HOLD. SIGN IS THE SIGN OF F0, AND DOES NOT CHANGE FROM ITS
      C INITIAL VALUE. THE TWO ROOTS OF THE QUADRATIC NECESSARILY LIE
      C BETWEEN X0 AND X1, AND BEYOND X0 OR X2 IF (A) SIGN*F1 < 0
      C BETWEEN X1 AND X2, AND BEYOND X0 OR X2 IF (B) SIGN*F1 > 0
      C (THAT IS, FOR EXAMPLE, IN CASE (A) THERE IS NO ROOT BETWEEN X1 AND X2.)
      C CHOOSE THE ROOT INSIDE THE INTERVAL.
      C XL IS THE LARGEST X FOR WHICH F*SIGN > 0
      C XU IS THE SMALLEST X FOR WHICH F*SIGN < 0
      C
      INTEGER SC

```

```

C      DIMENSION STORE(200,10),INDX(7),GD(7,7),C(7)
C      COMMON/8/STORE,INDX,GD,C
C      ID = 0
C      IER = 0
C      PN = X2 - X0
C      IF(X2.LE.X0)GO TO 290
C      W = ALOG(PN/ACC)/.693147
C      LST = W + 1.5
C
C      LST GIVES THE NUMBER OF ITERATIONS WHICH A HALF-INTERVAL SEARCH
C      WOULD REQUIRE FOR THESE INITIAL VALUES AND DESIRED ACCURACY. IF
C      MULLER'S ALGORITHM EXCEEDS THIS, EXIT WITH ERROR CONDITION 3.
C
C      700 FORMAT(* MULLR:*,I3.3F13.10,3E11.4,F13.10,2E11.4)
C      701 FORMAT(* MULLR:*,I3.3F13.10,1E11.4)
C
C      ASSUME THAT THE INITIAL VALUES ARE SUCH THAT X0 < X2
C      IF NOT, RETURN (IER = 4)
C
C      XL = X0
C      XU = X2
C      SIGN = F0/ABS(F0)
C
C      SPLIT THE INTERVAL AND EVALUATE THE DETERMINANT AT THE MIDPOINT
C
C      40 X = (XL + XU)/2.0
C      P2 = X
C      CALL CHEBY(NOP,SC)
C      CALL MATINV(GD,NOT,C,F,0)
C      ID = ID + 1
C      IF(F*SIGN)50,200,60
C      50 XU = X
C      GO TO 100
C      60 XL = X
C
C      EVALUATE THE NEW X FROM THE OLD X0, X1, AND X2
C
C      100 X1 = X
C      F1 = F
C      DD01 = (F1 - F0)/(X1 - X0)
C      DD12 = (F2 - F1)/(X2 - X1)
C      DD012 = (DD12 - DD01)/(X2 - X0)
C      W = DD01 + (X1 - X0)*DD012
C      U = DD012/W
C      GL = F1/W
C
C      U = 1.0 - 4.0*GL
C      IF(U.LT.0.0)GO TO 260
C      U = 2.0*GL/(1.0 + SORT(U))
C      X = X1 - U
C      IF(X.GT.X0.AND.X.LT.X2)GO TO 145
C      U = 1.0 - 4.0*GL*DD012/W
C      X = X1 - U
C      145 CONTINUE
C      IF(X.GT.1.0.OR.X.LT.0.0)GO TO 270
C      P2 = X
C      CALL CHEBY(NOP,SC)
C      ID = ID + 1
C      PN = ABS(XU - XL)
C      IF(PN.LT.ACC)GO TO 210
C      IF(ID.GT.LST)GO TO 280
C      CALL MATINV(GD,NOT,C,F,0)

```

```

C      IF (TAG.GE.3)WRITE(6,700)ID,X0,X1,X2,F0,F1,F2,X,F,PN
C      RESET THE UPPER OR LOWER BOUND ON THE ROOT
C
C      IF(F*SIGN)150,200,155
150 XU = X
    GO TO 160
155 XL = X
C
C      CHANGE X0 OR X2 TO THE PREVIOUS X1; THE NEW X1 IS X
C
160 IF(F1*SIGN)165,200,170
165 X2 = X1
    F2 = F1
    GO TO 175
170 X0 = X1
    F0 = F1
175 GL = ABS(U)
    IF((F*F1).LT.0.0)GO TO 100
    IF(GL.GT.ACC)GO TO 100
C
C      IF POINTS ACCUMULATE CLOSELY ON ONE SIDE OF THE TRUE ROOT, A SINGLE
C      SMALL JUMP MAY SPAN THE ROOT, THEREFORE CHANGE X BY A LITTLE LESS
C      THAN ACC AND EVALUATE THE DETERMINANT. IF THIS FAILS (THAT IS, IF THE
C      FUNCTION HAS THE SAME SIGN AGAIN) SPLIT THE INTERVAL IN HALF AND
C      CONTINUE.
C
C      LINEAR EXTRAPOLATION PAST THE ANTICIPATED ZERO
C      IF X = X1, DO THIS ANYWAY AND THEN EXIT.
C
      W = 1.0E-12
      IF(X.NE.X1)GO TO 180
      XT = W
      IF((F*SIGN).GT.0.0)XT = -W
      GO TO 185
180 XT = 2.0*F*(X1 - X)/(F1 - F)
      GL = ABS(XT)
      IF(GL.GT.ACC)GO TO 100
      IF(GL.GT.W)GO TO 185
      XT = W*XT/GL
185 XT = X - XT
      P2 = XT
      CALL CHEBY(NOP,SC)
      CALL MATINV(GO,NOT,C,FT,0)
      ID = ID + 1
      IF(IDIAG.GE.3)WRITE(6,701)ID,XT,FT
190 IF(FT*F)190,200,40
192 XU = XT
      X2 = XT
      F2 = FT
      IF(X-X1)100,205,100
193 XL = XT
      X0 = XT
      F0 = FT
      IF(X-X1)100,205,100
C
C      IF THE DETERMINANT IS EXACTLY ZERO, THE MATRIX (WHICH MAY HAVE BEEN
C      ALTERED IN MATINV) IS RECOMPUTED.
C
200 PN = 0.0
    GO TO 209
C
C      IF X = X1, TAKE THIS AS THE ROOT
C

```

```

208 P2 X
PN( ABS(XU - XL)
C
209 CALL CHEBY(NOP,SC)
ID = ID + 1
C
C NORMAL RETURN: PN,LT,ACC
C
210 IF(IDIAG.GE.3)WRITE(6,700)ID,X0,X1,X2,F0,F1,F2,X
RETURN
C
C ERROR RETURN 1: COMPLEX ROOT
C
260 IER = 1
GO TO 299
C
C ERROR RETURN 2: PROCEDURE IS OSCILLATING
C
270 IER = 2
GO TO 299
C
C ERROR RETURN 3: MORE ITERATIONS THAN HALF-INTERVAL SEARCH WOULD REQUIRE
C
280 IER = 3
GO TO 210
C
C ERROR RETURN 4: X0 > X2 INITIALLY
C
290 IER = 4
299 ID = ID + 1
CALL CHEBY(NOP,SC)
RETURN
END
SUBROUTINE HINTS(X1,X2,G1,G2,ACC,PN,NOP,SC,ID,IDIAG,IER)
C
C SUBROUTINE HINTS PERFORMS A HALF-INTERVAL SEARCH TO LOCATE THE ROOTS OF
C THE EQUATION DET(RLS) = 0 OR DET(TLS) = 0 TO WITHIN A DESIRED ACCURACY
C ACC. ID IS THE NUMBER OF BISECTIONS, AND PN IS THE ACTUAL MAXIMUM ERROR.
C
C INTEGER SC
C DIMENSION STORE(200,16),INDX(7),GD(7,7),C(7)
C COMMON/A/B,P2,P1,NOT
C COMMON/B/STORE,INDX,GD,C
C
700 FORMAT(= HINTS:*,13,13X,2F13.10,11X,2E11.4,F13.10,2E11.4)
C
ID = 0
IER = 0
100 PN = ABS(X2 - X1)
C
IF(PN.LT.ACC)RETURN
ID = ID + 1
IF(ID.GT.25)GO TO 230
P2 = (X1 + X2)/2.0
CALL CHEBY(NOP,SC)
CALL MATINV(GD,NOT,C,G,0)
C
IF(IDIAG.GE.3)WRITE(6,700)ID,X1,X2,G1,G2,P2,G,PN
C
IF(G*(G1/ABS(G1)))120,210,130
120 X2 = P2
G2 = G
GO TO 100
130 X1 = P2
G1 = G
GO TO 100

```



```

C      OBTAIN COS(L*PHI) AND SIN(L*PHI)
C      CL = STORE(UUU,2*1+1)
C      SL = STORE(UUU,2*1+2)
C      RETRIEVE THE K BESSEL FUNCTIONS OF ORDER L AND L-1
C      BK = BKST(L+1)
C      LL = L - 1
C      IF(L.EQ.0) LL = 1
C      BKM = BKST(LL+1)
C      DO 100 J = 1,NOT
C      S = INDX(J)
C      OBTAIN COS(S*PHI) AND SIN(S*PHI)
C      CS = STORE(UUU,2*J+1)
C      SS = STORE(UUU,2*J+2)
C      OBTAIN THE J BESSEL FUNCTION OF ORDER S AND S-1
C      BJ = BJST(S+1)
C      K = S - 1
C      IF(S.EQ.0) K = 1
C      BJM = BJST(K+1)
C      IF(S.EQ.0) BJM = -BJM
C      100 GD(I,J) = GD(I,J) + W*RLS(SC)
C      RETURN
C      END
C      FUNCTION RLS(SC)
C      EVALUATE THE INTEGRAND OF THE LINE INTEGRAL, WHICH BECOMES THE
C      MATRIX ELEMENT R(L,S) (COSINE EXPANSION) OR T(L,S) (SINE EXPANSION)
C      INTEGER S,SC
C      COMMON/A/B,P2,P1,NOT
C      COMMON/C/P,OMP,RH,DR,CL,SL,CS,SS,BJ,BJM,BK,BKM,L,S
C      XS = S
C      XL = L
C      EPS = 1.0
C      IF(L.GT.0) EPS = 2.0
C      TERM = BK*BJM*OMP + P*BJ*BKM
C      TERM = 2.0*B*RH*TERM
C      IF(L.EQ.S) GO TO 90
C      DO L = L-S
C      TERM = TERM + 2.0*DD*BK*BJ/P1
C      90 F = EPS*TERM
C      NOTE THAT I=-(L+1) IS NOT INCLUDED IN F
C      E = 0.0
C      IF(L.EQ.S) GO TO 100
C      E = 2.0*EPS*BK*BJ/DR/(RH*P1)
C      NOTE THAT I=-(L+1) IS NOT INCLUDED IN E
C      100 CONTINUE
C      IF(SC.EQ.2) GO TO 110
C      TERM = XS*CL*SS - XL*SL*CS

```

```

      RI = 4.0*(F*CL*CS + E*TERM)
      RETURN
C
110 TERM = XS*SL*CS - XL*SS*CL
    RLS = 4.0*(F*SL*SS - E*TERM)
    RETURN
    END
    SUBROUTINE PERIM(N,R,PHI,RHO,DRDP)
C
C  EVALUATE RHO(PHI) AND D(RHO)/D(PHI) ON THE PERIMETER
C
C      REAL N
C
C      IF(R.NE.1.0.OR.N.NE.1.0)GO TO 100
      RHO = 1.0
      DRDP = 0.0
      RETURN
C
C 100 CONTINUE
      CP = COS(PHI)
      SP = SIN(PHI)
      PN = 2.0*N
      DD = (CP/R)*PN + SP*PN
      PN = 1.0/PN
      RHO = 1.0/(DD*PN)
C
C      PN = 2.0*N - 2.0
      AA = SP*PN - ((CP/R)*PN)/(R*R)
      DRDP = -RHO*SP*CP*AA/DD
      RETURN
      END
    SUBROUTINE MATINV(A,N,B,DETER,M)
C
C  DIMENSION A(7,7),B(7,1),INDEX(7,3)
10 DETER = 1.0
15 DO 20 J = 1,N
20 INDEX(J,3) = 0
30 DO 50 I = 1,N
C
C  SEARCH FOR PIVOT ELEMENT
C
40 AMAX = 0.
45 DO 105 J = 1,N
    IF (INDEX(J,3)-1) 60,105,60
60 DO 100 K = 1,N
    IF (INDEX(K,3)-1) 80,100,740
80 IF(AMAX-ABS(A(J,K))) 85,100,100
85 IROW = J
90 ICOLUM = K
    AMAX = ABS(A(J,K))
100 CONTINUE
105 CONTINUE
    IF(AMAX.EQ.0.0)GO TO 750
    INDEX(ICOLUM,3) = INDEX(ICOLUM,3) + 1
260 INDEX(I,1) = IROW
270 INDEX(I,2) = ICOLUM
C
C  INTERCHANGE ROWS TO PUT PIVOT ELEM.IN DIAG.
C
130 IF(IROW-ICOLUM) 140,310,140
140 DETER = -DETER
150 DO 200 L = 1,N
160 SWAP = A(IROW,L)
170 A(IROW,L) = A(ICOLUM,L)
200 A(ICOLUM,L) = SWAP

```

```

210 DO 50 L = 1,M
220 SW = B(IROW,L)
230 B(IJROW,L) = B(ICOLUMN,L)
250 B(ICOLUMN,L) = SWAP
C
C   DIVIDE PIVOT ROW BY PIVOT ELEMENT
C
310 PIVOT = A(ICOLUMN,ICOLUMN)
   DETER = DETER*PIVOT
330 A(ICOLUMN,ICOLUMN) = 1.0
340 DO 350 L = 1,N
350 A(ICOLUMN,L) = A(ICOLUMN,L)/PIVOT
355 IF (M) 380,380,360
360 DO 370 L = 1,M
370 B(ICOLUMN,L) = B(ICOLUMN,L)/PIVOT
C
C   REDUCE NON PIVOTS ROWS
C
380 DO 550 L1 = 1,N
390 IF (L1-ICOLUMN) 400,550,400
400 T = A(L1,ICOLUMN)
420 A(L1,ICOLUMN) = 0.
430 DO 450 L = 1,N
450 A(L1,L) = A(L1,L)-A(ICOLUMN,L)*T
455 IF (M) 550,550,460
460 DO 500 L = 1,M
500 B(L1,L) = B(L1,L)-B(ICOLUMN,L)*T
550 CONTINUE
C
C   INTERCHANGE COLUMNS
C
600 DO 710 I = 1,N
610 L = N + 1 - I
620 IF (INDEX(L,1)-INDEX(L,2)) 630,710,630
630 JROW = INDEX(L,1)
640 JCOLUMN = INDEX(L,2)
650 DO 705 K = 1,N
660 SWAP = A(K,JROW)
670 A(K,JROW) = A(K,JCOLUMN)
700 A(K,JCOLUMN) = SWAP
705 CONTINUE
710 CONTINUE
740 RETURN
C
750 DETER = 0.0
   RETURN
   END

```

Program Listing

- IDGS -

(Reference: Vol. I, Section II.3.3)









```

      'F' MAX.NE.1.0.0R.PMIN.NE.0.0)WRITE(6,110)PMAX.PMIN
      (

```

```

      .....
      INITIALIZE STORAGE ARRAYS FOR THIS CASE
      .....

```

```

      DO 70 I = 7,14
      DO 69 K = 1,NOT
      CFS(JDEX,I-6,K) = 0.0
      CFA(JDEX,I-6,K) = 0.0
      ROOS(JDEX,I-6) = 0.0
      69 ROOT(JDEX,I) = 0.0
      70 ROOT(JDEX,3) = N
      ROOT(JDEX,4) = R
      ROOT(JDEX,5) = D
      ROOT(JDEX,6) = B
      IROOT(JDEX,1) = 0
      IROOT(JDEX,2) = SC - 1
      IROOT(JDEX,3) = NOT + SC - 2
      IROOT(JDEX,4) = 0
      .....

```

```

      IDENTIFY CROSS-SECTION FOR THIS CASE
      .....

```

```

      J = SC + 5
      I = 4
      IF(N.GT.1.0)GO TO 90
      IF(N-0.5)94,95,96
      94 I = 9
      GO TO 92
      95 I = 8
      GO TO 92
      96 IF(N.NE.1.0)GO TO 92
      I = 2
      IF(R.EQ.1.0)I = 1
      90 IF(N.LT.30.)GO TO 92
      I = 5
      IF(R.EQ.1.0)I = 3
      92 CONTINUE
      WRITE(6,109)PR(J),PR(1)
      ROOT(JDEX,2) = PR(J)
      ROOT(JDEX,1) = PR(1)
      IF(NOP.EQ.1.AND.1.NE.1)WRITE(6,131)
      .....

```

```

      FIND LST. THE CODE FOR DETERMINING WHEN TO REPEAT CALCULATIONS

```

```

      LST = 1: SKIP INDEX AND PERIMETER CALCULATIONS
      LST = 2: RECALCULATE INDICES
      LST = 3: RECALCULATE PERIMETER
      LST = 4: RECALCULATE BOTH INDICES AND PERIMETER

```

```

      CALL CODE(N,R,NL,RL,SC,LSC,LST)
      IF(LST.LT.3)GO TO 80

```

```

      CALCULATE AND STORE RHO(PHI) AS STORE(I,1), I = 1,NOP
      CALCULATE AND STORE D(RHO)/D(PHI) AS STORE(I,2), I = 1,NOP
      CALCULATE AND STORE COS(M*PHI) AND SIN(M*PHI) AS STORE(I,2*M+1)
      AND STORE(I,2*M+2), RESPECTIVELY.

```

```

      PN = NOP
      W = PI/(PN*2.0)
      DO 79 I = 1,NOP
      PN = 2*(NOP - I) + 1
      PHI = W*PN/2.0

```

```

CALL PERIM(N,R,PHI,RHO,DRDP)
S1 = (1.1) = RHO
S1 = (1.2) = DRDP
IF(JDEX.GT.1)GO TO 79
DO 75 M = 1,NT2
PN = M
PN = PN*PHI
STORE(1.2*M+1) = COS(PN)
75 STORE(1.2*M+2) = SIN(PN)
79 CONTINUE
IF(NOP.GT.1)GO TO 80
C IF NOP = 1, ASSUME A CIRCLE. SET TRIG FUNCTIONS EQUAL TO CONSTANTS
C CHOSEN SO THAT THE ANALYTICAL RESULTS FOR THE INTEGRALS ARE OBTAINED
C
DO 87 M = 1,NT2
STORE(1.2*M+1) = SORT(.5)
87 STORE(1.2*M+2) = SORT(.5)
C
80 IF(1ST.EQ.1.OR.1ST.EQ.3)GO TO 89
C
C DETERMINE THE INDEX PAIRS (L,S) OF ALL THE MATRIX ELEMENTS NEEDED
C BY CALLING SUBROUTINE REL.
C CALL REL(N,R,SC,IDIAG)
C
C .....
C
C CONSTRUCT TABLE OF VALUES OF DET(SYM) AND DET(ASY) VS P2. GIVEN B.
C TO SAVE TIME (SINCE R1(L,S) AND R2(L,S) ARE INDEPENDENT OF D) THIS
C IS DONE FOR EACH OF THE VALUES OF D READ INTO THE ARRAY DST WHEN THE
C COUNTER, ND, IS EQUAL TO 1. THIS SECTION IS SKIPPED WHEN ND.GT.1.
C
89 LST = NP2 + 1
IF(ND.GT.1)GO TO 205
IF(IDIAG.GT.0)WRITE(6,106)
PN = NP2
PN = (PN*MAX - PN*MIN)/PN
M = PN/100.
P2 = PMAX
IF(PMAX.EQ.1.0)P2 = 1.0 - W
IF(NMAX.GT.1.AND.IDIAG.GT.0)WRITE(6,132)((DST(I),DST(I)),I=1,NMAX)
NR = 2*NMAX + 1
DO 200 M = 1,LST
TABLE(M,1) = P2
DO 190 I = 1,NMAX
D = DST(I)
CALL MATR(NOP,SC,I)
CALL MATINV(SYM,NOT.C,DET,0)
TABLE(M,2+I) = DET
CALL MATINV(ASY,NOT.C,XXX,0)
190 TABLE(M,2+I+1) = XXX
IF(IDIAG.GT.0)WRITE(6,107)M,(TABLE(M,I),I=1,NR)
P2 = PMAX - M*PN
200 IF(P2.LT.W)P2 = W
IF(IDIAG.GT.0)WRITE(6,114)
D = DST(1)
C
CALL SECOND(T1)
T1 = T1 - T0
WRITE(6,115)T1
GO TO 209
C
C WHEN ND.GT.1, RECOVER THE TABLE OF VALUES OF DET(SYM) AND DET(ASY)
C VS P2 CALCULATED EARLIER.
C

```

[illegible]

```

XV 3M R2
DO 256 J = 1,NT2
WRITE(6,118)XXX,(R2(J,K),K=1,NOT)
WRITE(6,114)
XXX = 3HALP
DO 257 J = 1,NOT
WRITE(6,118)XXX,(ALPHA(J,K),K=1,NT2)
WRITE(6,114)
XXX = 3HR*A
DO 260 J = 1,NOT
DO 259 K = 1,NOT
M = 0
DO 258 LST = 1,NT2
DN = ALPHA(J,LST)*R2(LST,K)
IF(DN.EQ.0.0)GO TO 256
M = M + 1
CFS(M,8,10) = DN
258 CONTINUE
259 WRITE(6,118)XXX,(CFS(LST,8,10),LST=1,M)
260 WRITE(6,114)
IF(1DIAG.LT.5)GO TO 265
XXX = 3M*KN
J = NOT + NT2 + 1
WRITE(6,118)XXX,(BST(K),K=1,J)
WRITE(6,114)
.....
C C C C C
C PRINT THE FINAL MATRIX, IF DESIRED: STORE ROOT; CALCULATE A FINAL VALUE
C FOR THE DETERMINANT. ALSO PRINT THE COEFFICIENTS.
C
265 IF(1.LE.8)ROOT(JDEX,1+6) = P2
IF(1DIAG.LT.1)GO TO 253
XXX = 3MSYM
WRITE(6,114)
DO 252 J = 1,NOT
WRITE(6,118)XXX,(SYM(J,K),K=1,NOT)
253 WRITE(6,114)
CALL MATINV(SYM,NOT,C,DET,0)
WRITE(6,125)XXX,JDEX,1,P2,PM,1D,DET
IF(NOT.GT.1)WRITE(6,122)(C(K),K=1,NOT)
WRITE(6,114)
IF(DET.EQ.0.0)WRITE(6,113)
IF(1.GT.8)GO TO 250
DO 249 J = 1,NOT
249 CFS(JDEX,1,J) = C(J)
250 CONTINUE
C C C C C
.....
C C C C C
C PROCESS OF FINDING SYMMETRIC MODES IS COMPLETE: NOW CONSIDER
C ANTISYMMETRIC MODES: FIND THE ROOTS OF DET(ASY) = 0 PRECISELY
C
300 CONTINUE
IF(NR.GT.0)GO TO 321
WRITE(6,123)
IF(IND-NMAX)6,1,1
321 K = NR
IF(NR.LE.8)GO TO 351
K = K - 8
WRITE(6,128)K
K = B
351 IROOT(JDEX,4) = K
C
DO 350 I = 1,NR
M = INDEX(I,2)
C

```

```

C      CALL RTT(TABLE,ACC,PN,NOP,SC,3,M,ID,IDIAG)
C      R(      RETURNS A FINAL MATRIX, ASY, A SET OF COEFFICIENTS, d, ND
C      THE ROOT, P2. SEE THE SUBROUTINE FOR A DESCRIPTION OF PARAMETERS.
C
C      PRINT THE FINAL MATRIX, IF DESIRED: STORE ROOT: CALCULATE A FINAL VALUE
C      FOR THE DETERMINANT. ALSO PRINT THE COEFFICIENTS.
C
C      IF(1.LE.8)ROOTS(JDEX,I) = P2
C      IF(IDIAG.LT.1)GO TO 353
C      XXX = 3MAS,
C      WRITE(6,114)
C      DO 352 J = 1,NOT
C      352 WRITE(6,118)XXX,(ASY(J,K),K=1,NOT)
C      353 WRITE(6,114)
C      CALL MATINV(ASY,NOT,C,DET,0)
C      WRITE(6,125)XXX,JDEX,I,P2,PN,ID,DET
C      IF(NOT.GT.1)WRITE(6,122)(C(K),K=1,NOT)
C      WRITE(6,114)
C      IF(DET.EQ.0.0)WRITE(6,113)
C      IF(1.GT.8)GO TO 350
C      DO 349 J = 1,NOT
C      349 CFA(JDEX,I,J) = C(J)
C      350 CONTINUE
C
C      .....
C      CALL SECOND(T1)
C      T1 = T1 - TO
C      WRITE(6,126)T1
C      IF(ND-NMAX)5,1,1
C
C      RETURN TO EITHER PROCESS ANOTHER VALUE OF D FROM THE LAST DATA CARD,
C      OR TO READ A NEW CARD.
C
C      .....
C
C      AFTER ALL CASES (20 OR LESS) HAVE BEEN COMPLETED, PRINT A TABLE
C      OF RESULTS GIVING THE ROOTS AND THE COEFFICIENT RATIOS FOR EACH CASE.
C
C      SYMMETRIC ROOTS:
C
C      999 WRITE(6,123)
C      WRITE(6,123)
C      WRITE(6,123)
C      XXX = 4M
C      WRITE(6,116)XXX
C      DO 900 I = 1,JDEX
C      MAX = IROOT(I,1) + 6
C      IF(IDIAG.LE.1)WRITE(6,117)I,ROOT(I,1),(IROOT(I,M),M=2,3),
C      1(ROOT(I,J),J=2,MAX)
C      900 IF(IDIAG.GE.2)WRITE(6,127)I,ROOT(I,1),(IROOT(I,M),M=2,3),
C      1(ROOT(I,J),J=2,MAX)
C
C      IF(NOT.EQ.1)GO TO 911
C      IF(IDIAG.EQ.0)GO TO 911
C      WRITE(6,111)
C      WRITE(6,119)
C      DO 910 I = 1,JDEX
C      MAX = IROOT(I,1)
C      IF(MAX.EQ.0)GO TO 905
C      WRITE(6,117)I,ROOT(I,1),(IROOT(I,M),M=2,3),(ROOT(I,J),J=2,6),
C      1(CFS(I,K),K=1,MAX)
C      DO 902 K = 2,NOT

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```

902 WRITE(6,120)(CFS(I,M,K),(I,M,K),M=1,MAX)
DO 704 K = 1,NOT
C 03 M = 1,MAX
903 C(M) = CFS(I,M,K)*(-1.0)**(K+1)
904 WRITE(6,120)(C(M),M=1,MAX)
GO TO 910
905 WRITE(6,117)I,ROOT(I,1),(IROOT(I,M),M=2,3),(ROOT(I,U),U=2,6)
910 CONTINUE
C
C ANTISYMMETRIC ROOTS:
C
911 WRITE(6,111)
XXX = 4HANTI
WRITE(6,116)XXX
DO 920 I = 1,JDEX
MAX = IROOT(I,4)
IF(MAX.EQ.0)GO TO 915
IF(IDIAG.LE.1)WRITE(6,117)I,ROOT(I,1),(IROOT(I,M),M=2,3),
1(ROOT(I,U),U=2,6),(ROOTS(I,K),K=1,MAX)
IF(IDIAG.GE.2)WRITE(6,127)I,ROOT(I,1),(IROOT(I,M),M=2,3),
1(ROOT(I,U),U=2,6),(ROOTS(I,K),K=1,MAX)
GO TO 920
915 WRITE(6,117)I,ROOT(I,1),(IROOT(I,M),M=2,3),(ROOT(I,U),U=2,6)
920 CONTINUE
IF(NOT.EQ.1)GO TO 2
IF(IDIAG.EQ.0)GO TO 2
WRITE(6,111)
WRITE(6,119)
DO 930 I = 1,JDEX
MAX = IROOT(I,4)
IF(MAX.EQ.0)GO TO 925
WRITE(6,117)I,ROOT(I,1),(IROOT(I,M),M=2,3),(ROOT(I,U),U=2,6),
1(CFAL(M,1),M=1,MAX)
DO 922 K = 2,NOT
922 WRITE(6,120)(CFA(I,M,K),M=1,MAX)
DO 924 K = 1,NOT
DO 923 M = 1,MAX
923 C(M) = CFA(I,M,K)*(-1.0)**(K)
924 WRITE(6,120)(C(M),M=1,MAX)
GO TO 930
925 WRITE(6,117)I,ROOT(I,1),(IROOT(I,M),M=2,3),(ROOT(I,U),U=2,6)
930 CONTINUE
GO TO 2
END
SUBROUTINE CODE(N,R,NL,RL,SC,LSC,LST)
C
C REAL N,NL
C INTEGER SC
C
C FIND LST: CODE FOR DECIDING WHEN TO RECALCULATE INDICES (2).
C PERIMETER (3). OR BOTH (4).
C
C LST = 1
C IF(R.EQ.RL)GO TO 71
C
C FIND LST FOR CASES WITH R.NE.RL
C
C LST = 3
C IF(R.NE.1.0.AND.RL.NE.1.0)GO TO 73
C GO TO 72
C
C 71 IF(N.EQ.NL)GO TO 73
C
C FIND LST FOR CASES WITH R.EQ.RL
C
C LST = 3

```



```

C      MUJ _R'S METHOD TO GET THE ROOT PRECISELY
C
C      240 CALL MULLER(X1,X2,G1,G2,ACC,PN,NOP,SC,JJ,JD,IDIAG,IER)
C          IF(IER.GT.0)WRITE(6,701)IER
C
C      248 ID = ID + K
C          M = 1
C          IF(NOT.EQ.1)GO TO 280
C
C      CALCULATE THE COEFFICIENTS: SUBROUTINE LOAD COPIES (NOT-1) OF THE NOT
C      ROWS OF SYM OR ASY INTO ARRAY SINV AND VECTOR C. SUBROUTINE MATINV
C      SOLVES THE (NOT-1)X(NOT-1) SYSTEM FOR THE COEFFICIENTS AND RETURNS THEM
C      IN C
C
C      IF(JJ.EQ.3)GO TO 260
C      CALL LOAD(SYM,SINV,C,NOT,M)
C      GO TO 266
C
C      260 CONTINUE
C      CALL LOAD(ASY,SINV,C,NOT,M)
C      266 LST = NOT - 1
C      CALL MATINV(SINV,LST,C,DN,1)
C
C      STACK THE COEFFICIENTS IN THE PROPER ORDER IN VECTOR C. INCLUDING 1.000
C
C      IF(M.EQ.NOT)GO TO 280
C      DO 270 J = M,LST
C          K = NOT + M - J
C      270 C(K) = C(M-1)
C      280 C(M) = 1.0
C      RETURN
C      END
C
C      SUBROUTINE MULLER(X0,X2,F0,F2,ACC,PN,NOP,SC,JJ,JD,IDIAG,IER)
C
C      MULLER'S ALGORITHM FOR THE SOLUTION OF DET(RLS) = 0
C
C      INVERSE PARABOLIC INTERPOLATION USING POINTS
C      (X0,F0), (X1,F1), AND (X2,F2)
C      TO FIND THE NEW POINT (X,F), WHERE X0 < X1 < X2 AND F0*F2 < 0
C      ALWAYS HOLD. SIGN IS THE SIGN OF F0, AND DOES NOT CHANGE FROM ITS
C      INITIAL VALUE. THE TWO ROOTS OF THE QUADRATIC NECESSARILY LIE
C      BETWEEN X0 AND X1, AND BEYOND X0 OR X2 IF (A) SIGN*F1 < 0
C      BETWEEN X1 AND X2, AND BEYOND X0 OR X2 IF (B) SIGN*F1 > 0
C      (THAT IS, FOR EXAMPLE, IN CASE (A) THERE IS NO ROOT BETWEEN X1 AND X2.)
C      CHOOSE THE ROOT INSIDE THE INTERVAL.
C      X1 IS THE LARGEST X FOR WHICH F*SIGN > 0
C      X0 IS THE SMALLEST X FOR WHICH F*SIGN < 0
C
C      INTEGER SC
C      DIMENSION SYM(10,10),ASY(10,10),C(10)
C      COMMON/A/B,P2,P1,D,NOT,NT2
C      COMMON/E/SYM,ASY,C
C
C      ID = 0
C      IER = 0
C      PN = X2 - X0
C      IF(X2.LE.X0)GO TO 290
C      W = ALOG(PN/ACC)/.693147
C      LST = W + 1.5
C
C      LST GIVES THE NUMBER OF ITERATIONS WHICH A HALF-INTERVAL SEARCH
C      WOULD REQUIRE FOR THESE INITIAL VALUES AND DESIRED ACCURACY. IF
C      MULLER'S ALGORITHM EXCEEDS THIS, EXIT WITH ERROR CONDITION 3.
C
C

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700 FQ AT(0,MULLR:0.13,3F13,10.3E11,4,F13,10.2E11,4)
701 FQ AT(0,MULLR:0.13,72X,F13,10.E11,4)
C
C ASSUME THAT THE INITIAL VALUES ARE SUCH THAT X0 < X2
C IF NOT, RETURN (IER = 4)
C
C XL = X0
C XU = X2
C SIGN = F0/ABS(F0)
C
C SPLIT THE INTERVAL AND EVALUATE THE DETERMINANT AT THE MIDPOINT
C
40 X = (XL + XU)/2.0
P2 = X
CALL MATR(NOP,SC,1)
IF(JJ.EQ.2)CALL MATINV(SYM,NOT,C,F,0)
IF(JJ.EQ.3)CALL MATINV(ASY,NOT,C,F,0)
ID = ID + 1
IF(F*SIGN)50,200,60
50 XU = X
GO TO 100
60 XL = X
C
C EVALUATE THE NEW X FROM THE OLD X0, X1, AND X2
C
100 X1 = X
F1 = F
DD01 = (F1 - F0)/(X1 - X0)
DD12 = (F2 - F1)/(X2 - X1)
DD012 = (DD12 - DD01)/(X2 - X0)
W = DD01 + (X1 - X0)*DD012
U = DD012/W
GL = F1/W
U = 1.0 - 4.0*U*GL
IF(U.LT.0.0)GO TO 260
U = 2.0*GL/(1.0 + SORT(U))
X = X1 - U
IF(X.GT.X0.AND.X.LT.X2)GO TO 145
U = 1.0 - 4.0*GL*DD012/W
U = 2.0*GL/(1.0 - SORT(U))
X = X1 - U
145 CONTINUE
IF(X.GT.1.0.OR.X.LT.0.0)GO TO 270
P2 = X
CALL MATR(NOP,SC,1)
ID = ID + 1
PN = ABS(XU - XL)
IF(PN.LT.ACC)GO TO 210
IF(ID.GT.LST)GO TO 280
IF(JJ.EQ.2)CALL MATINV(SYM,NOT,C,F,0)
IF(JJ.EQ.3)CALL MATINV(ASY,NOT,C,F,0)
C
C IF(IDIAG.GE.3)WRITE(6,700)ID,X0,X1,X2,F0,F1,F2,X,F,PN
C
C RESET THE UPPER OR LOWER BOUND ON THE ROOT
C
IF(F*SIGN)150,200,155
150 XU = X
GO TO 160
155 XL = X
C
C CHANGE X0 OR X2 TO THE PREVIOUS X1; THE NEW X1 IS X
C
160 IF(F1*SIGN)165,200,170
165 X2 = X1

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```

F2 = F1
GO 3 175
170 AC = X1
F0 = F1
175 GL = ABS(U)
IF(((F-F1).LT.0.0)GO TO 100
IF(GL.GT.ACC)GO TO 100
C
C IF POINTS ACCUMULATE CLOSELY ON ONE SIDE OF THE TRUE ROOT, A SINGLE
C SMALL JUMP MAY SPAN THE ROOT, THEREFORE CHANGE X BY A LITTLE LESS
C THAN ACC AND EVALUATE THE DETERMINANT. IF THIS FAILS (THAT IS, IF THE
C FUNCTION HAS THE SAME SIGN AGAIN) SPLIT THE INTERVAL IN HALF AND
C CONTINUE.
C
C LINEAR EXTRAPOLATION PAST THE ANTICIPATED ZERO
C
W = 1.0E-12
IF(X.NE.X1)GO TO 180
XT = W
IF((F-SIGN).GT.0.0)XT = -W
GO TO 185
180 XT = 2.0*F*(X1 - X)/(F1 - F)
GL = ABS(XT)
IF(GL.GT.ACC)GO TO 100
IF(GL.GT.W)GO TO 185
XT = W*XT/GL
185 XT = X - XT
P2 = XT
CALL MATR(NOP,SC.1)
IF(JJ.EQ.2)CALL MATINV(SW,NOT.C,FT,0)
IF(JJ.EQ.3)CALL MATINV(ASY,NOT.C,FT,0)
ID = ID + 1
IF(IDIAG.GE.3)WRITE(6,701)ID,XT,FT
190 IF(FT.F)190,200,40
192 XU = XT
X2 = XT
F2 = FT
IF(X-X1)100,205,100
193 XL = XT
X0 = XT
F0 = FT
IF(X-X1)100,205,100
C
C IF THE DETERMINANT IS EXACTLY ZERO, THE MATRIX (WHICH MAY HAVE BEEN
C ALTERED IN MATINV) IS RECOMPUTED.
C
200 PN = 0.0
GO TO 209
C
C IF X = X1, TAKE THIS AS THE ROOT
C
205 P2 = X
PN = ABS(XU - XL)
C
C 209 CALL MATR(NOP,SC.1)
ID = ID + 1
C
C NORMAL RETURN: PN,LT,ACC
C
210 IF(IDIAG.GE.3)WRITE(6,700)ID,X0,X1,X2,F0,F1,F2,X
RETURN
C
C ERROR RETURN 1: COMPLEX ROOT

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C 260 IF = 1
C 265 GO TO 299
C 270 ERROR RETURN 2: PROCEDURE IS OSCILLATING
C 275 IER = 2
C 280 GO TO 299
C 285 ERROR RETURN 3: MORE ITERATIONS THAN HALF-INTERVAL SEARCH WOULD REQUIRE
C 290 IER = 3
C 295 GO TO 210
C 300 ERROR RETURN 4: X0 > X2 INITIALLY
C 305 IER = 4
C 310 ID = ID + 1
C 315 CALL MATR(NQP,SC,1)
C 320 RETURN
C 325 END
C 330 SUBROUTINE HINTS(X1,X2,G1,G2,ACC,PN,NOP,SC,JJ,ID,IDIAG,IER)
C 335 SUBROUTINE HINTS PERFORMS A HALF-INTERVAL SEARCH TO LOCATE THE ROOTS OF
C 340 THE EQUATION DET(RLS) = 0 OR DET(TLS) = 0 TO WITHIN A DESIRED ACCURACY
C 345 ACC. ID IS THE NUMBER OF BISECTIONS, AND PN IS THE ACTUAL MAXIMUM ERROR.
C 350
C 355 INTEGER SC
C 360 DIMENSION SYM(10,10),ASY(10,10),C(10)
C 365 COMMON/A,B,P2,P1,D,NOT,NT2
C 370 COMMON/E/SYM,ASY,C
C 375
C 380 FORMAT(= HINTS:,,I3,I3X,2F13.10,11X,2E11.4,F13.10,2E11.4)
C 385 ID = 0
C 390 IER = 0
C 395 PN = ABS(X2 - X1)
C 400
C 405 IF(PN.LT.ACC)RETURN
C 410 ID = ID + 1
C 415 IF(ID.GT.25)GO TO 230
C 420 P2 = (X1 + X2)/2.0
C 425 CALL MATR(NQP,SC,1)
C 430 IF(JJ.EQ.2)CALL MATINV(SYM,NOT,C,G,0)
C 435 IF(JJ.EQ.3)CALL MATINV(ASY,NOT,C,G,0)
C 440
C 445 IF(IDIAG.GE.3)WRITE(6,700)ID,X1,X2,G1,G2,P2,G,PN
C 450
C 455 IF(G*(G1/ABS(G1)))120,210,130
C 460 X2 = P2
C 465 G2 = G
C 470 GO TO 100
C 475 X1 = P2
C 480 G1 = G
C 485 GO TO 100
C 490
C 495 IF THE DETERMINANT IS EXACTLY ZERO, THE MATRIX (WHICH WILL HAVE BEEN
C 500 ALTERED IN MATINV) IS RECOMPUTED.
C 505
C 510 PN = 0.0
C 515 CALL MATR(NQP,SC,1)
C 520 ID = ID + 1
C 525 RETURN
C 530 ERROR RETURN: TOO MANY ITERATIONS

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230 IER = 6
      R(1,1) = 1.0
      SUBROUTINE LOAD(SS,SINV,C,NOT,ID)
      C
      C SUBROUTINE LOAD LOADS SS (NEE SYM OR ASY) INTO ARRAY SINV, ELIMINATING
      C THE ROW AND COLUMN CONTAINING THE DIAGONAL ELEMENT WHICH IS SMALLEST
      C IN MAGNITUDE. THE NEGATIVE OF THE COLUMN OF SS WHICH IS LEFT OUT IS
      C LOADED INTO ARRAY C, EXCEPT FOR THE ELEMENT IN THE ROW WHICH IS ELIM-
      C INATED. THUS, SINV IS (NOT-1) BY (NOT-1) IN DIMENSION, AND C IS A VEC-
      C TOR (NOT-1) IN LENGTH.
      C
      C NOTE THAT ONLY THE FIRST FOUR DIAGONAL ELEMENTS ARE SEARCHED FOR
      C THE SMALLEST VALUE.
      C
      DIMENSION SS(10,10),SINV(10,10),C(10)
      G = 1.0E+321
      ID = 1
      K = NOT
      IF(K.GT.4)K = 4
      DO 200 J = 1,K
      H = ABS(SS(J,J))
      IF(H.GT.G)GO TO 200
      G = H
      ID = J
200 CONTINUE
      C
      JJ = 0
      DO 300 I = 1,NOT
      IF(J.EQ.ID)GO TO 300
      JJ = JJ + 1
      C(JJ) = -SS(J,ID)
      KK = 0
      DO 299 K = 1,NOT
      IF(K.EQ.ID)GO TO 299
      KK = KK + 1
      SINV(JJ,KK) = SS(J,K)
299 CONTINUE
      C
      300 CONTINUE
      RETURN
      END
      SUBROUTINE REL(N,R,SC,IDIAG)
      C
      C SUBROUTINE REL IDENTIFIES THOSE MATRIX ELEMENTS R(I,L,S) (COSINE EX-
      C PANSION, SC = 1) OR T(L,S) (SINE EXPANSION, SC = 2) WHICH ARE NONZERO.
      C THE VALUES OF THE ROW INDICES, L, ARE STORED IN VECTOR ID(I), AND
      C THE VALUES OF THE INDICES, S, NBR(I) IN NUMBER, ARE THE ROWS OF
      C MATRIX IDMAT(I,J). FOR EXAMPLE, FOR A FIVE-TERM COSINE EXPANSION
      C WITH R = 1, THE NONZERO ELEMENTS ARE R(0,0), R(0,4)
      C R(1,1), R(1,3)
      C R(2,2)
      C R(3,1), R(3,3)
      C R(4,0), R(4,4)
      C
      AND ID(I) = 0 IDMAT(I,J) = 0, 4 1, 3 NBR(I) = 2
      1 3 1
      2 2 1
      3 1, 3 2
      4 0, 4 2
      C
      REAL N
      INTEGER SC
      DIMENSION STORE(200,42),IDMAT(20,10),ID(20),NBR(20)
      DIMENSION R1(10,10),R2(20,10)
      COMMON/A,B,P2,P1,D,NOT,NT2
      COMMON/B/STORE,IDMAT,ID,NBR
      COMMON/D/R1,R2

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C 180 W( 1, NOT
DO 200 I = 1, NT2
  I1 = I
  J2 = MOD(I1, 4) + 1
  NO = 0
GO TO(189, 179, 185, 179), J2
C 185 IDENTIFY ELEMENTS OF ODD ROWS (ALWAYS 1, 3, 5, ...)
C
179 J = -1
180 J = J + 2
  IF(J.GT.MAX)GO TO 200
  NO = NO + 1
  IDMAT(I, NO) = J
GO TO 180
C 185 IDENTIFY ELEMENTS OF EVEN ROWS
C
185 IF(R.NE.1.0)GO TO 189
C 185 ELEMENTS ARE 4, 8, 12, ...
C
  J = -2
GO TO 190
189 J = 0
C 185 ELEMENTS ARE 2, 4, 6, ... OR 2, 6, 10, ...
C
190 J = J + NDE
  IF(J.GT.MAX)GO TO 200
  NO = NO + 1
  IDMAT(I, NO) = J
GO TO 190
200 NBR(I) = NO
250 CONTINUE
C
  IF(IDIAG.LT.2)RETURN
  WRITE(6, 300)
  MAX = 1H
  JZ = 11 - SC
  IF(NT2.LT.JZ)JZ = NT2
DO 290 I = 1, JZ
  NO = NBR(I)
290 WRITE(6, 301)NO, ((MAX, ID(I), IDMAT(I, J)), J=1, NO)
C
  IF(JZ.EQ.NT2)RETURN
  JZ = JZ + 1
DO 295 I = JZ, NT2
  NO = NBR(I)
295 WRITE(6, 302)NO, ((MAX, ID(I), IDMAT(I, J)), J=1, NO)
C 300 FORMAT(/, ' TABLE OF INDICES (L.S) FOR REQUIRED ',
1. R(L.S)')
301 FORMAT(16, 3X, 10(A1, '( ', I1, ', ', I1, ', ', 3X))
302 FORMAT(16, 2X, 10(A1, '( ', I2, ', ', I1, ', ', 3X))
C
  RETURN
END
SUBROUTINE CHEBY(NOP, SC)
C
  INTEGER S, SC, SMAX
  DIMENSION STORE(200, 42), IDMAT(20, 10), ID(20), NBR(20)
  DIMENSION R1(10, 10), R2(20, 10)
  DIMENSION BUST(11), BIST(21), BKST(21)
  COMMON/A/B, P2, P1, D, NOT, NT2

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COMMON/B/STORE, IDMAT, ID, NBR
C ON/C/P,OMP,RH,DR,CL,SL,CS,SS,BJ,BJM,BK,BKM,BI,BIM,L,S (
C 3N/D/R1,R2
C
C CHEBYSHEV INTEGRATION TO PRODUCE R1(L,S) AND R2(L,S) (SC = 1)
C OR T1(L,S) AND T2(L,S) (SC = 2)
C
C PTS = NOP
C W = PI/(PTS*2.0)
C P = SORT(P2)
C OMP = SORT(1.0 - P2)
C LMAX = ID(NT2) + 1
C SMAX = ID(NT2) + 1
C ORD = 0.0
C NDR = 0
C
C ZERO ALL THE REQUIRED MATRIX ELEMENTS IN R1 AND R2
C
C DO 55 I = 1,NT2
C NO = NBR(I)
C IF(ND.EQ.0)GO TO 55
C DO 50 J = 1,NO
C JK = IDMAT(I,J) - SC + 2
C IF(I.GT.NOT)GO TO 50
C R1(I,JK) = 0.0
C 50 R2(I,JK) = 0.0
C 55 CONTINUE
C
C LOOP DO 101 DOES INTEGRATION USING NOP POINTS IN THE RANGE (0,PI/2)
C LOOP DO 101 I.... CORRESPONDS TO L
C LOOP DO 100 J.... CORRESPONDS TO S
C
C DO 101 JJJ = 1,NOP
C RH = STORE(JJJ,1)
C DR = STORE(JJJ,2)
C
C CALCULATE AND STORE THE J BESSEL FUNCTIONS NEEDED BY SUBROUTINE INT
C JO IS STORED AS BJST(1), J1 IS STORED AS BJST(2), ETC.
C
C ARG = PI*B*RH*SORT(1.0 - P2)
C CALL BESJ(ARG,ORD,SMAX,BJST,NZ)
C
C CALCULATE AND STORE THE I BESSEL FUNCTIONS NEEDED BY SUBROUTINE INT.
C IO IS STORED AS BIST(1), I1 IS STORED AS BIST(2), ETC.
C
C ARG = PI*B*RH*SORT(P2)
C CALL BESI(ARG,ORD,1,LMAX,BIST,NZ)
C
C CALCULATE AND STORE THE K BESSEL FUNCTIONS NEEDED IN SUBROUTINE INT.
C KO IS STORED AS BKST(1), K1 IS STORED AS BKST(2), ETC.
C
C CALL BESKN(ARG,NOR,1,LMAX,BKST,NZ)
C
C CALCULATE R1(L,S) AND R2(L,S) OR T1(L,S) AND T2(L,S)
C
C DO 101 I = 1,NT2
C NO = NBR(I)
C IF(ND.EQ.0)GO TO 101
C L = ID(I)
C LM = L - 1
C IF(L.EQ.0)LM = 1
C BK = BKST(L+1)
C BKM = BKST(LM+1)
C BI = BIST(L+1)
C BIM = BIST(LM+1)
C

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```

IF(L.GT.0)GO TO 70
C 1.0
S 0.0
GO TO 72
70 CL = STORE(JJJ,2*L+1)
SL = STORE(JJJ,2*L+2)
72 CONTINUE
C
DO 100 J = 1,NO
S = 10MAT(I,J)
JK = S - SC + 2
BJ = BJST(S+1)
BJM = -BJST(2)
IF(S.GT.0)GO TO 80
CS = 1.0
SS = 0.0
GO TO 82
80 CS = STORE(JJJ,2*S+1)
SS = STORE(JJJ,2*S+2)
BJM = BJST(S)
82 CONTINUE
C
CALL INT(SC,RA,RB)
R2(I,JK) = R2(I,JK) + RB*W
IF(I.GT.NOT)GO TO 100
R1(I,JK) = R1(I,JK) + RA*W
100 CONTINUE
101 CONTINUE
C
END
SUBROUTINE MATR(NOP,SC,ND)
END
C
INTEGER SC
DIMENSION R1(10,10),R2(20,10)
DIMENSION SYM(10,10),ASY(10,10),C(10)
DIMENSION ALPHA(10,20),BST(31)
COMMON/A/B,P2,P1,D,NOT,NT2
COMMON/O/R1,R2
COMMON/E/SYM,ASY,C
COMMON/F/ALPHA,BST
C
CALCULATE THE COUPLING COEFFICIENTS ALPHA (SC = 1) OR BETA (SC = 2):
STORE AS ALPHA(I,J)
ARG = PI*B*SQRT(P2)*D
C
CALCULATE THE K BESSEL FUNCTIONS OF ORDER 0 THROUGH MAX = NOT + NT2,
AND STORE THEM WITH INDICES 1 THROUGH MAX + 1
MAX = NOT + NT2 + 1
I = 0
CALL BESKN(ARG,I,1,MAX,BST,J)
C
IF(SC.GT.1)GO TO 150
C
THIS SECTION CALCULATES ALPHA(K,L)
WE NEED THE K-BESSEL FUNCTIONS OF ORDER K - L AND ORDER K + L. FOR THE
COSINE SOLUTION, 0 <= K,L <= (NOT-1). TAKING K = I - 1, L = J - 1,
WE GET K+L = I + J - 2, K - L = I - J. HOWEVER SINCE THESE START AT
ZERO, WE NEED MM = I + J - 1 AND NN = I - J + 1 AS RETRIEVAL INDICES
FOR THE ARRAY BST CONTAINING THE BESSEL FUNCTIONS.
DO 100 I = 1,NOT
EPS = 2.0
IF(1.EQ.1)EPS = 1.0

```

```

DO 100 J = 1,NT2
  I = J - 1
  IF(I.GE.J)NN = I - J + 1
  IF(J.GT.I)NN = J - I + 1
  ALPHA(I,J) = EPS*(BST(MM) + BST(MN))/PI
GO TO 200

C
C
C THIS SECTION CALCULATES BETA(K,L).
C WE NEED THE K-BESSEL FUNCTIONS OF ORDER K - L AND K + L. FOR THE SINE
C EXPANSION, 1 <= K,L <= NOT. TAKING K = I AND L = J SO THAT K + L
C = I + J AND K - L = I - J, WE NEED MM = I + J + 1 AND MN = I - J + 1
C AS RETRIEVAL INDICES FOR THE ARRAY BST CONTAINING THE BESSEL FUNCTIONS.
C
C
C NOTE THAT EVEN THOUGH WE CALCULATE BETA(K,L) (SINE SOLUTION), WE STORE
C THE RESULTS IN ARRAY ALPHA.
C
150 CONTINUE
DO 170 I = 1,NOT
  DO 170 J = 1,NT2
    MM = I + J + 1
    IF(I.GE.J)NN = I - J + 1
    IF(J.GT.I)NN = J - I + 1
    ALPHA(I,J) = 2.0*(BST(MM) - BST(MN))/PI
170 CONTINUE
IF(MD.GT.1)GO TO 280

C
C CALL SUBROUTINE CHEBY TO DO THE LINE INTEGRALS TO FIND R1 AND R2
C
C CALL CHEBY(NOP,SC)
C
C CALCULATE THE ELEMENTS OF THE MATRIX FROM THE ELEMENTS OF R1 AND
C SUMS OF ELEMENTS OF R2.
C
280 CONTINUE
DO 290 I = 1,NOT
  DO 290 J = 1,NT2
    SYM(I,J) = R1(I,J)
    ASY(I,J) = R1(I,J)
    SIGN = 1.0
    IF(SC.EQ.2)SIGN = -1.0
    DO 300 I = 1,NOT
      DO 300 J = 1,NT2
        SUM = 0.0
        DO 295 MM = 1,NT2
          SUM = SUM + ALPHA(I,MM)*R2(MM,J)
        SYM(I,J) = SYM(I,J) + SUM*SIGN
        ASY(I,J) = ASY(I,J) - SUM*SIGN
      RETURN
    END
  SUBROUTINE INT(SC,R1,R2)
C
C EVALUATE THE INTEGRANDS OF THE LINE INTEGRALS WHICH BECOME THE
C MATRIX ELEMENTS R1(L,S) AND R2(L,S)
C
C
C INTEGER S,SC
C COMMON/A,B,P2,PI,D,NOT,NT2
C COMMON/C/P,OMP,RH,DR,CL,SL,CS,SS,BJ,BJM,BK,BKM,BI,BIM,L,S
C
C XS = S
C XL = L
C EPS = 1.0
C IF(L.GT.0)EPS = 2.0
C TERM = BK*BJM*OMP + P*BJ*BKM
C TERM = 2.0*B*RH*TERM
C TERM = BI*BJM*OMP - P*BJ*BIM

```

```

110  = PI*BR*H*HERM
11  .EQ.S)GO TO 90
DD = L - S
TERM = TERM + 2.0*DD*BK*BJ/PI
HERM = HERM + DD*BI*BJ
90 F = EPS*TERM
H = EPS*HERM
C
C NOTE THAT I=-(L+1) IS NOT INCLUDED IN F
C NOTE THAT I=0L IS NOT INCLUDED IN H
C
E = 0.0
G = 0.0
IF(L.EQ.S)GO TO 100
IF(L.EQ.S)GO TO 100
G = 2.0*EPS*BK*BJ*DR/(RH*PI)
H = EPS*BI*BJ*DR/RH
C
C NOTE THAT I=-(L+1) IS NOT INCLUDED IN E
C NOTE THAT I=0L IS NOT INCLUDED IN G
C
100 CONTINUE
C
IF(SC.EQ.2)GO TO 110
TERM = XS*CL*SS - XL*SL*CS
R1 = 4.0*(P*CL*CS + E*TERM)
R2 = 4.0*(M*CL*CS + G*TERM)
RETURN
C
110 TERM = XS*SL*CS - XL*SS*CL
R1 = 4.0*(P*SL*SS - E*TERM)
R2 = 4.0*(M*SL*SS - G*TERM)
RETURN
END
SUBROUTINE MATINV(A,N,B,DETER,M)
C
DIMENSION A(10,10),B(10,1),INDEX(10,3)
10 DETER = 1.0
15 DO 20 J = 1,N
20 INDEX(J,3) = 0
30 DO 550 I = 1,N
C
C SEARCH FOR PIVOT ELEMENT
C
40 AMAX = 0.
45 DO 105 J = 1,N
IF (INDEX(J,3)-1) 60,105,60
60 DO 100 K = 1,N
IF (INDEX(K,3)-1) 80,100,740
80 IF(AMAX-ABS(A(J,K))) 85,100,100
85 IROW = J
90 ICOL = K
AMAX = ABS(A(J,K))
100 CONTINUE
105 CONTINUE
IF(AMAX.EQ.0.0)GO TO 750
INDEX(ICOL,3) = INDEX(ICOL,3) + 1
260 INDEX(I,1) = IROW
270 INDEX(I,2) = ICOL
C
C INTERCHANGE ROWS TO PUT PIVOT ELEM.IN DIAG.
C
130 IF(IROW=ICOL) 140,310,140
140 DETER = -DETER
150 DO 200 L = 1,N
160 SWAP = A(IROW,L)
170 A(IROW,L) = A(ICOL,L)

```

```

200 A(I,COLUMN,L) = SWAP
   I( ) 310,310,210
210 DO 350 L = 1,M
220 SWAP = B(IRON,L)
230 B(IRON,L) = B(ICOLUMN,L)
250 B(ICOLUMN,L) = SWAP
C
C   DIVIDE PIVOT ROW BY PIVOT ELEMENT
C
310 PIVOT = A(ICOLUMN,ICOLUMN)
   DETER = DETER-PIVOT
330 A(ICOLUMN,ICOLUMN) = 1.0
340 DO 350 L = 1,N
350 A(ICOLUMN,L) = A(ICOLUMN,L)/PIVOT
355 IF (M) 360,360,360
360 DO 370 L = 1,M
370 B(ICOLUMN,L) = B(ICOLUMN,L)/PIVOT
C
C   REDUCE NON PIVOTS ROWS
C
380 DO 550 LI = 1,N
390 IF (LI-ICOLUMN) 400,550,400
400 I = A(LI,ICOLUMN)
420 A(LI,ICOLUMN) = 0.
430 DO 450 L = 1,N
450 A(LI,L) = A(LI,L)-A(ICOLUMN,L)*I
455 IF (M) 550,550,460
460 DO 500 L = 1,M
500 B(LI,L) = B(LI,L)-B(ICOLUMN,L)*I
550 CONTINUE
C
C   INTERCHANGE COLUMNS
C
600 DO 710 I = 1,N
610 L = N + 1 - I
620 IF (INDEX(L,1)-INDEX(L,2)) 630,710,630
630 JROW = INDEX(L,1)
640 JCOLUMN = INDEX(L,2)
650 DO 705 K = 1,N
660 SWAP = A(K,JROW)
670 A(K,JROW) = A(K,JCOLUMN)
700 A(K,JCOLUMN) = SWAP
705 CONTINUE
710 CONTINUE
740 RETURN
C
750 DETER = 0.0
   RETURN
   END
C
C   SUBROUTINE PERIM(N,R,PHI,RHO,DRDP)
C
C   EVALUATE RHO(PHI) AND D(RHO)/D(PHI) ON THE PERIMETER.
C
C
C   REAL N
C
C   IF (R.NE.1.0.OR.N.NE.1.0)GO TO 100
   RHO = 1.0
   DRDP = 0.0
   RETURN
C
C   100 CONTINUE
   CP = COS(PHI)
   SP = SIN(PHI)
   PN = 2.0*N

```

DD = (CP/R)\*\*PN + SP\*\*PN  
 PN = 1.0/PN  
 R = 1.0/(DD\*\*PN)  
 PN = 2.0\*N - 2.0  
 AA = SP\*\*PN - ((CP/R)\*\*PN)/(R\*R)  
 ORDP = -RHO\*SP\*CP\*AA/DD  
 RETURN  
 END

C

**Program Listing**

**- TWOGS-**

**(Reference: Vol.I,, Section II.3.3)**



```

108 F( AT(1, NP2 = NUMBER OF INITIAL TABLE POINTS = 12) )
109 F( AT(9X, CROSS-SECTION = 12, IS = A, A10)
110 FORMAT(5X, PMAX = F6.4, 10X, PMIN = F6.4, /)
111 FORMAT(//)
112 FORMAT(18X, CASE = 13, // 17X, D = F7.3, // 17X, N1 = F7.3, //
113 17X, R1 = F7.3, // 17X, N2 = F7.3, // 17X, R2 = F7.3, // 17X,
114 2*52 = F7.3, // 17X, B = F7.3, /)
115 FORMAT(1X)
116 FORMAT(5X, EXEC TIME FOR TABLE = F7.3, SEC)
117 FORMAT(1X, SUMMARY OF TAG RESULTS: //)
118 16X, SHAPE = 19X, 52, 6X, N1, 6X, N2, 6X, D, 7X, B, 7X, ROOTS ... )
119 FORMAT(13, 1X, R9, 12, 13, 2X, A6, 1, 000, F8.3, 2F7.3, F8.3, 8F9.5)
120 FORMAT(14, A3, // 10E13.5, // 6X, 10E13.5, // 7X, 10E13.5, // 8X, 9E13.5)
121 19X, SHAPE = 19X, 52, 6X, N1, 6X, N2, 6X, D, 7X, B, 7X, ROOTS ... )
122 COEFFICIENTS
123 120 FORMAT(63X, 8F9.5)
124 121 FORMAT(15, 15F5.2)
125 122 FORMAT(15, 15F5.2)
126 113X, 81E13.5, // 1X)
127 123 FORMAT(16, ROOT(S) IDENTIFIED FOR CASE #, 13, /)
128 124 FORMAT(1X, A3, // CASE #, 13, // ROOT #, 12, // P2 = F10.8, // -)
129 125 169.3, // 13, NEW FN EVALS: DET = E15.8)
130 126 FORMAT(1X, EXEC TIME THIS CASE = F7.3, SEC, /)
131 127 FORMAT(13, 1X, R9, 12, 13, 2X, A6, 1, 000, F8.3, 2F7.3, F8.3, 7F10.8,
132 1/ 64X, F10.8)
133 128 FORMAT(1X, $$$$$$ MORE THAN 8 ROOTS, LAST, 13, NOT LISTED IN,
134 1 SUMMARY, /)
135 129 FORMAT(1X, N12 = MAX VAL OF INDEX L IN R2(L, S) = 13)
136 130 FORMAT(1X)
137 131 FORMAT(1X, $$$$$$ ERROR --- NOP = 1 BUT CROSS-SECTION IS NOT A,
138 1 CIRCLE $$$$$$, /)
139 132 FORMAT(25X, D = F5.2, 9X, 6(F5.2, 9X), F5.2)
140 133 FORMAT(8X, A6, EXPANSION IS USED HERE, /)
141 134 FORMAT(4X, R9, 13X, F7.3, F8.3, F7.3, 15X, 8F9.5)

```

# LIST OF VARIABLES WITH SINGLE ASSIGNED PURPOSES:

```

ACC ACCURACY TO WITHIN WHICH HALF-INTERVAL SEARCH IS PERFORMED
ALPHA ARRAY OF COUPLING COEFFICIENTS ALPHA (SC = 1) OR BETA (SC = 2)
B NORMALIZED FREQUENCY (READ IN)
BST ARRAY FOR STORING K BESSEL FUNCTIONS USED TO CALCULATE ALPHA
C VECTOR OF COEFFICIENTS IN THE FIELD EXPANSION (DESIRED RESULT)
CFF ARRAY FOR STORING VALUES OF C FOR DIFFERENT CASES
D DISTANCE SEPARATING CENTERS OF TWO GUIDES (UNITS OF SEMIMINOR AXIS)
DET DETERMINANT OF RLS
DRDP DIRHO/DIRPHI ON THE PERIMETER OF THE GUIDES
IDT ARRAY FOR STORING DIFFERENT VALUES OF D TO BE USED (READ IN)
ID # OF FUNCTION EVALUATIONS NECESSARY TO FIND A ROOT
ID1AG DIAGNOSTIC CODE TO PRINT EXTRA RESULTS (*) (READ IN)
ID1AT ARRAY FOR STORING VALUES OF S NEEDED WHEN R(L, S) OR T(L, S) IS COMPUTED
INDEX ARRAY FOR STORING TABLE INDICES NEAR WHICH ROOTS OCCUR
INROOT ARRAY FOR STORING INFORMATION USED IN PRINTING RESULTS AT THE END
JD ARRAY FOR STORING VALUES OF L NEEDED WHEN R(L, S) OR T(L, S) IS COMPUTED
JDEX COUNTER TO KEEP TRACK OF THE NUMBER OF SEPARATE CASES COMPLETED
LSC VALUE OF SC FROM PREVIOUS CASE (HELPS AVOID REPEATED CALCULATIONS)
N1 SUPERELLIPSE INDEX, GUIDE # 1 (REAL) (READ IN)
N2 SUPERELLIPSE INDEX, GUIDE # 2 (REAL) (READ IN)
NBR ARRAY GIVING NUMBER OF VALUES OF S (IN IDMAT) FOR EACH L (IN JD)
ND COUNTER IDENTIFYING WHICH ELEMENT OF DST IS TAKEN FOR D
NDH 2*NOT A DIMENSION OF MATRIX (RLS)
NG INDEX INDICATING WHICH GUIDE (1 OR 2) IS BEING CONSIDERED
NL1 VALUE OF N1 FROM PREVIOUS CASE (HELPS AVOID REPEATING CALCULATIONS)

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```

C C NL2 VALUE OF 1/2 FROM PREVIOUS CASE (HELPS AVOID REPEATING CALCULATIONS)
C C NM4 # OF NONZERO VALUES OF D IN DST (LE.6)
C C NOT # OF INTEGRATION POINTS (READ IN)
C C NP2 # OF TERMS IN EXPANSIONS (READ IN)
C C NR # OF VALUES OF P2 TAKEN TO CONSTRUCT TABLE (READ IN)
C C NT2 # OF ROOTS OF DET(RLS) LOCATED
C C PHI HIGHEST ALLOWED VALUE OF INDEX L IN R2(L,S) AND R3(L,S)
C C P1 ANGLE IN (RHO,PHI) COORDINATE SYSTEM
C C P2 LARGEST VALUE OF P2 USED IN TABLE (READ IN)
C C PMAX SMALLEST VALUE OF P2 USED IN TABLE (READ IN)
C C PMIN ARRAY FOR STORING ALPHANUMERIC PROGRAM DATA
C C PR PROPAGATION CONSTANT 1 DESIRED RESULT)
C C R1 ASPECT RATIO. GUIDE # 1 (READ IN)
C C R2 ASPECT RATIO. GUIDE # 2 (READ IN)
C C RHO RADIUS IN (RHO,PHI) COORDINATE SYSTEM
C C RHST ARRAY FOR STORING RHO AND DRDP (AS FNS OF PHI) FOR EACH PERIMETER
C C RL1 VALUE OF R1 FROM PREVIOUS CASE (HELPS AVOID REPEATING CALCULATIONS)
C C RL2 VALUE OF R2 FROM PREVIOUS CASE (HELPS AVOID REPEATING CALCULATIONS)
C C RLS ARRAY OF COEFFICIENTS WHOSE DETERMINANT IS TO BE FOUND
C C ROOM ARRAY FOR STORING INFORMATION USED IN PRINTING RESULTS AT THE END
C C ROOT ARRAY FOR STORING OUTPUT INFORMATION, INCLUDING VALUES OF P2
C C RS1 * ARRAY CONTAINING THE SUBMATRIX R1(L,S)
C C RS2 * ARRAY CONTAINING THE SUBMATRIX R2(L,S)
C C RS3 * ARRAY CONTAINING THE SUBMATRIX R3(L,S)
C C RS4 * ARRAY CONTAINING THE SUBMATRIX R4(L,S)
C C SC INDICATES SINE OR COSINE EXPANSION (INTEGER) (READ IN)
C C STORE ARRAY FOR STORING FUNCTIONS USED REPEATEDLY AND DEPENDING ONLY ON PHI
C C SZ RATIO OF Y-DIMENSION OF GUIDE # 2 TO THAT OF GUIDE # 1 (READ IN)
C C SZL VALUE OF SZ FROM PREVIOUS CASE (HELPS AVOID REPEATING CALCULATIONS)
C C T0 TABLE INITIAL TABLE OF VALUES OF ROOTS FOR DIFFERENT VALUES OF P2
C C T1 INITIAL CP TIME
C C T2 LATER CP TIME
C C * IDENTIFIES VARIABLES NOT USED IN MAIN PROGRAM, ONLY PRINTED
C C (IF IDIAG > 3)
C C *****
C C INITIALIZE CONSTANTS FOR ENTIRE JOB
C C *****
C C PI = 3.1415926535898
C C 2 CALL DATE(PN)
C C CALL TIME(W)
C C RL1 = 0.0
C C RL2 = 0.0
C C NL1 = 0.0
C C NL2 = 0.0
C C SZL = 0.0
C C LSC = 0
C C NOT = # OF TERMS TAKEN BEFORE TRUNCATION
C C NT2 = LARGEST VALUE TAKEN FOR L IN R2(L,S)
C C ACCURACY OF HALF-INTERVAL SEARCH IS 10**(-1)
C C NP2 = # OF VALUES OF P2 IN THE INITIAL TABLE
C C NP2 = # OF INTEGRATION POINTS IN THE RANGE (0,PI/2)
C C IDIAG = DIAGNOSIS CODE: 0 = NORMAL CONDITION (DEFAULT)
C C 1 = INITIAL TABLE, FINAL MATRIX PRINTED
C C 2 = SUMMARY PRINTED WITH MORE SIG FIGURES
C C 3 = DETAILS OF ROOT-LOCATION PRINTED
C C 4 = R1, R2, ALPHA, R2*ALPHA PRINTED
C C 5 = KN(D) PRINTED AFTER ROOTS FOUND
C C DEF: NOT
C C DEF: 1.0E-3
C C DEF: 10
C C DEF: 50
C C * IF THE CROSS-SECTION IS CIRCULAR, THE PHI-INTEGRATIONS CAN BE DONE IN
C C * CLOSED FORM. TO SAVE TIME ONE CAN SET NOP = 1, LEAVING ONLY ONE "INTE-
C C * GRATION POINT". IN THIS CASE, ADJUSTMENTS IN THE VALUES OF THE TRIG
C C * FUNCTIONS ARE MADE AT STATEMENT 87 SO THAT THE INTEGRAL WILL COME OUT

```



```

IF(R.LE.0.0)B = 1.0
  SC = 1
  C.GT.2)SC = 2
  IF(SZ.LE.0.0)SZ = 1.0
.....
COUNT THE NUMBER, NMAX, OF CASES (DIFFERENT D) ON THE LAST DATA CARD
ND = 0
D = R1 + R2*SZ
DO 5 I = 1,6
  IF(DST(I).EQ.0.0)GO TO 6
  IF(DST(I).LT.D)DST(I) = D
  NMAX = I
5 CONTINUE
6 ND = ND + 1
D = DST(ND)
IF(JDEX.GE.20)GO TO 999
JDEX = JDEX + 1
.....
PRINT INPUT DATA
WRITE(6,111)
WRITE(6,123)
WRITE(6,111)
WRITE(6,112).DEX,D.M1,R1,N2,R2,SZ,B
IF(PMAX.NE.1.0.OR.PMIN.NE.0.0)WRITE(6,110)PMAX,PMIN
.....
INITIALIZE STORAGE ARRAYS FOR THIS CASE
DO 70 I = 7,14
  DO 69 K = 1,NDM
    CFF(JDEX,I-6,K) = 0.0
70 ROOT(JDEX,I) = 0.0
    ROOT(JDEX,3) = N1
    ROOT(JDEX,4) = R1
    ROOM(JDEX,2) = SZ
    ROOM(JDEX,3) = N2
    ROOM(JDEX,4) = R2
    ROOT(JDEX,5) = D
    ROOT(JDEX,6) = B
    IROOT(JDEX,1) = 0
    IROOT(JDEX,2) = SC - 1
    IROOT(JDEX,3) = NOT + SC - 2
.....
IDENTIFY CROSS-SECTIONS FOR THIS CASE
NG = 1
R = R1
N = N1
J = SC + 5
89 I = 4
  IF(N.GT.1.0)GO TO 90
  IF(N-.5)94,95,96
94 I = 9
  GO TO 92
95 I = 8
  GO TO 92
96 IF(N.NE.1.0)GO TO 92

```

```

      I( 2 ..EQ.1.0)I = 1
90 IF(N.LT.30)GO TO 92
      I = 5
      IF(R.EQ.1.0)I = 3
92 WRITE(6,109)NG,PR(I)
      IF(NG.EQ.2)GO TO 98
      ROOT(JDEX,2) = PR(J)
      ROOT(JDEX,1) = PR(I)
      IF(NOP.EQ.1.AND.I.NE.1)WRITE(6,131)
      NG = 2
      R = R2
      N = N2
      GO TO 89
98 WRITE(6,133)PR(J)
      ROOM(JDEX,1) = PR(I)
      IF(NOP.EQ.1.AND.I.NE.1)WRITE(6,131)
      .....
      CALCULATE AND STORE COS(M*PHI) AND SIN(M*PHI) AS STORE(I,2*M-1)
      AND STORE(I,2*M), RESPECTIVELY.
      IF(JDEX.GT.1)GO TO 76
      PN = NOP
      W = PI/(PN*2.0)
      DO 74 I = 1,NOP
      PN = 2*(NOP - I) + 1
      PHI = W*PN/2.0
      DO 74 M = 1,NT2
      PN = M
      PN = PN*PHI
      STORE(I,2*M-1) = COS(PN)
      STORE(I,2*M) = SIN(PN)
74 IF(NOP.GT.1)GO TO 76
      IF NOP = 1, ASSUME TWO CIRCLES, SET TRIG FUNCTIONS EQUAL TO CONSTANTS
      CHOSEN SO THAT THE ANALYTICAL RESULTS FOR THE INTEGRALS ARE OBTAINED.
      DO 75 M = 1,NT2
      STORE(I,2*M-1) = SORT(.5)
      STORE(I,2*M) = SORT(.5)
75 DETERMINE THE INDEX PAIRS (L,S) OF ALL THE MATRIX ELEMENTS R(L,S) NEEDED
      BY CALLING SUBROUTINE REL
      76 NG = 1
      I = LSC
      CALL CODE(N1,R1,NL1,RL1,SC,LSC,LST)
      LSC = I
      IF(LST.EQ.1.OR.LST.EQ.3)GO TO 78
      CALL REL(N1,R1,SC,NG,IDIAG)
      78 NG = 2
      CALL CODE(N2,R2,NL2,RL2,SC,LSC,J)
      IF(J.EQ.1.OR.J.EQ.3)GO TO 80
      CALL REL(N2,R2,SC,NG,IDIAG)
      80 IF(SZ.EQ.SZL)GO TO 81
      SZL = SZ
      IF(J.GE.3)GO TO 81
      J = J + 2
      81 CONTINUE
      IF(LST.LT.3.AND.J.LT.3)GO TO 87
      PN = NOP
      W = PI/(PN*2.0)

```

```

C          CALCULATE AND STORE RHO1(PH1) AS RHST(1,1), I = 1,NOP
C          CALCULATE AND STORE RHO2(PH1) AS RHST(1,3), I = 1,NOP
C          CALCULATE AND STORE DIRHO1(D,PH1) AS RHST(1,2), I = 1,NOP
C          CALCULATE AND STORE DIRHO2(D,PH1) AS RHST(1,4), I = 1,NOP
C
C      DO 85 I = 1,NOP
C      PN = 2*(NOP - I) + 1
C      PHI = W*PN/2.0
C      CALL PERIN(N1,R1,PH1,RHO,DRDP)
C      RHST(1,1) = RHO
C      RHST(1,2) = DRDP
C      CALL PERIN(N2,R2,PH1,RHO,DRDP)
C      RHST(1,3) = RHO*SZ
C      85 RHST(1,4) = DRDP*SZ
C
C      .....
C      CONSTRUCT TABLE OF VALUES OF DET(RLS) VS P2, GIVEN B. TO SAVE TIME,
C      (SINCE RS1(L,S) - RS4(L,S) ARE INDEPENDENT OF D) THIS
C      IS DONE FOR EACH OF THE VALUES OF D READ INTO ARRAY DST WHEN THE
C      COUNTER, ND, IS EQUAL TO 1. THIS SECTION IS SKIPPED WHEN ND.GT.1.
C
C      87 LST = NP2 + 1
C      IF(ND.GT.1)GO TO 205
C      IF(IDIAG.GT.0)WRITE(6,106)
C      PN = NP2
C      W = (PMAX - PMIN)/PN
C      P2 = PMAX
C      IF(PMAX.EQ.1.0)P2 = 1.0 - W
C      IF(NMAX.GT.1.AND.IDIAG.GT.0)WRITE(6,132)(DST(I),I=1,NMAX)
C      NR = NMAX + 1
C      DO 200 M = 1,LST
C      TABLE(M,1) = P2
C      DO 190 I = 1,NMAX
C      D = DST(I)
C      CALL MATR(NOP,SC,I)
C      CALL MATINV(RLS,NOM,C,DET,0)
C      190 TABLE(M,I+1) = DET
C      IF(IDIAG.GT.0)WRITE(6,107)M,(TABLE(M,I),I=1,NR)
C      P2 = PMAX - W*PN
C      200 IF(P2.LT.W)P2 = W
C      IF(IDIAG.GT.0)WRITE(6,114)
C      D = DST(I)
C
C      CALL SECOND(T1)
C      T1 = T1 - TO
C      WRITE(6,115)T1
C      GO TO 209
C
C      WHEN ND.GT.1, RECOVER THE TABLE OF VALUES OF DET(RLS)
C      VS P2 CALCULATED EARLIER.
C
C      205 DO 206 M = 1,LST
C      206 TABLE(M,2) = TABLE(M,ND+1)
C
C      .....
C      MAKE TENTATIVE IDENTIFICATION OF ROOTS OF EQUATION DET(RLS) = 0
C
C      209 NR = 0
C      IF(TABLE(1,2).NE.0.0)GO TO 210
C      NR = 1
C      INDEX(NR) = 1001
C      210 DO 220 M = 2,LST
C      PN = ABS(TABLE(M,2))

```

```

      17,GT,0.0)GO TO 215
      N = NR + 1
      INDEX(NR) = M + 1000
      GO TO 220
215 PN = (TABLE(M,2)/PN)*TABLE(M-1,2)
      IF(PN,GE,0.0)GO TO 220
      NR = NR + 1
      INDEX(NR) = M - 1
220 IF(NR,GE,20)GO TO 221
221 WRITE(6,124)NR,JDEX
      IF(NR,GT,0)GO TO 231
      WRITE(6,123)
      IF(ND-NMAX)6,1,1
      .....
      C LOCATE THE ROOTS OF DET(RLS) = 0 PRECISELY.
      C LOOP DO 250 IS COMPLETED ONCE FOR EACH ROOT (I) IDENTIFIED ABOVE.
      C IF THERE ARE MORE THAN 8 ROOTS, MAKE NOTE OF IT. ONLY THE FIRST 8
      C ROOTS WILL BE LISTED IN THE SUMMARY AT THE END. (IF THERE ARE MORE
      C THAN 20 ROOTS, THE LAST ONES WILL HAVE BEEN MISSED COMPLETELY.)
      C
231 K = NR
      IF(NR,LE,8)GO TO 251
      K = K - 8
      WRITE(6,128)K
      K = 8
251 IROOT(JDEX,1) = K
      DO 250 J = 1,NR
      M = INDEX(I)
      C
      C CALL RTT(TTABLE,ACC,PN,NOP,SC,M,IDIAG)
      C RTT RETURNS A FINAL MATRIX, RLS, A SET OF COEFFICIENTS, C, AND
      C THE ROOT, P2. SEE THE SUBROUTINE FOR A DESCRIPTION OF PARAMETERS.
      C IF IDIAG > 3, PRINT VARIOUS ARRAYS AND VARIABLES FOUND DURING LOCATION
      C OF THE ROOTS (FOR PURPOSES OF CHECKING PROGRAM FUNCTIONS)
      C
      IF(IDIAG,LT,4)GO TO 265
      WRITE(6,114)
      XXX = 3M R1
      DO 242 J = 1,NOT
242 WRITE(6,118)XXX,(RS1(J,K),K=1,NOT)
      XXX = 3M R2
      DO 243 J = 1,NT2
243 WRITE(6,118)XXX,(RS2(J,K),K=1,NOT)
      WRITE(6,114)
      XXX = 3M R3
      DO 254 J = 1,NT2
254 WRITE(6,118)XXX,(RS3(J,K),K=1,NOT)
      WRITE(6,114)
      XXX = 3M R4
      DO 255 J = 1,NOT
255 WRITE(6,118)XXX,(RS4(J,K),K=1,NOT)
      WRITE(6,114)
      XXX = 3HALP
      DO 257 J = 1,NOT
257 WRITE(6,118)XXX,(ALPHA(J,K),K=1,NT2)
      WRITE(6,114)
      XXX = 3HRA2
      DO 260 J = 1,NOT
      DO 259 K = 1,NOT

```

```

LST = 0
258 MAX = 1,NT2
259 ALPHA(J,MAX)*RS2(MAX,K)*(-1.0)**(MAX*SC)
IF(DN.EQ.0.0)GO TO 258
LST = LST + 1
CFF(LST,8,20) = DN
258 CONTINUE
259 WRITE(6,118)XXX,(CFF(MAX,8,20),MAX=1,LST)
260 WRITE(6,114)
XXX = 3HRA3
DO 263 J = 1,NOT
DET = (-1.0)**(J*SC-1)
DO 262 K = 1,NOT
LST = 0
DO 261 MAX = 1,NT2
DN = ALPHA(J,MAX)*RS3(MAX,K)*DET
IF(DN.EQ.0.0)GO TO 261
LST = LST + 1
CFF(LST,8,20) = DN
261 CONTINUE
262 WRITE(6,118)XXX,(CFF(MAX,8,20),MAX=1,LST)
263 WRITE(6,114)
IF(IDIAG.LT.5)GO TO 265
XXX = 3HKN
J = NOT + NT2 + 1
WRITE(6,118)XXX,(BST(K),K=1,J)
WRITE(6,114)
C .....
C PRINT THE FINAL MATRIX, IF DESIRED: STORE ROOT: CALCULATE A FINAL VALUE
C FOR THE DETERMINANT. PRINT THE COEFFICIENTS, AND ALSO CONVERT THE
C COEFFICIENTS FOR THE FIELD EXPANSION IN THE 2ND GUIDE TO THE
C PROPER FORM: THAT IS, CHANGE THE SIGN OF THE ODD COEFFICIENTS.
C .....
265 IF(1.LE.8)ROOT(JDEX,I*6) = P2
IF(IDIAG.LT.1)GO TO 253
XXX = 3HRLS
WRITE(6,114)
DO 252 J = 1,NDM
252 WRITE(6,118)XXX,(RLS(J,K),K=1,NDM)
253 WRITE(6,114)
CALL MATINV(RLS,NDM,C,DET,0)
WRITE(6,125)XXX,JDEX,I,P2,PN,ID,DET
J = NOT + 3 - SC
DO 248 K = J,NDM,2
248 C(K) = -C(K)
WRITE(6,122)(C(K),K=1,NDM)
WRITE(6,114)
IF(DET.EQ.0.0)WRITE(6,113)
IF(1.GT.8)GO TO 250
DO 249 J = 1,NDM
249 CFF(JDEX,I,J) = C(J)
250 CONTINUE
C .....
C CALL SECOND(T1)
C T1 = T1 - TO
C WRITE(6,126)T1
C IF(ND-NMAX)6,1,1
C .....
C RETURN TO EITHER PROCESS ANOTHER VALUE OF D FROM THE LAST DATA CARD,
C OR TO READ A NEW CARD.
C .....

```

```
C      .[ .....  
C      .[ .....  
C  
C      AFTER ALL CASES (20 OR LESS) HAVE BEEN COMPLETED, PRINT A TABLE  
C OF RESULTS GIVING THE ROOTS AND THE COEFFICIENT RATIOS FOR EACH CASE.  
C  
C      999 WRITE(6,123)  
C        WRITE(6,123)  
C        WRITE(6,123)  
C        DO 900 I = 1,JDEX  
C          MAX = IROOT(I,1) + 6  
C          IF(IDIAG.LE.1)WRITE(6,117)I,ROOT(I,1),(IROOT(I,M),M=2,3),  
C            1(ROOT(I,J),J=2,MAX)  
C          IF(IDIAG.GE.2)WRITE(6,127)I,ROOT(I,1),(IROOT(I,M),M=2,3),  
C            1(ROOT(I,J),J=2,MAX)  
C      900 WRITE(6,134)(ROOM(I,M),M=1,4)  
C  
C      IF(IDIAG.EQ.0)GO TO 2  
C        WRITE(6,111)  
C        WRITE(6,119)  
C        DO 910 I = 1,JDEX  
C          MAX = IROOT(I,1)  
C          IF(MAX.EQ.0)GO TO 905  
C          WRITE(6,117)I,ROOT(I,1),(IROOT(I,M),M=2,3),(ROOT(I,J),J=2,6),  
C            1(CFF(I,K,1),K=1,MAX)  
C          WRITE(6,134)(ROOM(I,M),M=1,4),(CFF(I,J,2),J=1,MAX)  
C          LST = NDM - 2  
C          IF(LST.LE.2)GO TO 910  
C          DO 902 K = 3,NDM  
C            GO TO 910  
C      902 WRITE(6,120)(CFF(I,M,K),M=1,MAX)  
C      905 WRITE(6,117)I,ROOT(I,1),(IROOT(I,M),M=2,3),(ROOT(I,J),J=2,6)  
C      910 CONTINUE  
C  
C      GO TO 2  
C      END  
C      SUBROUTINE CODE(N,R,NL,RL,SC,LSC,LST)  
C  
C      REAL N,NL  
C      INTEGER SC  
C  
C      FIND LST: CODE FOR DECIDING WHEN TO RECALCULATE INDICES (2),  
C      PERIMETER (3), OR BOTH (4).  
C  
C      LST = 1  
C      IF(R.EQ.RL)/GO TO 71  
C  
C      FIND LST FOR CASES WITH R.NE.RL  
C  
C      LST = 3  
C      IF(R.NE.1.0.AND.RL.NE.1.0)GO TO 73  
C      GO TO 72  
C  
C      71 IF(N.EQ.NL)GO TO 73  
C  
C      FIND LST FOR CASES WITH R.EQ.RL  
C  
C      LST = 3  
C      IF(R.NE.1.0)GO TO 73  
C      IF(N.NE.1.0.AND.NL.NE.1.0)GO TO 73  
C      LST = 4  
C      GO TO 74  
C  
C      73 IF(SC.EQ.LSC)GO TO 74
```

```

C      Q      GE LST WHEN SC.NE.LSC
C      LST = LST + 1
74 LSC = SC
   RL = R
   NL = N
   RETURN
END
SUBROUTINE RTTY(TABLE,ACC,PN,NOP,SC,M,ID,IDIAG)
C
C SUBROUTINE RTTY PRECISELY LOCATES THE ROOT SPECIFIED BY M (USING
C ELEMENTS M AND M+1 IN THE ARRAYS TABLE(M,1) AND TABLE(M,2), AND THEN
C DETERMINES THE FIELD EXPANSION COEFFICIENTS C(J) CORRESPONDING TO
C THAT ROOT. ACC IS THE DESIRED ACCURACY OF THE ROOT, AND PN IS THE
C DIFFERENCE IN THE ACTUAL BOUNDS OBTAINED. ID IS THE NUMBER OF FUN-
C CTION EVALUATIONS NECESSARY. SC, IDIAG, AND NOP ARE AS IN MAIN. RLS, THE
C FINAL MATRIX AT THE ROOT P2, IS ALSO RETURNED. (C, RLS, AND P2
C ARE ALL IN COMMON BLOCKS.)
C
C INTEGER SC
C DIMENSION RLS(20,20),C(20)
C DIMENSION TABLE(51,7),SINV(20,20)
C COMMON/A,B,P2,P1,D,NOT,NT2
C COMMON/E/RLS,C
C
C 700 FORMAT(/,' SEARCH ID=7X,'X0=,11X,'X1=,11X,'X2=,10X,'F0=,9X,'F1=,
C 19X,'F2=,11X,'X=,10X,'F=,10X,'DX=')
C 701 FORMAT(/,' $$$$ ERROR IN HINTS OR MULLER, IER =,13,' $$$$$')
C
C IF(IDIAG.GE.3)WRITE(6,700)
C K = 0
C ID = 0
C IF(M.LT.1000)GO TO 230
C P2 = TABLE(M-1000,1)
C CALL MATR(NOP,SC,1)
C ID = 1
C GO TO 248
C
C 230 G1 = TABLE(M+1,2)
C G2 = TABLE(M,2)
C X1 = TABLE(M+1,1)
C X2 = TABLE(M,1)
C DN = X2 - X1
C
C IF G1 AND G2 ARE TWO OR MORE ORDERS OF MAGNITUDE DIFFERENT, USE
C A HALF-INTERVAL SEARCH TO NARROW THE RANGE X1 TO X2
C
C 235 NW = -G1/G2
C IF(NW.LT.100..AND.NW.GT..01)GO TO 240
C IF(DN.LT.ACC)GO TO 240
C DN = DN/3.0
C
C CALL HINTS(X1,X2,G1,G2,DN,PN,NOP,SC,ID,IDIAG,IER)
C IF(IER.GT.0)WRITE(6,701)IER
C
C K = K + ID
C IF(PN.EQ.0.0)GO TO 248
C GO TO 235
C
C MULLER'S METHOD TO GET THE ROOT PRECISELY
C
C 240 CALL MULLER(X1,X2,G1,G2,ACC,PN,NOP,SC,ID,IDIAG,IER)
C IF(IER.GT.0)WRITE(6,701)IER
C
C 248 ID = ID + K

```



```

C 40 X = (XL + XU)/2.0
P = X
CALL MATR(NOP,SC,1)
CALL MATINV(RLS,NDM,C,F,0)
ID = ID + 1
IF(F*SIGN)50,200,60
50 XU = X
60 XL = X
GO TO 100
C
C EVALUATE THE NEW X FROM THE OLD X0, X1, AND X2
C
100 X1 = X
F1 = F
DD01 = (F1 - F0)/(X1 - X0)
DD12 = (F2 - F1)/(X2 - X1)
DD012 = (DD12 - DD01)/(X2 - X0)
W = DD01 + (X1 - X0)*DD012
U = DD012/W
GL = F1/W
U = 1.0 - 4.0*U*GL
IF(U.LT.0.0)GO TO 260
U = 2.0*GL/(1.0 + SORT(U))
X = X1 - U
IF(X.GT.X0.AND.X.LT.X2)GO TO 145
U = 1.0 - 4.0*GL*DD012/W
U = 2.0*GL/(1.0 - SORT(U))
X = X1 - U
145 CONTINUE
IF(X.GT.1.0.OR.X.LT.0.0)GO TO 270
P2 = X
CALL MATR(NOP,SC,1)
ID = ID + 1
PN = ABS(XU - XL)
IF(PN.LT.ACC)GO TO 210
IF(ID.GT.LST)GO TO 280
CALL MATINV(RLS,NDM,C,F,0)
C IF(IDIAG.GE.3)WRITE(8,700)ID,X0,X1,X2,F0,F1,F2,X,F,PN
C
C RESET THE UPPER OR LOWER BOUND ON THE ROOT
C
150 XU = X
GO TO 160
155 XL = X
C
C CHANGE X0 OR X2 TO THE PREVIOUS X1; THE NEW X1 IS X
C
160 IF(F1*SIGN)150,200,155
165 X2 = X1
F2 = F1
GO TO 175
170 X0 = X1
F0 = F1
175 GL = ABS(U)
IF((F*F1).LT.0.0)GO TO 100
IF(GL.GT.ACC)GO TO 100
C
C IF POINTS ACCUMULATE CLOSELY ON ONE SIDE OF THE TRUE ROOT, A SINGLE
C SMALL JUMP MAY SPAN THE ROOT. THEREFORE CHANGE X BY A LITTLE LESS
C THAN ACC AND EVALUATE THE DETERMINANT. IF THIS FAILS (THAT IS, IF THE
C FUNCTION HAS THE SAME SIGN AGAIN) SPLIT THE INTERVAL IN HALF AND
C CONTINUE.

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```

C
C
C      LINEAR EXTRAPOLATION PAST THE ANTICIPATED ZERO
C
      W = 1.0E-12
      IF(X.NE.X1)GO TO 180
      XT = W
      IF((F*SIGN).GT.0.0)XT = -W
      GO TO 185
180  XT = 2.0*F*(X1 - X1)/(F1 - F)
      GL = ABS(XT)
      IF(GL.GT.ACC)GO TO 100
      IF(GL.GT.W)GO TO 185
      XT = W*XT/GL
185  XT = X - XT
      P2 = XT
      CALL MATR(NOP,SC,1)
      CALL MATINV(RLS,NOM,C,FT,0)
      ID = ID + 1
      IF(IDIAG.GE.3)WRITE(6,701)ID,XT,FT
190  IF(FT.F)190,200,40
192  XU = XT
      X2 = XT
      F2 = FT
      IF(X-X1)100,205,100
193  XL = XT
      X0 = XT
      F0 = FT
      IF(X-X1)100,205,100
C
C      IF THE DETERMINANT IS EXACTLY ZERO, THE MATRIX (WHICH MAY HAVE BEEN
C      ALTERED IN MATINV) IS RECOMPUTED.
C
200  PN = 0.0
      GO TO 209
C
C      IF X = X1, TAKE THIS AS THE ROOT
C
205  P2 = X
      PN = ABS(XU - XL)
C
C
209  CALL MATR(NOP,SC,1)
      ID = ID + 1
      NORMAL RETURN: PN,LT,ACC
C
210  IF(IDIAG.GE.3)WRITE(6,700)ID,X0,X1,X2,F0,F1,F2,X
      RETURN
C
C      ERROR RETURN 1: COMPLEX ROOT
C
260  IER = 1
      GO TO 299
C
C      ERROR RETURN 2: PROCEDURE IS OSCILLATING
C
270  IER = 2
      GO TO 299
C
C      ERROR RETURN 3: MORE ITERATIONS THAN HALF-INTERVAL SEARCH WOULD REQUIRE
C
280  IER = 3
      GO TO 210
C

```

```

C      ERROR RETURN 4: X0 > X2 INITIALLY
C      290 ILL = 4
C      299 ID = ID + 1
C      CALL MATR(NOP,SC,1)
C      RETURN
C      END
C      SUBROUTINE HINTS(X1,X2,G1,G2,ACC,PN,NOP,SC,ID,IDIAG,IER)
C      C
C      C      SUBROUTINE HINTS PERFORMS A HALF-INTERVAL SEARCH TO LOCATE THE ROOTS OF
C      C      THE EQUATION DET(RLS) = 0 OR DET(ITS) = 0 TO WITHIN A DESIRED ACCURACY
C      C      ACC. ID IS THE NUMBER OF BISECTIONS, AND PN IS THE ACTUAL MAXIMUM ERROR.
C      C
C      C      INTEGER SC
C      C      DIMENSION RLS(20,20),C(20)
C      C      COMMON/A,B,P2,P1,D,NOT,NT2
C      C      COMMON/E/RLS,C
C      C
C      C      700 FORMAT(= HINTS:0,13,13X,2F13.10,11X,2E11.4,F13.10,2E11.4)
C      C
C      C      ID = 0
C      C      IER = 0
C      C      NDM = 2*NOT
C      C      100 PN = ABS(X2 - X1)
C      C
C      C      IF(PN.LT.ACC)RETURN
C      C      ID = ID + 1
C      C      IF(ID.GT.25)GO TO 230
C      C      P2 = (X1 + X2)/2.0
C      C      CALL MATR(NOP,SC,1)
C      C      CALL MATINV(RLS,NDM,C,G,0)
C      C
C      C      IF(IDIAG.GE.3)WRITE(6,700)ID,X1,X2,G1,G2,P2,G,PN
C      C
C      C      IF(G*(G1/ABS(G1)))120,210,130
C      C      120 X2 = P2
C      C      G2 = G
C      C      GO TO 100
C      C      130 X1 = P2
C      C      G1 = G
C      C      GO TO 100
C      C
C      C      IF THE DETERMINANT IS EXACTLY ZERO, THE MATRIX (WHICH WILL HAVE BEEN
C      C      ALTERED IN MATINV) IS RECOMPUTED.
C      C
C      C      210 PN = 0.0
C      C      CALL MATR(NOP,SC,1)
C      C      ID = ID + 1
C      C      RETURN
C      C
C      C      ERROR RETURN: TOO MANY ITERATIONS
C      C
C      C      230 IER = 6
C      C      RETURN
C      C      END
C      C      SUBROUTINE LOAD(SS,SINV,C,NOT,ID)
C      C
C      C      SUBROUTINE LOAD LOADS SS (NEE RLS) INTO ARRAY SINV, ELIMINATING
C      C      THE ROW AND COLUMN CONTAINING THE DIAGONAL ELEMENT WHICH IS SMALLEST
C      C      IN MAGNITUDE. THE NEGATIVE OF THE COLUMN OF SS WHICH IS LEFT OUT IS
C      C      LOADED INTO ARRAY C, EXCEPT FOR THE ELEMENT IN THE ROW WHICH IS ELIM-
C      C      INATED, THUS, SINV IS (NOT-1) BY (NOT-1) IN DIMENSION, AND C IS A VEC-
C      C      TOR (NOT-1) IN LENGTH.
C      C
C      C      REMEMBER, NOT IS WHAT IS CALLED NDM IN TWOGS. ALSO, ONLY THE FIRST
C      C      FOUR ELEMENTS OF EACH HALF OF THE DIAGONAL ARE SEARCHED FOR THE

```

```

C C
C C      SMALLEST VALUE.
C C      DIMENSION SS(20,20),SINV(20,20),C(20)
C C      G = 1.0E+321
C C      ID = 1
C C      K = NOT
C C      IF(K.GT.4)K = 4
C C      DO 200 J = 1,K
C C      M = ABS(SS(J,J))
C C      IF(M.GT.G)GO TO 200
C C      G = M
C C      ID = J
C C      200 CONTINUE
C C
C C      JJ = NOT/2 + 1
C C      K = JJ + 3
C C      IF(K.GT.NOT)K = NOT
C C      DO 210 J = JJ,K
C C      M = ABS(SS(J,J))
C C      IF(M.GT.G)GO TO 210
C C      G = M
C C      ID = J
C C      210 CONTINUE
C C
C C      JJ = 0
C C      DO 300 J = 1,NOT
C C      IF(J.EQ.ID)GO TO 300
C C      JJ = JJ + 1
C C      C(JJ) = -SS(J,ID)
C C      KK = 0
C C      DO 299 K = 1,NOT
C C      IF(K.EQ.ID)GO TO 299
C C      KK = KK + 1
C C      SINV(JJ,KK) = SS(J,K)
C C      299 CONTINUE
C C      300 CONTINUE
C C      RETURN
C C      END
C C      SUBROUTINE REL(N,R,SC,NG,IDIAG)
C C
C C      SUBROUTINE REL IDENTIFIES THOSE MATRIX ELEMENTS R(L,S) (COSINE EX-
C C      PANSION, SC = 1) OR T(L,S) (SINE EXPANSION, SC = 2) WHICH ARE NONZERO.
C C      THE VALUES OF THE ROW INDICES, L, ARE STORED IN VECTOR ID(1), AND
C C      THE VALUES OF THE INDICES S, NBR(1,NG) IN NUMBER, ARE THE ROWS OF
C C      MATRIX IDMAT(1,J,NG). NG REFERS TO GUIDE # 1 OR GUIDE # 2.
C C
C C      AS AN EXAMPLE, CONSIDER A FIVE-TERM COSINE EXPANSION.
C C      WITH R = 1, THE NONZERO ELEMENTS ARE R(0,0), R(0,4)
C C      R(1,1), R(1,3)
C C      R(2,2)
C C      R(3,1), R(3,3)
C C      R(4,0), R(4,4)
C C      NBR(1) = 2
C C      AND ID(1) = 0 IDMAT(1,J) = 0, 4
C C      1 3
C C      2 2
C C      3 1, 3
C C      4 0, 4
C C
C C      INTEGER SC
C C      REAL N
C C      DIMENSION STORE(200,40),RHST(200,4),IDMAT(20,10,2),ID(20)
C C      DIMENSION NBR(20,2)
C C      DIMENSION R1(10,10),R2(20,10),R3(20,10),R4(10,10)
C C      COMMON/A,B,P2,P1,D,NOT,M12
C C      COMMON/B/STORE,RHST,IDMAT,ID,NBR
C C      COMMON/C/R1,R2,R3,R4

```

```

C      { 10.EQ.2)GO TO 56
C
DO 50 I = 1,NT2
  DO 50 J = 1,NOT
    R3(I,J) = 0.0
  50 R2(I,J) = 0.0
  DO 55 I = 1,NOT
    DO 55 J = 1,NOT
      R4(I,J) = 0.0
  55 R1(I,J) = 0.0
C
DO 60 I = 1,NT2
  II = I + SC - 2
  60 ID(II) = II
C
C      FIND INDICES FOR A CIRCULAR GUIDE
C
56 IF(N.NE.1.0.OR.R.NE.1.0)GO TO 70
DO 62 I = 1,NT2
  NBR(I,NG) = 1
  62 IDMAT(I,NG) = I + SC - 2
  IF(NT2.EQ.NOT)GO TO 250
  II = NOT + 1
  DO 68 I = II,NT2
    68 NBR(I,NG) = 0
  GO TO 250
C
70 NDE = 2
  IF(R.EQ.1.0)NDE = 4
  IF(SC.GT.1)GO TO 150
C
C      COSINE EXPANSIONS; HIGHEST INDEX USED = MAX = NOT - 1
C
MAX = NOT - 1
DO 100 I = 1,NT2
  II = I - 1
  J2 = MOD(II,4) + 1
  ND = 0
  GO TO (85,79,89,79),J2
C
C      IDENTIFY ELEMENTS OF ODD ROWS (ALWAYS 1,3,5,...)
C
79 J = -1
80 J = J + 2
  IF(J.GT.MAX)GO TO 100
  ND = ND + 1
  IDMAT(I,NG,NG) = J
  GO TO 80
C
C      IDENTIFY ELEMENTS OF EVEN ROWS
C
85 IF(R.NE.1.0)GO TO 89
  ELEMENTS ARE 0,4,8,...
  J = -4
  GO TO 90
C
C      ELEMENTS ARE 0,2,4,... OR 2,6,10...
C
89 J = -2
90 J = J + NDE
  IF(J.GT.MAX)GO TO 100
  ND = ND + 1

```



```

C      I( GER 5, SC, SMAX,
C      DIMENSION STORE(200,40), RHST(200,4), IDMAT(20,10,2), ID(20)
C      DIMENSION NBR(20,2)
C      DIMENSION R1(10,10), R2(20,10), R3(20,10), R4(10,10)
C      DIMENSION BJST(11), BIST(21), BMST(21)
C      COMMON/A/B,P2,P1,D,NOT,NT2
C      COMMON/B/STORE,RHST,IDMAT,ID,NBR
C      COMMON/C/P,OMP,RH,DR,CL,SL,CS,SS,BJ,BJM,BK,BKM,B1,B1M,L,S
C      COMMON/D/R1,R2,R3,R4
C
C      CHEBYSCHEV INTEGRATION TO PRODUCE R1(L,S) THROUGH R4(L,S) (SC = 1)
C      OR T1(L,S) THROUGH T4(L,S) (SC = 2)
C
C      PTS = NOP
C      W = PI/(PTS+2.0)
C      P = SORT(P2)
C      LMAX = SORT(1.0 - P2)
C      SMAX = ID(INT2) + 1
C      ORD = 0.0
C      NOR = 0
C
C      ZERO ALL THE REQUIRED MATRIX ELEMENTS IN R1, R2, R3, AND R4
C
C      DO 60 I = 1,NT2
C      NO = NBR(I,1)
C      IF(NO.EQ.0)GO TO 53
C      DO 50 J = 1,NO
C      JK = IDMAT(I,J,1) - SC + 2
C      IF(I.GT.NOT)GO TO 50
C      R1(I,JK) = 0.0
C      R3(I,JK) = 0.0
C      50 NO = NBR(I,2)
C      IF(NO.EQ.0)GO TO 60
C      DO 55 J = 1,NO
C      JK = IDMAT(I,J,2) - SC + 2
C      IF(I.GT.NOT)GO TO 55
C      R4(I,JK) = 0.0
C      55 R2(I,JK) = 0.0
C      60 CONTINUE
C
C      THE INTEGRATION LOOP DO 101 JJJ = 1,NOP IS ENTERED TWICE, ONCE
C      FOR THE FIRST GUIDE (NG = 1) AND ONCE FOR THE SECOND (NG = 2). THE
C      LOOP PRODUCES TWO COMPLETE ARRAYS EACH TIME: R1(L,S) AND R2(L,S)
C      OR R3(L,S) AND R4(L,S).
C
C      NG = 0
C      65 NG = NG + 1
C      IF(NG.GT.2)RETURN
C
C      LOOP DO 101 DOES INTEGRATION USING NOP POINTS IN THE RANGE (0,PI/2)
C      LOOP DO 101 I.... CORRESPONDS TO L
C      LOOP DO 100 J.... CORRESPONDS TO S
C
C      DO 101 JJJ = 1,NOP
C      RH = RHST(JJJ,2*NG-1)
C      DR = RHST(JJJ,2*NG)
C
C      CALCULATE AND STORE THE J BESSEL FUNCTIONS NEEDED BY SUBROUTINE INT
C      JO IS STORED AS BJST(1), J1 IS STORED AS BJST(2), ETC.
C      ARG = PI*B*RH*OMP
C      CALL BESJ(ARG,ORD,SMAX,BJST,NZ)

```

```

C      CALCULATE AND STORE THE I BESSEL FUNCTIONS NEEDED BY SUBROUTINE INT.
C      ;( 3 STORED AS B1ST(1), I1 IS STORED AS B1ST(2), ETC.
C      ARG = PI*8*RH*P
C      CALL BES1(ARG,ORD,1,LMAX,B1ST,NZ)
C
C      CALCULATE AND STORE THE K BESSEL FUNCTIONS NEEDED IN SUBROUTINE INT.
C      K0 IS STORED AS BKST(1), K1 IS STORED AS BKST(2), ETC.
C      CALL BESKN(ARG,NOR,1,LMAX,BKST,NZ)
C
C      CALCULATE R1(L,S) AND R2(L,S) (NG = 1), OR R3(L,S) AND R4(L,S) (NG = 2)
C
      DO 101 I = 1,NT2
      NO = NBR(I,NG)
      IF(NG.EQ.0)GO TO 101
      L = ID(I)
      LM = L - 1
      IF(L.EQ.0)LM = 1
      BK = BKST(L+1)
      BKM = BKST(LM+1)
      B1 = B1ST(L+1)
      B1M = B1ST(LM+1)
C
      IF(L.GT.0)GO TO 70
      CL = 1.0
      SL = 0.0
      GO TO 72
      70 CL = STORE(JJJ,2*L-1)
      SL = STORE(JJJ,2*L)
      72 CONTINUE
C
      DO 100 J = 1,NO
      S = IDMAT(I,J,NG)
      JK = S - SC + 2
      BJ = BJST(S+1)
      BJM = -BJST(2)
      IF(S.GT.0)GO TO 80
      CS = 1.0
      SS = 0.0
      GO TO 82
      80 CS = STORE(JJJ,2*S-1)
      SS = STORE(JJJ,2*S)
      BJM = BJST(S)
      82 CONTINUE
C
      CALL INT(SC,RA,RB)
      IF(NG.EQ.2)GO TO 90
      R2(I,JK) = R2(I,JK) + RB*W
      IF(I.GT.0)GO TO 100
      R1(I,JK) = R1(I,JK) + RA*W
      GO TO 100
      90 R3(I,JK) = R3(I,JK) + RB*W
      IF(I.GT.0)GO TO 100
      R4(I,JK) = R4(I,JK) + RA*W
      100 CONTINUE
      101 CONTINUE
C
      GO TO 65
      END
C
      SUBROUTINE MATR(NOP,SC,NO)
C
      INTEGER SC
      DIMENSION R1(10,10),R2(20,10),R3(20,10),R4(10,10)
      DIMENSION RLS(20,20),C(20)
      DIMENSION ALPHA(10,20),BST(31)

```

```

COMMON/A/B,P2,P1,D,NOT,NT2
C  DN/D/R1,R2,R3,R4
C  COMMON/E/R1,R2,C
COMMON/F/ALPHA,BST
C
C  CALCULATE THE COUPLING COEFFICIENTS ALPHA (SC = 1) OR BETA (SC = 2):
C  STORE AS ALPHA(I,J)
C
C  ARG = PI*B*SQRT(P2)*D
C
C  CALCULATE THE K BESSEL FUNCTIONS OF ORDER 0 THROUGH MAX = NOT + NT2,
C  AND STORE THEM WITH INDICES 1 THROUGH MAX + 1
C
C  MAX = NOT + NT2 + 1
C  I = 0
C  CALL BESKN(ARG,I,1,MAX,BST,J)
C
C  IF(SC.GT.1)GO TO 150
C
C  THIS SECTION CALCULATES ALPHA(K,L)
C  WE NEED THE K-BESSEL FUNCTIONS OF ORDER K = L AND ORDER K + L. FOR THE
C  COSINE SOLUTION, 0 <= K,L <= (NOT-1). TAKING K = I - 1, L = J - 1,
C  WE GET K+L = I + J - 2, K - L = I - J. HOWEVER SINCE THESE START AT
C  ZERO, WE NEED MM = I + J - 1 AND NN = I - J + 1 AS RETRIEVAL INDICES
C  FOR THE ARRAY BST CONTAINING THE BESSEL FUNCTIONS.
C
C  DO 100 I = 1,NOT
C  EPS = 2.0
C  IF(I.EQ.1)EPS = 1.0
C  DO 100 J = 1,NT2
C  MM = I + J - 1
C  IF(I.GE.J)NN = I - J + 1
C  IF(J.GT.1)NN = J - 1 + 1
C  ALPHA(I,J) = EPS*(BST(MM) + BST(NN))/PI
C  GO TO 200
C
C  100 ALPHA(I,J) = EPS*(BST(MM) + BST(NN))/PI
C
C  THIS SECTION CALCULATES BETA(K,L).
C  WE NEED THE K-BESSEL FUNCTIONS OF ORDER K = L AND K + L. FOR THE SINE
C  EXPANSION, 1 <= K,L <= NOT. TAKING K = I AND L = J SO THAT K + L
C  = I + J AND K - L = I - J, WE NEED MM = I + J + 1 AND NN = I - J + 1
C  AS RETRIEVAL INDICES FOR THE ARRAY BST CONTAINING THE BESSEL FUNCTIONS.
C
C  NOTE THAT EVEN THOUGH WE CALCULATE BETA(K,L) (SINE SOLUTION), WE STORE
C  THE RESULTS IN ARRAY ALPHA.
C
C  150 CONTINUE
C  DO 170 I = 1,NOT
C  DO 170 J = 1,NT2
C  MM = I + J + 1
C  IF(I.GE.J)NN = I - J + 1
C  IF(J.GT.1)NN = J - 1 + 1
C  ALPHA(I,J) = 2.0*(BST(MM) - BST(NN))/PI
C  170 CONTINUE
C  IF(NOT.GT.1)GO TO 280
C
C  CALL SUBROUTINE CHEBY TO DO THE LINE INTEGRALS TO FIND R1 AND R2
C  R3 AND R4
C
C  CALL CHEBY(NOP,SC)
C
C  CALCULATE THE ELEMENTS OF THE MATRIX FROM THE ELEMENTS OF R1 AND
C  R4, AND SUMS OF ELEMENTS OF R2 AND R3
C
C  280 CONTINUE
C  NN = SC - 1
C  DO 300 I = 1,NOT

```

```

DO 300 J = 1,NOT
  R(I,J) = R(I,I,J)
  RLS(I+NOT,J+NOT) = R4(I,J)
  S2 = 0.0
  S3 = 0.0
DO 295 MM = 1,NT2
  S2 = S2 + ALPHA(I,MM)*R2(MM,J)
  S3 = S3 + ALPHA(I,MM)*R3(MM,J)
295 S3 = S3 + ALPHA(I,MM)*R3(MM,J)
  RLS(I,J+NOT) = S2
300 RLS(I+NOT,J) = S3
  RETURN
END
SUBROUTINE INT(SC,R1,R2)
C
C EVALUATE THE INTEGRANDS OF THE LINE INTEGRALS WHICH BECOME THE
C MATRIX ELEMENTS R1(L,S) THROUGH R4(L,S)
C
  INTEGER S,SC
  COMMON/A/B,P2,P1,D,NOT,NT2
  COMMON/C/P,DMP,RH,DR,CL,SL,CS,SS,BJ,BJM,BK,BKM,BI,BIM,L,S
  XS = S
  XL = L
  EPS = 1.0
  IF(L.GT.0)EPS = 2.0
  TERM = BK*BJM*DMP + P*BJ*BKM
  TERM = 2.0*B*RH*TERM
  HERM = B*BJM*DMP - P*BJ*BIM
  HERM = P*B*RH*HERM
  IF(L.EQ.S)GO TO 90
  DD = L - S
  TERM = TERM + 2.0*DD*BK*BJ/PI
  HERM = HERM + DD*B1*BJ
90 F = EPS*TERM
  H = EPS*HERM
C
C NOTE THAT I**-(L+1) IS NOT INCLUDED IN F
C NOTE THAT I**L IS NOT INCLUDED IN H
C
  E = 0.0
  G = 0.0
  IF(L.EQ.S)GO TO 100
  E = 2.0*EPS*BK*BJ*DR/(RH*PI)
  G = EPS*B1*BJ*DR/RH
C
C NOTE THAT I**-(L+1) IS NOT INCLUDED IN E
C NOTE THAT I**L IS NOT INCLUDED IN G
C
  100 CONTINUE
C
  IF(SC.EQ.2)GO TO 110
  TERM = XS*CL*SS - XL*SL*CS
  R1 = 4.0*(F*CL*CS + E*TERM)
  R2 = 4.0*(H*CL*CS + G*TERM)
  RETURN
C
110 TERM = XS*SL*CS - XL*SS*CL
  R1 = 4.0*(F*SL*SS - E*TERM)
  R2 = 4.0*(H*SL*SS - G*TERM)
  RETURN
END
SUBROUTINE MATINV(A,N,B,DETER,M)
C
  DIMENSION A(20,20),B(20,1),INDEX(20,3)
  10 DETER = 1.0
  15 DO 20 J = 1,N

```

```

20 INDEX(J,3) = 0
30 IF .50 I = 1,N
C SEARCH FOR PIVOT ELEMENT
C
40 AMAX = 0.
45 DO 105 J = 1,N
IF (INDEX(J,3)-1) 60,105,60
60 DO 100 K = 1,N
IF (INDEX(K,3)-1) 80,100,740
80 IF (AMAX-ABS(A(J,K))) 85,100,100
85 IROW = J
90 ICOLUM = K
AMAX = ABS(A(J,K))
100 CONTINUE
105 CONTINUE
IF (AMAX.EQ.0.0) GO TO 750
INDEX(ICOLUM,3) = INDEX(ICOLUM,3) + 1
260 INDEX(I,1) = IROW
270 INDEX(I,2) = ICOLUM
C INTERCHANGE ROWS TO PUT PIVOT ELEM. IN DIAG.
C
130 IF (IROW-ICOLUM) 140,310,140
140 DETER = -DETER
150 DO 200 L = 1,N
160 SWAP = A(IROW,L)
170 A(IROW,L) = A(ICOLUM,L)
200 A(ICOLUM,L) = SWAP
IF (M) 310,310,210
210 DO 250 L = 1,M
220 SWAP = B(IROW,L)
230 B(IROW,L) = B(ICOLUM,L)
250 B(ICOLUM,L) = SWAP
C DIVIDE PIVOT ROW BY PIVOT ELEMENT
C
310 PIVOT = A(ICOLUM,ICOLUM)
DETER = DETER*PIVOT
330 A(ICOLUM,ICOLUM) = 1.0
340 DO 350 L = 1,N
350 A(ICOLUM,L) = A(ICOLUM,L)/PIVOT
355 IF (M) 360,380,360
360 DO 370 L = 1,M
370 B(ICOLUM,L) = B(ICOLUM,L)/PIVOT
C REDUCE NON PIVOTS ROWS
C
380 DO 550 LI = 1,N
390 IF (LI-ICOLUM) 400,550,400
400 T = A(LI,ICOLUM)
420 A(LI,ICOLUM) = 0.
430 DO 450 L = 1,N
450 A(LI,L) = A(LI,L)-A(ICOLUM,L)*T
455 IF (M) 550,550,460
460 DO 500 L = 1,M
500 B(LI,L) = B(LI,L)-B(ICOLUM,L)*T
550 CONTINUE
C INTERCHANGE COLUMNS
C
600 DO 710 I = 1,N
610 L = N + 1 - I
620 IF (INDEX(L,1)-INDEX(L,2)) 630,710,630
630 JROW = INDEX(L,1)
640 JCOLUM = INDEX(L,2)

```

```

650 D= 705 K = 1.N
660 I = A(K,JROW)
670 A(I,JROW) = A(K,JCOLUMN)
700 A(K,JCOLUMN) = SWAP
705 CONTINUE
710 CONTINUE
740 RETURN
C
750 DETER = 0.0
RETURN
END
C
SUBROUTINE PERIM(N,R,PHI,RHO,DRDP)
C
C
C
C
C
C
C
C
EVALUATE RHO(PHI) ON THE PERIMETER.
EVALUATE D(RHO)/D(PHI) ON THE PERIMETER
C
REAL N
C
IF(R.NE.1.0.OR.N.NE.1.0)GO TO 100
RHO = 1.0
DRDP = 0.0
RETURN
C
100 CONTINUE
CP = COS(PHI)
SP = SIN(PHI)
PN = 2.0*N
DD = (CP/R)**PN + SP**PN
PN = 1.0/PN
RHO = 1.0/(DD**PN)
C
PN = 2.0*N - 2.0
AA = SP**PN - ((CP/R)**PN)/(R*R)
DRDP = -RHO*SP*CP*AA/DD
RETURN
END

```

Program Listing

- LIDGS -

(Reference: Vol. I, Section II.3.5)



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23  *SHAPE*.20X,B*.6X,N*.6X,D*.6X,B*.8X,9X,ROOTS (P2),.*)
117  IAT/(1X,R9,12,12,2X,6,F7,2,F8,2,F7,2,F8,3,9F9,4,/.53( '9.4)
118  FUMAT(5X,PMAX,*,F6,3,10X,PMIN,*,F6,3,/)
119  FUMAT(3X,*SHAPE*.20X,R*.6X,N*.6X,D*.6X,B*.8X,9X,
1  *COEFFICIENTS .....*)
120  FUMAT(53X,9F9,4,/.52X,2F9,4)
121  FUMAT(10 NOD = NUMBER OF INTEGRATION POINTS =.13)
122  FUMAT(4X,6(A2,C(*.12,*) =.E12.5),/4X,4(A2,C(*.12,*) =.
1E12.5))
123  FUMAT(1X,A4,*SYMMETRIC CASE:*.14,* ROOT(S) IDENTIFIED*)
124  FUMAT(1X,A3,1X,10E12,4, 6X,10E12,4,/.7X,10E12,4)
125  FUMAT(1X,A3,1X,10E12,4, 6X,10E12,4,/.7X,10E12,4)
126  FUMAT(1X,A3,1X,10E12,4, 6X,10E12,4,/.7X,10E12,4)
127  FUMAT(1X,A3,1X,10E12,4, 6X,10E12,4,/.7X,10E12,4)
128  FUMAT(1X,A3,1X,10E12,4, 6X,10E12,4,/.7X,10E12,4)
129  FUMAT(1X,A3,1X,10E12,4, 6X,10E12,4,/.7X,10E12,4)
130  FUMAT(1X,A3,1X,10E12,4, 6X,10E12,4,/.7X,10E12,4)
131  FUMAT(1X,A3,1X,10E12,4, 6X,10E12,4,/.7X,10E12,4)
132  FUMAT(1X,A3,1X,10E12,4, 6X,10E12,4,/.7X,10E12,4)
133  FUMAT(1X,A3,1X,10E12,4, 6X,10E12,4,/.7X,10E12,4)
134  FUMAT(1X,A3,1X,10E12,4, 6X,10E12,4,/.7X,10E12,4)
135  FUMAT(1X,A3,1X,10E12,4, 6X,10E12,4,/.7X,10E12,4)

```

# LIST OF VARIABLES WITH SINGLE ASSIGNED PURPOSES:

```

C ACC ACCURACY TO WITHIN WHICH THE ROOTS ARE FOUND
C ALPHA * ARRAY OF COUPLING COEFFICIENTS ALPHA (SC = 1) OR BETA (SC = 2)
C ASY * ARRAY OF COEFFICIENTS WHOSE DETERMINANT IS TO BE FOUND (ANTISYMM)
C B NORMALIZED FREQUENCY (READ IN)
C C VECTOR OF COEFFICIENTS IN THE FIELD EXPANSION (DESIRED RESULT)
C CFA * ARRAY FOR STORING VALUES OF C FOR DIFFERENT CASES (ANTISYMM CASES)
C CFS * ARRAY FOR STORING VALUES OF C FOR DIFFERENT CASES (SYMMETRIC CASES)
C D DISTANCE SEPARATING CENTERS OF TWO GUIDES (UNITS OF SEMIMINOR AXIS)
C DET VALUE OF DETERMINANT (EQUIVALENCED WITH PHI)
C DK ACCURACY TO WITHIN WHICH THE J AND I BESSEL FUNCTIONS ARE FOUND
C DRDP D(RHO)/D(PHI) OF THE GUIDE PERIMETERS (EQUIVALENCED WITH XXX)
C ED INDICATOR SHOWING WHEN WDG IS EVEN (0) OR ODD (1); FOR USE IN MATR
C IDIAG DIAGNOSIS CODE, TO PRINT EXTRA RESULTS (*) (READ IN)
C IDIAT * ARRAY FOR STORING VALUES OF S NEEDED WHEN R(L,S) OR T(L,S) IS COMPUTED
C INDEX * ARRAY FOR STORING TABLE INDICES NEAR WHICH ROOTS OCCUR
C IROOT * ARRAY FOR STORING INFORMATION USED IN PRINTING RESULTS AT THE END
C JD * ARRAY FOR STORING VALUES OF L NEEDED WHEN R(L,S) OR T(L,S) IS COMPUTED
C JDEX * COUNTER TO KEEP TRACK OF THE NUMBER OF SEPARATE CASES COMPLETED
C LSC VALUE OF SC FROM PREVIOUS CASE (HELPS AVOID REPEATED CALCULATIONS)
C N SUPERELLIPSE INDEX (READ IN)
C NBR * ARRAY GIVING NUMBER OF VALUES OF S (IN IDIAT) FOR EACH L (IN JD)
C NO * COUNTER IDENTIFYING WHICH ELEMENT OF DST IS TAKEN FOR D
C NDM DIMENSION OF SYM AND ASY (= NSM*NOT)
C NL VALUE OF N FROM PREVIOUS CASE (HELPS AVOID REPEATING CALCULATIONS)
C NMAX * # OF NONZERO VALUES OF D IN DST (LE-4)
C NOG * # OF GUIDES IN THE LINEAR ARRAY
C NOD * # OF TERMS IN EXPANSIONS (READ IN)
C NP2 * # OF VALUES OF P2 TAKEN TO CONSTRUCT TABLE (READ IN)
C NT2 HIGHEST ALLOWED VALUE OF INDEX L USED IN R2(L,S)
C NR * # OF ROOTS OF DET(ASY)
C NRS * # OF ROOTS OF DET(SYM)
C NSM SORT OF # OF SUBMATRICES IN SYM AND ASY
C PHI ANGLE IN (RHO,PHI) COORDINATE SYSTEM
C PI
C PMAX LARGEST VALUE OF P2 USED IN TABLE (READ IN)
C PMIN SMALLEST VALUE OF P2 USED IN TABLE (READ IN)

```



```

DK = 10.0*(-K)
IF OP.GT.200INOP = 200
  I.,NOP.LT.20INOP = 20

PRINT THE FIXED INFORMATION READ FROM THE FIRST DATA CARD

WRITE(6,135)NOP
WRITE(6,103)NOT
WRITE(6,127)NT2
WRITE(6,104)K
WRITE(6,105)I
WRITE(6,108)NP2
WRITE(6,121)NOP
WRITE(6,133)IDIAG
WRITE(6,125)
JDEX = 0

.....
SC = 1 IF COSINE TERMS ARE TO BE USED; 2 IF SINE TERMS ARE DESIRED
D = CENTER SEPARATION OF GUIDES, IN UNITS OF SEMIMINOR AXIS
N = INDEX OF OVOID (1 = ELLIPSE, >30 = SQUARE)
R = ASPECT RATIO
B = ASSUMED VALUE OF THE INDEPENDENT PARAMETER, THE BINDING CONSTANT, B
P2 = THE PROPAGATION CONSTANT, TO BE FOUND
PMAX, PMIN = RANGE OF VALUES OF P2 TO BE SEARCHED FOR ROOTS

ONE DATA CARD CORRESPONDS TO ONE INDIVIDUAL CASE
RETURN TO STATEMENT 1 IMMEDIATELY AFTER EACH CASE IS COMPLETE. A BLANK
CARD READ AT STATEMENT 1 CAUSES RESULTS FOR ALL CASES TO BE DUMPED
(PRINTED) AND CONTROL WILL THEN GO TO STATEMENT 2. A BLANK CARD THERE
TERMINATES THE JOB EXECUTION, SO TWO BLANK CARDS SHOULD FOLLOW THE
LAST DATA CARD. IF A DATA CARD CONTAINS MORE THAN ONE NONZERO VALUE OF
DST (THE GUIDE SPACINGS, D) THE PROGRAM WILL RETURN TO STATEMENT 6 TO
PROCESS THOSE PAST THE FIRST. THIS IS A MORE EFFICIENT PROCEDURE THAN
READING EACH VALUE OF D SEPARATELY FROM OTHERWISE IDENTICAL DATA CARDS.

1 READ(5,102)SC,DST(1),N,R,B,PMAX,PMIN,(DST(I),I=2,4)
  IF(R.LE.0.0)GO TO 999
  D = 2.0*R
  ND = 0

COUNT THE NUMBER, NMAX, OF CASES (DIFFERENT D) ON THE LAST DATA CARD

DO 5 I = 1,4
  IF(DST(I).EQ.0.0)GO TO 6
  IF(DST(I).LT.D)DST(I) = D
  NMAX = I
5 CONTINUE
6 ND = ND + 1
  D = DST(ND)
  CALL SECOND(TO)
  IF(PMAX.EQ.0.0)PMAX = 1.0
  IF(N.LE.0.0IN = 1.0
  IF(B.LE.0.0)B = 1.0
  IF(SC.LE.0)SC = 1
  IF(SC.GT.2)SC = 2
  IF(JDEX.EQ.15)GO TO 999
  JDEX = JDEX + 1

INITIALIZE STORAGE ARRAYS FOR THIS CASE

DO 70 I = 7,10
  CFS(JDEX,I=6,K) = 0.0

```

```

C          C
70 R = ((JDEX,I-6,K) = 0.0
    RL((JDEX,I-6) = 0.0
    ROOT(JDEX,3) = R
    ROOT(JDEX,4) = N
    ROOT(JDEX,5) = D
    ROOT(JDEX,6) = B
    IROOT(JDEX,2) = SC - 1
    IROOT(JDEX,3) = NOT + SC - 2
C          C
FIND LST, THE CODE FOR DETERMINING WHEN TO REPEAT CALCULATIONS
LST = 1: SKIP INDEX AND PERIMETER CALCULATIONS
LST = 2: RECALCULATE INDICES
LST = 3: RECALCULATE PERIMETER
LST = 4: RECALCULATE BOTH INDICES AND PERIMETER
C          C
CALL CODEIN,R,N,L,RL,SC,LS,C,LST)
C          C
CALCULATE AND STORE RHO(PHI) AS STORE(I,1), I = 1,NOP
CALCULATE AND STORE D(RHO)/D(PHI) AS STORE(I,2), I = 1,NOP
CALCULATE AND STORE COS(M*PHI) AND SIN(M*PHI) AS STORE(I,2*M+1)
AND STORE(I,2*M+2), RESPECTIVELY.
C          C
IF(LST,LT,3)GO TO 80
ON = NOP
DN = PI/(DN*2.0)
DO 79 I = 1,NOP
    PN = 2*(NOP - I) + 1
    PHI = DN*PN/2.0
    CALL PERIM(PHI,RHO,DROP)
    STORE(I,1) = RHO
    STORE(I,2) = DROP
    IF(JDEX.GT,1)GO TO 79
    DO 75 M = 1,NT2
        PN = M
        PN = PN*PHI
        STORE(I,2*M+1) = COS(PN)
        STORE(I,2*M+2) = SIN(PN)
    79 CONTINUE
C          C
80 IF(LST.EQ,1,OR,LST.EQ,3)GO TO 89
C          C
DETERMINE THE INDEX PAIRS (L,S) OF ALL THE MATRIX ELEMENTS NEEDED
BY CALLING SUBROUTINE REL.
C          C
CALL REL(SC)
WRITE(6,125)
C          C
DO 85 I = 1,NT2
    IF(NBR(I).GT,0)GO TO 85
    NBR(I) = 1
    IOMAT(I,1) = 1
    WRITE(6,134)I
    85 CONTINUE
C          C
*****
PRINT INPUT DATA
C          C
89 CONTINUE
C          C
WRITE(6,112).DEX,D,N,R,B
IF(PMAX.NE,1.0,OR,PMIN.NE,0.0)WRITE(6,118)PMAX,PMIN
C          C
J = SC + 5
I = 4

```

```

      IF (ND.GT.1.0) GO TO 90
      I = 9
      GO TO 92
94 I = 8
      GO TO 92
95 I = 7
      GO TO 92
96 IF (N.NE.1.0) GO TO 92
      I = 2
      IF (R.EQ.1.0) I = 1
90 IF (N.LT.30.0) GO TO 92
      I = 5
      IF (R.EQ.1.0) I = 3
92 CONTINUE
      WRITE(6,115) PR(J), PR(I)
      ROOT(JDEX,2) = PR(J)
      ROOT(JDEX,1) = PR(I)
      .....
      IF (ND.GT.1) GO TO 205
      DN = NP2
      DN = (PMAX - PMIN)/DN
      P2 = PMAX - DN/2.0
      WRITE(6,106)
      IF (NMAX.GT.1) WRITE(6,132)((DST(I),DST(I)),I=1,NMAX)
      .....
      C CONSTRUCT TABLE OF VALUES OF DET(SYM) AND DET(ASY) VS P2, GIVEN B
      C TO SAVE TIME (SINCE R1(L,S) AND R2(L,S) ARE INDEPENDENT OF D) THIS
      C IS DONE FOR EACH OF THE VALUES OF D READ INTO ARRAY DST WHEN THE
      C COUNTER, ND, IS EQUAL TO 1. THIS SECTION IS SKIPPED WHEN ND.GT.1.
      C
      NR = 2*NMAX + 1
      DO 200 M = 1,NP2
      TABLE(M,1) = P2
      DO 190 I = 1,NMAX
      D = DST(I)
      CALL MATRINDP(SC,I)
      CALL MATINV(SYM,NDM,C,DET.O)
      TABLE(M,2*I) = DET
      CALL MATINV(ASY,NDM,C,XXX.O)
190 TABLE(M,2*I+1) = XXX
      IF (IDIA.GE.0) WRITE(6,107)M,(TABLE(M,I),I=1,NR)
      IF (IDIA.GE.1) WRITE(6,126)M,(TABLE(M,I),I=1,NR)
200 P2 = P2 - DN
      D = DST(I)
      C
      CALL SECOND(T1)
      T1 = T1 - T0
      WRITE(6,110)T1
      GO TO 211
      C
      C WHEN ND.GT.1, RECOVER THE TABLE OF VALUES OF DET(SYM) AND DET(ASY)
      C VS P2 CALCULATED EARLIER.
      C
205 DO 210 M = 1,NP2
      TABLE(M,2) = TABLE(M,2*ND)
210 TABLE(M,3) = TABLE(M,2*ND+1)
      C
      .....
      C MAKE TENTATIVE IDENTIFICATION OF ROOTS OF EQUATIONS DET(SYM) = 0 AND
      C DET(ASY) = 0
      C
211 NR = 0
      C
      DO 220 K = 1,2
      NRS = NR

```



```

IF(DN.EQ.0.0)GO TO 258
  LST = LST + 1
  C(15,4,LST) = DN
258 CONTINUE
259 WRITE(6,129)XXX,(CFS(15,4,MAX).MAX = 1,LST)
260 WRITE(6,114)
C
C PRINT THE FINAL MATRIX: STORE THE ROOT
C
265 WRITE(6,114)
XXX = 3HASY
DO 252 J = 1,NDM
  IF(IDIAG.LE.0)WRITE(6,124)XXX,(SYM(J,K).K=1,NDM)
  IF(IDIAG.GE.1)WRITE(6,129)XXX,(SYM(J,K).K=1,NDM)
252 CONTINUE
  ROOT(JDEX,1+6) = P2
C
C STORE THE VECTOR OF EXPANSION COEFFICIENTS
C
DO 270 J = 1,NDM
270 CFS(JDEX,1,J) = C(J)
C
C MAKE A FINAL DETERMINATION OF DET FOR THIS ROOT; PRINT IT AND THE EX-
C PANSION COEFFICIENTS
C
CALL MATINV(SYM,NDM,C,DET,0)
WRITE(6,114)
IF(IDIAG.LE.0)WRITE(6,109)XXX,1,P2,PN,ID,DET
IF(IDIAG.GT.0)WRITE(6,130)XXX,1,P2,PN,ID,DET
WRITE(6,114)
XXX = 2H
DO 272 J = 1,NSM
  MM = (J-1)*NOT + 1
  NN = J*NOT
  ID = J + SC - 2
272 WRITE(6,122)((XXX,ID,C(K)).K=MM,NN)
  WRITE(6,114)
  IF(DET.EQ.0.0)WRITE(6,113)
250 CONTINUE
C
C .....
C PROCESS OF FINDING SYMMETRIC MODES IS COMPLETE; NOW CONSIDER
C ANTISYMMETRIC MODES: FIND THE ROOTS OF DET(ASY) = 0 PRECISELY
C
300 CONTINUE
  IF(NR.GT.0)GO TO 321
  IROOT(JDEX,4) = 0
  WRITE(6,125)
  IF(ND-NMAX)/6,1,1
321 CONTINUE
  IROOT(JDEX,4) = NR
C
C LOOP DO 350... IS EXECUTED ONCE FOR EACH ROOT (I) IDENTIFIED ABOVE
C SUBROUTINE ROOT LOCATES EACH ROOT PRECISELY
C
DO 350 I = 1,NR
  M = INDEX(I,2)
  CALL RTT(TABLE,ACC,PN,NOP,SC,3,M,ID,IDIAG)
C
C PRINT FINAL MATRIX: STORE ROOT
C
XXX = 3HASY
DO 352 J = 1,NDM
  IF(IDIAG.LE.0)WRITE(6,124)XXX,(ASY(J,K).K=1,NDM)
  IF(IDIAG.GE.1)WRITE(6,129)XXX,(ASY(J,K).K=1,NDM)
352 CONTINUE

```



```

IF (EQ.EQ.1) NN = 2
D 04 K = NN, NSM
D 04 M = 1, NOT
DO 903 ID = 1, MAX
J = (NSM - K) * NOT + M
903 C(ID) = CFS(I, ID, J) * (-1.0) ** (M+1)
904 WRITE(6, 120) (C(ID), ID=1, MAX)
GO TO 910
905 WRITE(6, 117) ROOT(I, 1), (ROOT(I, M), M=2, 3), (ROOT(I, J), J=2, 6)
910 CONTINUE
C
WRITE(6, 111)
911 XXX = 4HANTI
WRITE(6, 116) PN, NOG, XXX
DO 920 I = 1, JDEX
MAX = IROOT(I, 4)
IF (MAX.EQ.0) GO TO 915
IF (IOTAG.LE.0) WRITE(6, 117) ROOT(I, 1), (ROOT(I, M), M=2, 3),
1(ROOT(I, J), J=2, 6), (ROOT(I, K), K=1, MAX)
IF (IOTAG.GT.0) WRITE(6, 131) ROOT(I, 1), (ROOT(I, M), M=2, 3),
1(ROOT(I, J), J=2, 6), (ROOT(I, K), K=1, MAX)
GO TO 920
915 WRITE(6, 117) ROOT(I, 1), (ROOT(I, M), M=2, 3), (ROOT(I, J), J=2, 6)
920 CONTINUE
WRITE(6, 114)
WRITE(6, 114)
WRITE(6, 114)
WRITE(6, 114)
WRITE(6, 119)
DO 930 I = 1, JDEX
MAX = IROOT(I, 4)
IF (MAX.EQ.0) GO TO 925
WRITE(6, 117) ROOT(I, 1), (ROOT(I, M), M=2, 3), (ROOT(I, J), J=2, 6),
1(CFA(I, ID, 1), ID=1, MAX)
DO 922 K = 2, NDM
922 WRITE(6, 120) (CFA(I, ID, K), ID=1, MAX)
NN = 1
IF (EQ.EQ.1) NN = 2
DO 924 K = NN, NSM
DO 921 M = 1, NOT
DO 923 ID = 1, MAX
J = (NSM - K) * NOT + M
923 C(ID) = CFA(I, ID, J) * (-1.0) ** (M)
924 WRITE(6, 120) (C(ID), ID=1, MAX)
GO TO 930
925 WRITE(6, 117) ROOT(I, 1), (ROOT(I, M), M=2, 3), (ROOT(I, J), J=2, 6)
930 CONTINUE
GO TO 2
END
SUBROUTINE REL(SC)
C
C SUBROUTINE REL IDENTIFIES THOSE MATRIX ELEMENTS R(L,S) (COSINE EX-
C PANSION, SC = 1) OR I(L,S) (SINE EXPANSION, SC = 2) WHICH ARE NONZERO.
C THE VALUES OF THE ROW INDICES, L, ARE STORED IN VECTOR ID(I), AND
C THE VALUES OF THE INDICES, S, NBR(I) IN NUMBER, ARE THE ROWS OF
C MATRIX IDMAT(I,J). FOR EXAMPLE, FOR A FIVE-TERM COSINE EXPANSION
C WITH R = 1, THE NONZERO ELEMENTS ARE R(0,0), R(0,4)
C R(1,1), R(1,3)
C R(2,2)
C R(3,1), R(3,3)
C R(4,0), R(4,4)
C
C AND ID(I) = 0 IDMAT(I,J) = 0, 4 1, 3 NBR(I) = 2
C 1 2 2 1
C 2 3 1, 3 2
C 3 4 0, 4 2
C 4

```

```

C      A      N
      INTEGER SC,EO
      DIMENSION STORE(200,42),IDMAT(20,10),ID(20),NBR(20)
      DIMENSION R1(10,10),R2(20,10)
      COMMON/A/B,P2,P1,R,N,D,DK,NDM,NOT,NSM,NT2,EO
      COMMON/B/STORE,IDMAT,ID,NBR
      COMMON/D/R1,R2

C      DO 50 I = 1,NT2
      DO 50 J = 1,NOT
      50 R2(I,J) = 0.0
      DO 55 I = 1,NOT
      DO 55 J = 1,NOT
      55 R1(I,J) = 0.0

C      DO 60 I = 1,NT2
      60 ID(I) = I + SC - 2

C      C      FIND INDICES FOR A CIRCULAR GUIDE
C      IF(N.NE.1.0.OR.R.NE.1.0)GO TO 70
      DO 62 I = 1,NT2
      NBR(I) = 1
      62 IDMAT(I,1) = I + SC - 2
      IF(NT2.EQ.NOT)GO TO 250
      II = NOT + 1
      DO 68 I = 1,NT2
      68 NBR(I) = 0
      GO TO 250

C      70 NOE = 2
      IF(R.EQ.1.0)NDE = 4
      IF(SC.GT.1)GO TO 150

C      C      COSINE EXPANSIONS; HIGHEST INDEX USED = MAX = NOT - 1
C      MAX = NOT - 1
      DO 100 I = 1,NT2
      II = I - 1
      JZ = MOD(II,4) + 1
      ND = 0
      GO TO (85,79,89,79),JZ

C      C      IDENTIFY ELEMENTS OF ODD ROWS (ALWAYS 1,3,5,...)
C      79 J = -1
      80 J = J + 2
      IF(J.GT.MAX)GO TO 100
      ND = ND + 1
      IDMAT(I,ND) = J
      GO TO 80

C      C      IDENTIFY ELEMENTS OF EVEN ROWS
C      85 IF(R.NE.1.0)GO TO 89
      C      ELEMENTS ARE 0,4,8,...
      J = -4
      GO TO 90

C      C      ELEMENTS ARE 0,2,4,... OR 2,6,10,...
C      89 J = -2
      90 J = J + NOE

```

```

      I = J.GT.MAX)GO TO 100
      NO = NO + 1
      IDMAT(I,NO) = J
      GO TO 90
100 NBR(I) = NO
      GO TO 250
C
C   SINE EXPANSIONS: HIGHEST INDEX USED = MAX = NOT
C
150 MAX = NOT
      DO 200 I = 1,NT2
      II = I
      JZ = MOD(II,4) + 1
      NO = 0
      GO TO(189,179,185,179),JZ
C
C   IDENTIFY ELEMENTS OF ODD ROWS (ALWAYS 1,3,5,...)
C
179 J = -1
180 J = J + 2
      IF(J.GT.MAX)GO TO 200
      NO = NO + 1
      IDMAT(I,NO) = J
      GO TO 180
C
C   IDENTIFY ELEMENTS OF EVEN ROWS
C
185 IF(R.NE.1.0)GO TO 189
C
C   ELEMENTS ARE 4,8,12,...
C
      J = -2
      GO TO 190
189 J = 0
C
C   ELEMENTS ARE 2,4,6... OR 2,6,10,...
C
190 J = J + NDE
      IF(J.GT.MAX)GO TO 200
      NO = NO + 1
      IDMAT(I,NO) = J
      GO TO 190
200 NBR(I) = NO
250 CONTINUE
C
      WRITE(6,300)
      MAX = 1H
      JZ = 11 - SC
      IF(NT2.LT.JZ)JZ = NT2
      DO 290 I = 1,JZ
      NO = NBR(I)
290 WRITE(6,301)NO,((MAX,ID(I),IDMAT(I,J)),J=1,NO)
C
      IF(JZ.EQ.NT2)RETURN
      JZ = JZ + 1
      DO 295 I = JZ,NT2
      NO = NBR(I)
295 WRITE(6,302)NO,((MAX,ID(I),IDMAT(I,J)),J=1,NO)
C
300 FORMAT(/,*,TABLE OF INDICES (L,S) FOR REQUIRED *,
1*,R(L,S),*)
301 FORMAT(16,3X,10(A1,*,*,11,*,*,11,*,*,3X))
302 FORMAT(16,2X,10(A1,*,*,12,*,*,11,*,*,2X))
C
      RETURN
      END

```

```

C      SUBROUTINE CODE(N,R,NL,RL,SC,LSC,LST)
C      I = N,NL
C      INTEGER SC
C      FIND LST: CODE FOR DECIDING WHEN TO RECALCULATE INDICES (2),
C      PERIMETER (3), OR BOTH (4).
C
C      LST = 1
C      IF(R.EQ.RL)GO TO 71
C
C      FIND LST FOR CASES WITH R.NE.RL
C
C      LST = 3
C      IF(R.NE.1.0.AND.RL.NE.1.0)GO TO 73
C      GO TO 72
C
C      71 IF(N.EQ.NL)GO TO 73
C
C      FIND LST FOR CASES WITH R.EQ.RL
C
C      LST = 3
C      IF(R.NE.1.0)GO TO 73
C      IF(N.NE.1.0.AND.NL.NE.1.0)GO TO 73
C      72 LST = 4
C      GO TO 74
C
C      73 IF(SC.EQ.LSC)GO TO 74
C
C      CHANGE LST WHEN SC.NE.LSC
C
C      LST = LST + 1
C      74 LSC = SC
C      RL = R
C      NL = N
C      RETURN
C      END
C
C      SUBROUTINE RTT(TABLE,ACC,PN,NOP,SC,JJ,M,ID,IDIAG)
C
C      SUBROUTINE ROOT PRECISELY LOCATES THE ROOT SPECIFIED BY M AND
C      JJ (USING ELEMENTS M AND M+1 IN THE ARRAY, TABLE(M,JJ)) AND THEN
C      DETERMINES THE FIELD EXPANSION COEFFICIENTS C(J) CORRESPONDING TO
C      THAT ROOT
C
C      REAL N
C      INTEGER SC,EO
C      DIMENSION SYM(30,30),ASY(30,30),C(30)
C      DIMENSION TABLE(51,9)
C      DIMENSION SIN(30,30)
C      COMMON/A/B,P2,P1,R,N,D,DK,NDM,NOT,NSM,NT2,EO
C      COMMON/E/SYM,ASY,C
C
C      700 FORMAT(/, 'SEARCH ID= 7X, *X0*, 11X, *X1*, 11X, *X2*, 10X, *F0*, 9X, *F1*,
C      19X, *F2*, 11X, *X*, 10X, *F*, 10X, *DX*')
C      701 FORMAT(/, ' $$$$ ERROR IN HINTS OR MULLER, IER =, 13, $$$$$$')
C
C      IF(IDIAG.GE.3)WRITE(6,700)
C      IER = 0
C      ID = 0
C      IF(M.LT.100)GO TO 230
C      P2 = TABLE(M-100,1)
C      CALL MATR(NOP,SC,1)
C      GO TO 248
C
C      HALF-INTERVAL SEARCH
C

```

```

230 G1 = TABLE(M+1,JJ)
C  G1 = TABLE(M,JJ)
C  X1 = TABLE(M+1,1)
C  X2 = TABLE(M,1)
C  DN = X2 - X1
C  K = 0
C
C  IF G1 AND G2 ARE TWO OR MORE ORDERS OF MAGNITUDE DIFFERENT, USE
C  A HALF-INTERVAL SEARCH TO NARROW THE RANGE X1 TO X2
C
235 W = -G1/G2
IF(WM.LT.100..AND.WM.GT..01)GO TO 240
IF(DN.LT.ACC)GO TO 240
DN = DN/3.0
CALL HINTS(X1,X2,G1,G2,DN,PN,NOP,SC,JJ,ID,IDIAG,IER)
IF(IER.GT.0)WRITE(6,701)IER
K = K + 1
IF(PN.EQ.0.0)GO TO 248
GO TO 235
C
C  MULLER'S METHOD TO GET THE ROOT PRECISELY
C
240 CALL MULLER(X1,X2,G1,G2,ACC,PN,NOP,SC,JJ,ID,IDIAG,IER)
C
C  ID = ID + K
248 CONTINUE
IF(IER.GT.0)WRITE(6,701)IER
C
C  CALCULATE CFS, THE RATIO OF COEFFICIENTS C3, C3,... TO C1, OR
C  D3, D3,... TO D1
C
IF(JJ.EQ.3)GO TO 260
CALL LOAD(SYM,SINV,C,JJ,SC,M)
C
C  GO TO 266
C
260 CONTINUE
CALL LOAD(ASY,SINV,C,JJ,SC,M)
266 LST = NDM - 1
C
C  CALL MATINV(SINV,LST,C,G,1)
C
C  STACK THE COEFFICIENTS IN THE PROPER ORDER IN VECTOR C, INCLUDING 1.000
C
IF(M.EQ.NDM)GO TO 280
DO 270 J = M,LST
K = NDM + M - J
270 C(K) = C(K-1)
280 C(M) = 1.0
C
C  CHECK THE NORMALIZATION OF THE COEFFICIENTS
C
IF(NSM.EQ.1)RETURN
LST = MOD(M,MOT)
G = 0.0
DO 290 K = 1,NSM
DN = ABS(C(LST+K-1))
IF(DN.LT.G)GO TO 290
J = LST + K - 1
G = DN
290 CONTINUE
IF(J.EQ.M)RETURN
G = C(J)
DO 295 K = 1,NDM
295 C(K) = C(K)/G
RETURN

```

```

END
S( ROUTINE MULLER(X0,X2,F0,F2,ACC,PN,NOP,SC,JJ,IO,IDIAG,IER)
MULLER'S ALGORITHM FOR THE SOLUTION OF DET(RLS) = 0
INVERSE PARABOLIC INTERPOLATION USING POINTS
(X0,F0), (X1,F1), AND (X2,F2)
TO FIND THE NEW POINT (X,F), WHERE X0 < X1 < X2 AND F0*F2 < 0
ALWAYS HOLD. SIGN IS THE SIGN OF F0, AND DOES NOT CHANGE FROM ITS
INITIAL VALUE. THE TWO ROOTS OF THE QUADRATIC NECESSARILY LIE
BETWEEN X0 AND X1, AND BEYOND X0 OR X2 IF (A) SIGN*F1 < 0
BETWEEN X1 AND X2, AND BEYOND X0 OR X2 IF (B) SIGN*F1 > 0
(THAT IS, FOR EXAMPLE, IN CASE (A) THERE IS NO ROOT BETWEEN X1 AND X2.)
CHOOSE THE ROOT INSIDE THE INTERVAL.
XL IS THE LARGEST X FOR WHICH F*SIGN > 0
XU IS THE SMALLEST X FOR WHICH F*SIGN < 0
INTEGER SC,EO
REAL N
DIMENSION SYM(30,30),ASY(30,30),C(30)
COMMON/A,B,P2,PI,R,N,D,DM,NDM,NOT,NSM,NT2,EO
COMMON/E/SYM,ASY,C
C
ID = 0
IER = 0
IF(X2.LE.X0)GO TO 290
PN = X2 - X0
W = ALOG(PN/ACC)/-.693147
LST = W + 1.5
C
LST GIVES THE NUMBER OF ITERATIONS WHICH A HALF-INTERVAL SEARCH
WOULD REQUIRE FOR THESE INITIAL VALUES AND DESIRED ACCURACY. IF
MULLER'S ALGORITHM EXCEEDS THIS, EXIT WITH ERROR CONDITION 3.
700 FORMAT(' MULLR:',I3,3F13.10,3E11.4,F13.10,2E11.4)
702 FORMAT(' MULLR:',I3,72X,F13.10,E11.4)
C
ASSUME THAT THE INITIAL VALUES ARE SUCH THAT X0 < X2
IF NOT, RETURN (IER = 4)
XL = X0
XU = X2
SIGN = F0/ABS(F0)
C
SPLIT THE INTERVAL AND EVALUATE THE DETERMINANT AT THE MIDPOINT
40 X = (X0 + X2)/2.0
P2 = X
CALL MATR(NOP,SC,1)
IF(JJ.EQ.2)CALL MATINV(SYM,NDM,C,F,0)
IF(JJ.EQ.3)CALL MATINV(ASY,NDM,C,F,0)
ID = ID + 1
IF(F*SIGN)50,200,60
50 XU = X
GO TO 100
60 XL = X
C
EVALUATE THE NEW X FROM THE OLD X0, X1, AND X2
100 X1 = X
F1 = F
DD01 = (F1 - F0)/(X1 - X0)
DD12 = (F2 - F1)/(X2 - X1)
DD012 = (DD12 - DD01)/(X2 - X0)
W = DD01 + (X1 - X0)*DD012

```

```

C
U = DD012/M
G = F1/M
I
U = 1.0 - 4.0*U*GL
IF(U.LT.0.0)GO TO 260
U = 2.0*GL/(1.0 + SORT(U))
X = X1 - U
IF(X.GT.X0.AND.X.LT.X2)GO TO 145
U = 1.0 - 4.0*GL*DD012/M
U = 2.0*GL/(1.0 - SORT(U))
X = X1 - U
145 CONTINUE
IF(X.GT.1.0.OR.X.LT.0.0)GO TO 270
P2 = X
CALL MATR(NOP,SC,1)
ID = ID + 1
PN = ABS(XU - XL)
IF(PN.LT.ACC)GO TO 210
IF(ID.GT.LST)GO TO 280
IF(JJ.EQ.2)CALL MATINV(SYM,NDM,C,F,0)
IF(JJ.EQ.3)CALL MATINV(ASY,NDM,C,F,0)
C
IF(IDIAG.GE.3)WRITE(6,700)ID,X0,X1,X2,F0,F1,F2,X,F,PN
C
C RESET THE UPPER OR LOWER BOUND ON THE ROOT
C
IF(F*SIGN)150,200,155
150 XU = X
GO TO 160
155 XL = X
C
C CHANGE X0 OR X2 TO THE PREVIOUS X1; THE NEW X1 IS X
C
160 IF(F1*SIGN)165,200,170
165 X2 = X1
F2 = F1
GO TO 175
170 X0 = X1
F0 = F1
175 GL = ABS(U)
IF((F*F1).LT.0.0)GO TO 100
IF(GL.GT.ACC)GO TO 100
C
C IF POINTS ACCUMULATE CLOSELY ON ONE SIDE OF THE TRUE ROOT, A SINGLE
C SMALL JUMP MAY SPAN THE ROOT. THEREFORE CHANGE X BY A LITTLE LESS
C THAN ACC AND EVALUATE THE DETERMINANT. IF THIS FAILS (THAT IS, IF THE
C FUNCTION HAS THE SAME SIGN AGAIN) SPLIT THE INTERVAL IN HALF AND
C CONTINUE.
C
C
C LINEAR EXTRAPOLATION PAST THE ANTICIPATED ZERO
C
W = 1.0E-12
IF(X.NE.X1)GO TO 180
XT = W
IF((F*SIGN).GT.0.0)XT = -W
GO TO 185
180 XT = 2.0*F*(X1 - X)/(F1 - F)
GL = ABS(XT)
IF(GL.GT.W)GO TO 185
XT = W*XT/GL
185 XT = X - XT
P2 = XT
CALL MATR(NOP,SC,1)
IF(JJ.EQ.2)CALL MATINV(SYM,NDM,C,FT,0)
IF(JJ.EQ.3)CALL MATINV(ASY,NDM,C,FT,0)

```

```

10 = ID + 1
{  DIAG.GE.3)WRITE(6,702)ID,X1,FT
  I,FT,F)190,200,40
190 IF(FT-SIGN)192,200,193
192 XU = XT
   X2 = XT
   F2 = FT
   GO TO 100
193 XL = XT
   X0 = XT
   F0 = FT
   GO TO 100

C  IF THE DETERMINANT IS EXACTLY ZERO, THE MATRIX (WHICH MAY HAVE BEEN
C  ALTERED IN MATINV) IS RECOMPUTED.
C  200 PN = 0.0
C  CALL MATR(NOP.SC,1)
   ID = ID + 1
C  NORMAL RETURN:
C  210 IF(IDIAG.GE.3)WRITE(6,700)ID,X0,X1,X2,F0,F1,F2,X
   RETURN
C  ERROR RETURN 1: COMPLEX ROOT
C  260 IER = 1
   RETURN
C  ERROR RETURN 2: PROCEDURE IS OSCILLATING
C  270 IER = 2
   RETURN
C  ERROR RETURN 3: MORE ITERATIONS THAN HALF-INTERVAL SEARCH WOULD REQUIRE
C  280 IER = 3
   GO TO 210
C  ERROR RETURN 4: X0 > X2 INITIALLY
C  290 IER = 4
   RETURN
END
SUBROUTINE HINTS(X1,X2,G1,G2,ACC,PN,NOP,SC,JJ,ID,IDIAG,IER)
C  SUBROUTINE HINTS PERFORMS A HALF-INTERVAL SEARCH TO LOCATE THE ROOTS OF
C  THE EQUATION DET(RLS) = 0 OR DET(TLS) = 0 TO WITHIN A DESIRED ACCURACY
C  ACC. ID IS THE NUMBER OF BISECTIONS, AND PN IS THE ACTUAL MAXIMUM ERROR.
C  INTEGER SC,EO
C  REAL N
C  DIMENSION SYM(30,30),ASY(30,30),C(30)
C  COMMON/A/B,P2,P1,R,N,D,DK,NDM,NOT,NSM,NT2,EO
C  COMMON/E/SYM,ASY,C
C  700 FORMAT(* HINTS:*,I3,I3X,2F13.10,11X,2E11.4,24X,E11.4)
C  ID = 0
   IER = 0
C  100 PN = ABS(X2 - X1)
C  IF(IDIAG.GE.3)WRITE(6,700)ID,X1,X2,G1,G2,PN
   IF(PN.LT.ACC)RETURN

```

```

ID = ID + 1
IF (ID.GT.25) GO TO 230
  (X1 + X2)/2.0
  CALL MATR(NOP,SC,1)
  IF (JQ.EQ.2) CALL MATINV(SYM,NOM,C,G,0)
  IF (JQ.EQ.3) CALL MATINV(ASY,NOM,C,G,0)
  IF (G.GT.1) 120,210,130
120 X2 = P2
    G2 = G
    GO TO 100
130 X1 = P2
    G1 = G
    GO TO 100
210 PN = 0.0
  RETURN
C
C ERROR RETURN: TOO MANY ITERATIONS
C
230 IER = 6
  RETURN
  END
  SUBROUTINE LOAD(SS,SINV,C,MM,SC,ID)
C
C SUBROUTINE LOAD LOADS SS (SEE SYM OR ASY) INTO ARRAY SINV, ELIMINATING
C THE ROW AND COLUMN CONTAINING THE DIAGONAL ELEMENT WHICH IS SMALLEST
C IN MAGNITUDE. THE NEGATIVE OF THE COLUMN OF SS WHICH IS LEFT OUT IS
C LOADED INTO ARRAY C, EXCEPT FOR THE ELEMENT IN THE ROW WHICH IS ELIM-
C INATED. THUS, SINV IS (NDM-1) BY (NDM-1) IN DIMENSION, AND C IS A
C VECTOR (NDM-1) IN LENGTH.
C
C ONLY THE FIRST THREE ELEMENTS OF EACH SECTION OF THE DIAGONAL ARE
C SEARCHED FOR THE SMALLEST VALUE.
C
  INTEGER SC,EO
  REAL N
  DIMENSION SS(30,30),SINV(30,30),C(30)
  COMMON/A/B,P2,P1,R,N,D,DK,NDM,NOT,NSM,NT2,EO
C
C FIRST FIND THE SMALLEST DIAGONAL ELEMENT
C
  G = 1.0E+90
  ID = 1
  K = NDM
  IF (K.GT.3) K = 3
  DO 200 I = 1,NSM
    DO 200 M = 1,K
      J = (I-1)*NOT + M
      H = ABS(SS(J,J))
      IF (H.GT.G) GO TO 200
      G = H
      KK = ID
      ID = J
200 CONTINUE
C
C PRECAUTION: AVOID CHOOSING FOR ID A NUMBER SUCH THAT C(ID) IS IDENTI-
C CALLY ZERO. THIS CAN HAPPEN WITH AN ODD NUMBER OF GUIDES IF C(ID)
C IS FROM THE FIELD EXPANSION OF THE MIDDLE GUIDE. IF IT DOES, USE
C THE SECOND SMALLEST DIAGONAL ELEMENT (KK) FOR ID
C
  IF (EO.EQ.0) GO TO 250
  JJ = NDM - NOT
  IF (ID.LE.JJ) GO TO 250
  G = (-1.0)**(ID-JJ+SC)
  H = (-1.0)**(MM+SC+1)
C

```

```

C IF G > 0. WE HAVE SELECTED AN EVEN COEFFICIENT
C I. A < 0. WE HAVE SELECTED AN ODD COEFFICIENT
C IF A > 0. WE WANT AN EVEN COEFFICIENT
C IF A < 0. WE WANT AN ODD COEFFICIENT
C IF((G*H).GT.0.0)GO TO 250
C ID = KK
C ELIMINATE THE ROW AND COLUMN IDENTIFIED BY ID
C
250 JJ = 0
DO 300 J = 1,NDM
IF(J.EQ.ID)GO TO 300
JJ = JJ + 1
C(JJ) = -SS(J,ID)
KK = 0
DO 299 K = 1,NDM
IF(K.EQ.ID)GO TO 299
KK = KK + 1
SINV(JJ,KK) = SS(J,K)
299 CONTINUE
300 RETURN
END
SUBROUTINE CHEBY(NOP,SC)
C
INTEGER S,SC,SMIN,SMAX,E0
REAL N
DIMENSION STORE(200,42),IDMAT(20,10),ID(20),NBR(20)
DIMENSION R1(10,10),R2(20,10)
DIMENSION BJST(11),B1ST(21),BKST(31)
COMMON/A/B,P2,PI,R,N,D,DK,NDM,NOT,NSM,NT2,E0
COMMON/B/STORE,IDMAT,ID,NBR
COMMON/C/RH,DR,CL,SL,CS,SS,BJ,BJN,BK,BKM,BT,BIM,L,S
COMMON/D/R1,R2
C
CHEBYSHEV INTEGRATION TO PRODUCE R1(L,S) AND R2(L,S) (SC = 1)
OR T1(L,S) AND T2(L,S) (SC = 2)
C
PTS = NOP
W = PI/(PTS*2.0)
DO 50 I = 1,NT2
NO = NBR(I)
DO 50 J = 1,NO
JK = IDMAT(I,J) - SC + 2
R1(I,JK) = 0.0
R2(I,JK) = 0.0
LMAX = ID(NT2)
SMAX = ID(NOT)
LMIN = 0
SMIN = 0
50 R2(I,JK) = 0.0
C
LOOP DO 100 DOES INTEGRATION USING NOP POINTS IN THE RANGE (0,PI/2)
C LOOP DO 100 I... CORRESPONDS TO L
C LOOP DO 100 J... CORRESPONDS TO S
C
DO 100 JJJ = 1,NOP
RH = STORE(JJJ,1)
DR = STORE(JJJ,2)
C
C CALCULATE AND STORE THE J BESSEL FUNCTIONS NEEDED BY SUBROUTINE INT
C J0 IS STORED AS BJST(1), J1 IS STORED AS BJST(2), ETC.
C
ARG = PI*B*RH*SQRT(1.0 - P2)
CALL JRECUR(ARG,SMIN,SMAX,BJST,DK)
C

```

```

C      CALCULATE AND STORE THE I BESSEL FUNCTIONS NEEDED BY SUBROUTINE INT.
C      IF 'S STORED AS BIST(1), I1 IS STORED AS BIST(2), ETC.
C      (
C      ARG = PI*B*RH*SQRT(P2)
C      CALL KRECUR(ARG,LMIN,LMAX,BIST,DK)
C
C      CALCULATE AND STORE THE K BESSEL FUNCTIONS NEEDED IN SUBROUTINE INT.
C      K0 IS STORED AS BKST(1), K1 IS STORED AS BKST(2), ETC.
C      CALL KRECUR(ARG,LMAX,BKST,DK)
C
C      CALCULATE R1(L,S) AND R2(L,S) OR T1(L,S) AND T2(L,S)
C      DO 100 I = 1,NTZ
C      L = ID(I)
C      LM = L - 1
C      IF(L.EQ.0) LM = 1
C      BK = BKST(L+1)
C      BKM = BKST(LM+1)
C      B1 = BIST(L+1)
C      B1M = BIST(LM+1)
C
C      NO = NBR(I)
C      IF(L.GT.0) GO TO 70
C      CL = 1.0
C      SL = 0.0
C      GO TO 72
C
C      70 CL = STORE(JJJ,2*L+1)
C      SL = STORE(JJJ,2*L+2)
C      72 CONTINUE
C
C      DO 100 J = 1,NO
C      S = IDMAT(I,J)
C      JK = S - SC + 2
C      BJ = BJST(S+1)
C      BJM = -BJST(2)
C      IF(S.GT.0) GO TO 80
C      CS = 1.0
C      SS = 0.0
C      GO TO 82
C
C      80 CS = STORE(JJJ,2*S+1)
C      SS = STORE(JJJ,2*S+2)
C      BJM = BJST(S)
C      82 CONTINUE
C
C      CALL INT(SC,RA,RB)
C      R2(I,JK) = R2(I,JK) + RB*W
C      IF(I.GT.NOT) GO TO 100
C      R1(I,JK) = R1(I,JK) + RA*W
C      100 CONTINUE
C
C      RETURN
C      END
C
C      SUBROUTINE KRECUR(X,NMAX,B,DK)
C
C      SUBROUTINE KRECUR CALCULATES THE MODIFIED BESSEL FUNCTIONS KN(X)
C      OF ORDER N AND ARGUMENT X FOR N BETWEEN 0 AND NMAX. IT CALLS
C      SUBROUTINE BESK TO FIND K0(X) AND K1(X), AND THEN
C      USES A RECURSION RELATION TO CALCULATE THE HIGHER ORDERS. THE
C      VECTOR B CONTAINS THE RESULTANT BESSEL FUNCTIONS, WITH THE VECTOR
C      INDEX BEING ONE GREATER THAN THE ORDER OF THE FUNCTION STORED AT
C      THAT LOCATION. THAT IS, K0 IS STORED AS B(1), K1 AS B(2), ETC.
C      DIMENSION B(31)

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C
1  I MAX.GT.30)NMAX = 30
2  I..4.GT.0.0)GO TO 5
3  DO 4 N = 1,NMAX
4  B(N+1) = .99E+99
5  B(1) = .99E+99
6  RETURN
7
8  N = 0
9  CALL BESK(X,N,BKM,K)
10 B(1) = BKM
11 IF(NMAX.EQ.N)RETURN
12 N = 1
13 CALL BESK(X,N,BK,K)
14 B(N+1) = BK
15 IF(N.GE.NMAX)RETURN
16 F = N
17 BKP = BKM + 2.0*F*BK/X
18 BKM = BK
19 BK = BKP
20 N = N + 1
21 GO TO 10
22 END
23 SUBROUTINE JRECUR(X,NMIN,NMAX,B,DK)
24
25 SUBROUTINE JRECUR CALCULATES THE MODIFIED BESSEL FUNCTIONS JN(X)
26 OF ORDER N AND ARGUMENT X FOR N BETWEEN NMIN AND NMAX. IT CALLS
27 SUBROUTINE BESJ TO FIND JN(X) FOR N = NMAX AND NMAX - 1, AND THEN
28 USES A RECURSION RELATION TO CALCULATE THE LOWER ORDERS. THE
29 VECTOR B CONTAINS THE RESULTANT BESSEL FUNCTIONS, WITH THE VECTOR
30 INDEX BEING ONE GREATER THAN THE ORDER OF THE FUNCTION STORED AT
31 THAT LOCATION. THAT IS, J0 IS STORED AS B(1), J1 AS B(2), ETC.
32
33 DIMENSION B(21)
34
35 IF(NMIN.LT.0)NMIN = 0
36 IF(NMAX.GT.20)NMAX = 20
37 IF(NMIN.GT.NMAX)NMIN = NMAX
38 IF(X.GT.0.0)GO TO 5
39 K = NMIN
40 IF(NMIN.EQ.0)K = 1
41 DO 4 N = K,NMAX
42 B(N+1) = 0.0
43 B(1) = 1.0
44 RETURN
45
46 EPS = DK/10.
47 IF(EPS.LT.1.0E-14)EPS = 1.0E-14
48 CALL BESJ(X,NMAX,BJP,EPS,K)
49 B(NMAX+1) = BJP
50 IF(NMIN.EQ.NMAX)RETURN
51 N = NMAX - 1
52 CALL BESJ(X,N,BJ,EPS,K)
53 B(N+1) = BJ
54 IF(NMIN.EQ.N)RETURN
55 F = N
56 BJM = 2.0*F*BJ/X - BJP
57 BJP = BJ
58 BJ = BJM
59 N = N - 1
60 GO TO 10
61 END
62 SUBROUTINE MATR(NOP,SC,NO)
63
64 REAL N
65 INTEGER SC,EO
66 DIMENSION B(10,10),B9(20,10)

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C      DIMENSION ALPHA(10,20,5),BST(31),SUM(5)
C      INSLON SYM(30,30),ASY(30,30),C(30)
C      COMMON/A/B,P2,P1,R,N,D,DK,NDM,NOT,NSM,NT2,E0
C      COMMON/D/R1,R2
C      COMMON/E/SYM,ASY,C
C      COMMON/F/ALPHA
C
C      CALCULATE THE COUPLING COEFFICIENTS ALPHA (SC = 1) OR BETA (SC = 2):
C      STORE AS ALPHA(I,J,K). THE INDEX K SPECIFIES WHICH PAIR OF GUIDES
C      IS CONSIDERED. THAT IS, THE ARGUMENT CONTAINS THE DISTANCE BETWEEN
C      GUIDE # 1 AND GUIDE # 2, # 1 AND # 3, ETC. FOR UNIFORM SPACINGS,
C      THE NUMBER OF PAIRS REQUIRED IS ONE LESS THAN NOG.
C
C      NOG = 2*NSM - E0
C      NS = NOG - 1
C      DO 200 K = 1,NS
C      SIGN = K
C      ARG = PI*B*SORT(P2)*SIGN-D
C
C      CALCULATE THE K BESSEL FUNCTIONS OF ORDER 0 THROUGH MAX = NOT + NT2,
C      AND STORE THEM WITH INDICES 1 THROUGH MAX + 1
C
C      MAX = NOT + NT2
C      CALL KRECUR(ARG,MAX,BST,DK)
C
C      IF(SC.GT.1)GO TO 150
C
C      THIS SECTION CALCULATES ALPHA(K,L)
C      WE NEED THE K-BESSEL FUNCTIONS OF ORDER K - L AND ORDER K + L. FOR THE
C      COSINE SOLUTION, 0 <= K,L <= (NOT-1). TAKING K = I - 1, L = J - 1,
C      WE GET K+L = I + J - 2, K - L = I - J. HOWEVER SINCE THESE START AT
C      ZERO, WE NEED MM = I + J - 1 AND NN = I - J + 1 AS RETRIEVAL INDICES
C      FOR THE ARRAY BST CONTAINING THE BESSEL FUNCTIONS.
C
C      DO 100 I = 1,NOT
C      EPS = 2.0
C      IF(I.EQ.1)EPS = 1.0
C      DO 100 J = 1,NT2
C      MM = I + J - 1
C      IF(I.GE.J)NN = I - J + 1
C      IF(J.GT.1)NN = J - I + 1
C      ALPHA(I,J,K) = EPS*(BST(MM) + BST(NN))/PI
C      GO TO 200
C
C      THIS SECTION CALCULATES BETA(K,L).
C      WE NEED THE K-BESSEL FUNCTIONS OF ORDER K - L AND K + L. FOR THE SINE
C      EXPANSION, 1 <= K,L <= NOT. TAKING K = I AND L = J SO THAT K + L
C      = I + J AND K - L = I - J, WE NEED MM = I + J + 1 AND NN = I - J + 1
C      AS RETRIEVAL INDICES FOR THE ARRAY BST CONTAINING THE BESSEL FUNCTIONS.
C
C      NOTE THAT EVEN THOUGH WE CALCULATE BETA(K,L) (SINE SOLUTION), WE STORE
C      THE RESULTS IN ARRAY ALPHA.
C
C      150 CONTINUE
C      DO 170 I = 1,NOT
C      DO 170 J = 1,NT2
C      MM = I + J + 1
C      IF(I.GE.J)NN = I - J + 1
C      IF(J.GT.1)NN = J - I + 1
C      ALPHA(I,J,K) = 2.0*(BST(MM) - BST(NN))/PI
C      170 CONTINUE
C      IF(ND.GT.1)GO TO 280
C
C      CALL SUBROUTINE CHEBY TO DO THE LINE INTEGRALS TO FIND R1 AND R2
C      CALL CHEBY(NOP,SC)

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C      CALCULATE THE ELEMENTS OF THE MATRIX FROM THE ELEMENTS OF R1 AND
C      SUMS OF ELEMENTS OF R2. FIRST FIND THE (1,1) ELEMENT OF EACH OF THE
C      SUBMATRICES. THE THE (1,2) ELEMENT OF EACH, AND SO ON.
C
C      280 CONTINUE
C      DO 300 I = 1,NOT
C      DO 300 J = 1,NOT
C      DO 285 K = 1,NS
C
C      CALCULATE THE SUMS OF ALPHA*R2 FOR THE (I,J) ELEMENT OF EACH OF
C      THE SUBMATRICES
C      SUM(K) = 0.0
C      DO 285 M = 1,NT2
C      285 SUM(K) = SUM(K) + R2(M,J)*ALPHA(I,M,K)
C
C      CALCULATE THE FIRST TERM IN THE (I,J) ELEMENT OF EACH OF THE SUBMATRICES
C
C      DO 290 K = 1,NSM
C      DO 290 M = 1,NSM
C      MM = (K - 1)*NOT
C      NN = (M - 1)*NOT
C      IF(K-M)/286,287,288
C
C      UPPER-RIGHT SUBMATRICES
C
C      286 SIGN = (-1.0)**(SC+J)
C      IF(M.EQ.NSM.AND.EQ.EQ.1)SIGN = 0.0
C      SYM(I+MM,J+NN) = SIGN*SUM(M-K)
C      ASY(I+MM,J+NN) = SIGN*SUM(M-K)
C      GO TO 290
C
C      DIAGONAL SUBMATRICES
C
C      287 SYM(I+MM,J+NN) = R1(I,J)
C      ASY(I+MM,J+NN) = R1(I,J)
C      GO TO 290
C
C      LOWER-LEFT SUBMATRICES
C
C      288 SIGN = (-1.0)**(SC+I)
C      SYM(I+MM,J+NN) = SIGN*SUM(K-M)
C      ASY(I+MM,J+NN) = SIGN*SUM(K-M)
C      290 CONTINUE
C
C      CALCULATE THE SECOND TERM IN THE (I,J) ELEMENT OF EACH OF THE SUBMATRICES
C      ADD THESE TO THE PREVIOUS RESULTS TO COMPLETE SYM(I,J) AND ASY(I,J).
C
C      SIGN = 1.0
C      IF(SC.EQ.2)SIGN = -1.0
C      DO 295 K = 1,NSM
C      DO 295 M = 1,NSM
C      IF(M.EQ.K.AND.K.EQ.NSM.AND.EQ.EQ.1)GO TO 295
C      MM = (K - 1)*NOT
C      NN = (M - 1)*NOT
C      SYM(I+MM,J+NN) = SYM(I+MM,J+NN) + SIGN*SUM(NOG-K-M+1)
C      ASY(I+MM,J+NN) = ASY(I+MM,J+NN) - SIGN*SUM(NOG-K-M+1)
C      295 CONTINUE
C      300 CONTINUE
C      RETURN
C      END
C      SUBROUTINE INT(SC,R1,R2)
C
C      EVALUATE THE INTEGRANDS OF THE LINE INTEGRALS WHICH BECOME THE
C      MATRIV ELEMENTS B1(I, 1) AND B2(I,1)

```

```

C C C
INTEGER S,SC,EO
REAL N
COMMON/A,B,P2,PI,R,N,D,DK,NDM,NOT,NSM,NT2,EO
COMMON/C/RH,DR,CL,SL,CS,SS,BJ,BJM,BK,BKM,B1,BIM,L,S
C
XS = S
XL = L
EPS = 1.0
IF(L.GT.0)EPS = 2.0
P = SQRT(P2)
ONP = SORT(1.0 - P2)
TERM = BK*BJM*ONP + P*BJ*BKM
TERM = 2.0*B-RH*TERM
HEM = B1*BJM*ONP - P*BJ*BIM
HEM = P1*B-RH*HEM
IF(L.EQ.S)GO TO 90
DD = L - S
TERM = TERM + 2.0*DD*BK*BJ/PI
HEM = HEM + DD*B1*BJ
90 F = EPS*TERM
H = EPS*HEM
C
C C C
NOTE THAT I=--(L+1) IS NOT INCLUDED IN F
NOTE THAT I=0 IS NOT INCLUDED IN K
C
E = 0.0
G = 0.0
IF(L.EQ.S)GO TO 100
IF(N.EQ.1.0.AND.R.EQ.1.0)GO TO 100
E = 2.0*EPS*BK*BJ/DR/(RH-PI)
G = EPS*B1*B-RH/DR/RH
C
C C C
NOTE THAT I=--(L+1) IS NOT INCLUDED IN E
NOTE THAT I=0 IS NOT INCLUDED IN G
C
100 CONTINUE
C
IF(SC.EQ.2)GO TO 110
TERM = XS*CL*SS - XL*SL*CS
R1 = 4.0*(F*CL*CS + E*TERM)
R2 = 4.0*(H*CL*CS + G*TERM)
GO TO 120
C
110 TERM = XS*SL*CS - XL*SS*CL
R1 = 4.0*(F*SL*SS - E*TERM)
R2 = 4.0*(H*SL*SS - G*TERM)
120 CONTINUE
RETURN
END
SUBROUTINE MATINV(A,N,B,DETER,M)
C
DIMENSION A(30,30),B(30,1),INDEX(30,3)
10 DETER = 1.0
15 DO 20 J = 1,N
20 INDEX(J,3) = 0
30 DO 550 I = 1,N
C
C C C
SEARCH FOR PIVOT ELEMENT
C
40 AMAX = 0.
45 DO 105 J = 1,N
IF (INDEX(J,3))1 60,105,60
60 DO 100 K = 1,N

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AD-A139 604

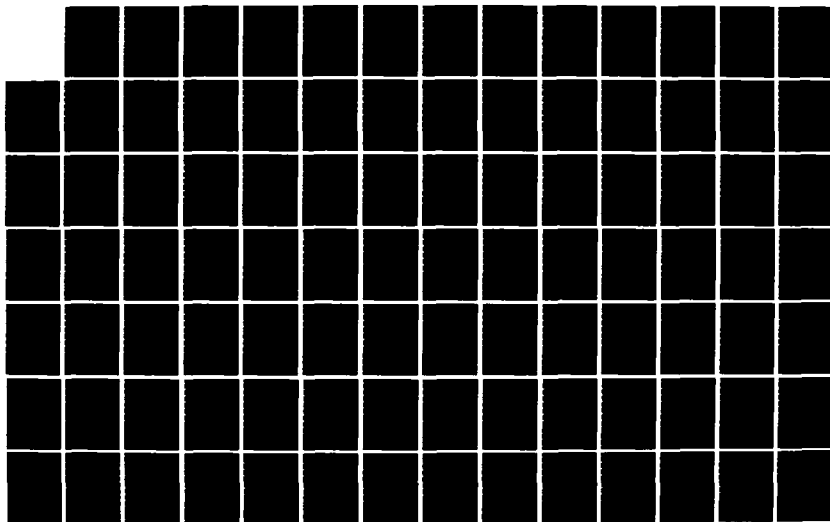
MATHEMATICAL MODELLING OF WAVEGUIDING TECHNIQUES AND  
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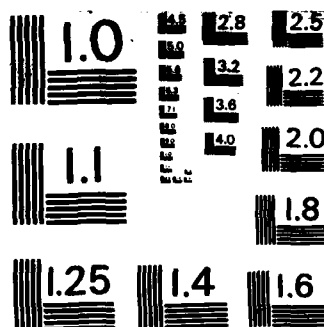
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MICROCOPY RESOLUTION TEST CHART  
NATIONAL BUREAU OF STANDARDS-1963-A

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      Y INDEX(K,3)-1) 80,100,740
80 I = MAX-ABS(A(J,K)) 85,100,100
85 IROW = J
90 ICOL = K
  AMAX = ABS(A(J,K))
100 CONTINUE
105 CONTINUE
  INDEX(ICOL,3) = INDEX(ICOL,3) + 1
260 INDEX(I,1) = IROW
270 INDEX(I,2) = ICOL
C
C INTERCHANGE ROWS TO PUT PIVOT ELEM. IN DIAG.
C
  IF(AMAX)130,999,130
130 IF(IROW-ICOL) 140,310,140
140 DETER = -DETER
150 DO 200 L = 1,N
160 SWAP = A(IROW,L)
170 A(IROW,L) = A(ICOL,L)
200 A(ICOL,L) = SWAP
  IF(M) 310,310,210
210 DO 250 L = 1,M
220 SWAP = B(IROW,L)
230 B(IROW,L) = B(ICOL,L)
250 B(ICOL,L) = SWAP
C
C DIVIDE PIVOT ROW BY PIVOT ELEMENT
C
310 PIVOT = A(ICOL,ICOL)
  DETER = DETER*PIVOT
330 A(ICOL,ICOL) = 1.0
340 DO 350 L = 1,N
350 A(ICOL,L) = A(ICOL,L)/PIVOT
355 IF (M) 380,380,360
360 DO 370 L = 1,M
370 B(ICOL,L) = B(ICOL,L)/PIVOT
C
C REDUCE NON PIVOTS ROWS
C
380 DO 550 L1 = 1,N
390 IF(L1-ICOL) 400,550,400
400 I = A(L1,ICOL)
420 A(L1,ICOL) = 0.
430 DO 450 L = 1,N
450 A(L1,L) = A(L1,L)-A(ICOL,L)*I
455 IF (M) 550,550,460
460 DO 500 L = 1,M
500 B(L1,L) = B(L1,L)-B(ICOL,L)*I
550 CONTINUE
C
C INTERCHANGE COLUMNS
C
600 DO 710 I = 1,N
610 L = N + 1 - I
620 IF(INDEX(L,1)-INDEX(L,2)) 630,710,630
630 JROW = INDEX(L,1)
640 JCOL = INDEX(L,2)
650 DO 705 K = 1,N
660 SWAP = A(K,JROW)
670 A(K,JROW) = A(K,JCOL)
700 A(K,JCOL) = SWAP
705 CONTINUE
710 CONTINUE
740 RETURN
999 DETER = 0.0
  RETURN

```

```

      SUBROUTINE IRECUR(X,NMIN,NMAX,B,OK)
      SUBROUTINE IRECUR CALCULATES THE MODIFIED BESSEL FUNCTIONS IN(X)
      OF ORDER N AND ARGUMENT X FOR N BETWEEN NMIN AND NMAX. IT CALLS
      SUBROUTINE IBESS TO FIND IN(X) FOR N = NMAX AND NMAX - 1, AND THEN
      USES A RECURSION RELATION TO CALCULATE THE LOWER ORDERS. THE
      VECTOR B CONTAINS THE RESULTANT BESSEL FUNCTIONS, WITH THE VECTOR
      INDEX BEING ONE GREATER THAN THE ORDER OF THE FUNCTION STORED AT
      THAT LOCATION.

      DIMENSION B(21)

      IF(NMIN.LT.0)NMIN = 0
      IF(NMAX.GT.20)NMAX = 20
      IF(NMIN.GT.NMAX)NMIN = NMAX
      IF(X.GT.0.0)GO TO 5
      K = NMIN
      IF(NMIN.EQ.0)K = 1
      DO 4 N = K,NMAX
      B(N+1) = 0.0
      IF(NMIN.EQ.0)B(1) = 1.0
      RETURN
      5 EPS = OK/10.
      IF(EPS.LT.1.0E-14)EPS = 1.0E-14
      CALL IBESS(X,NMAX,EPS,BIP,K)
      B(NMAX+1) = BIP
      IF(NMIN.EQ.NMAX)RETURN
      N = NMAX - 1
      CALL IBESS(X,N,EPS,BI,K)
      10 B(N+1) = BI
      IF(NMIN.EQ.N)RETURN
      F = N
      BIM = 2.0*F*BI/X + BIP
      BIP = BI
      BI = BIM
      N = N - 1
      GO TO 10
      END

```

```

      SUBROUTINE IBESS(X,N,EPS,BI,K)
      PETER P. WINTERSTEINER, ARCON CORP., WALTHAM, MA.      8/2/79

      SUBROUTINE IBESS, USED TO CALCULATE THE MODIFIED BESSEL FUNCTION
      IN(X) FROM EITHER AN ASYMPTOTIC SERIES OR AN ASCENDING SERIES,
      DEPENDING ON THE VALUES OF THE ARGUMENT, ORDER, AND ACCURACY DE-
      Sired. THE ASCENDING SERIES IS EQUATION 9.6.10 ON P. 375 OF THE
      NBS HANDBOOK; THE ASYMPTOTIC SERIES IS EQUATION 9.7.1 ON P. 377.

      THE ASCENDING SERIES IS GENERALLY USED FOR SMALL ARGUMENTS. THE
      ASYMPTOTIC SERIES FOR LARGE ARGUMENTS. SPECIFICALLY, THE ASCENDING
      SERIES IS USED IF AND ONLY IF:

```

```

      1) X < 5
      OR 2) X < LEXP + 1      (N = 0 ONLY)
      X < LEXP + ABS(N)

```

```

      WHERE
      X = ARGUMENT OF BESSEL FUNCTION (POSITIVE, REAL)
      N = ORDER OF BESSEL FUNCTION (-100 < N < 100)
      EPS = 10**(-LEXP) = DESIRED ACCURACY (LEXP.LE.14)

```

```

      OTHERWISE THE ASYMPTOTIC SERIES IS FASTER. OTHER ELEMENTS IN THE

```



```

      AM = 1.0D+00
10  K = K + 1
      IS = -IS
      FK = K
      SIGN = IS
      FKN = MU - (2*K-1)**2
      AK = AK*FKN/(FK*8.0D+00*XD)
      SUM = SUM + SIGN*AK
      IF(K.LE.NN)GO TO 10
      FKN = NP - K
      E = DABS(AK*FKN/SUM)
      IF(E.LT.EPS)GO TO 20
      IF(NP.GT.K)GO TO 10
20  IF(E.EQ.0.0)K = K + 400
      FK = DSORT(2.0*PI*XD)
      BI = SUM/FK
      IF(MFF.EQ.0)BI = BI*EXP(X)
      RETURN
C
C
C   ASCENDING SERIES
C
2  NP = 0.5*(SORT(X*X + E*E) - E) + 2.5
   AK = 1.0D+00
   SUM = 1.0D+00
   X2 = XD*XD/4.
C
C   SUM POWER SERIES AT LEAST TO POINT OF DIMINISHING TERMS
C
DO 30 K = 1,NP
   FKN = K
   FKN = K + NN
   AK = AK*X2/(FK*FKN)
30  SUM = SUM + AK
C
C   SUM SERIES UNTIL FRACTIONAL ERROR IS LESS THAN EPS
C
K = NP
35  K = K + 1
   IF(K.GT.500)GO TO 40
   FK = K
   FKN = K + NN
   AK = AK*X2/(FK*FKN)
   SUM = SUM + AK
   E = AK*FK*FKN/((FK*FKN - X2)*SUM)
   IF(E.GE.EPS)GO TO 35
40  CONTINUE
   FK = 1.0D+00
   IF(NN.LE.0)GO TO 50
   DO 45 J = 1,NN
      FKN = J
45  FK = FK*FKN
50  CONTINUE
   BI = SUM*(XD/2.0)**NN/FK
   IF(MFF.EQ.1)BI = BI*EXP(-X)
   RETURN
   END
C
C   SUBROUTINE PERIM(PHI,RHO,DRDP)
C
C   EVALUATE RHO(PHI) ON THE PERIMETER.
C   EVALUATE D(RHO)/D(PHI) ON THE PERIMETER
C
REAL N

```

```

C          IGER EO
C          COMMON/A,B,P2,P1,R,N,D,DK,NOM,NOT,NSM,NT2,EO
C          IF(R.NE.1.0.OR.N.NE.1.0)GO TO 100
C          RHO = 1.0
C          DROP = 0.0
C          RETURN
C
C 100 CONTINUE
C          CP = COS(PH1)
C          SP = SIN(PH1)
C          PN = 2.0*N
C          IF(PN.LT.85.1)GO TO 110
C          DD = PN*ALOG(CP/R)/2.302585
C          IF(DO.LT.-290.1)CP = 0.0
C          DD = PN*ALOG(SP)/2.302585
C          IF(DO.LT.-290.1)SP = 0.0
C 110 CONTINUE
C          DD = (CP/R)*PN + SP*PN
C          PN = 1.0/PN
C          RHO = 1.0/(DO*PN)
C
C          PN = 2.0*N - 2.0
C          AA = SP*PN - ((CP/R)*PN)/(R*R)
C          DRDP = -RHO*SP*CP*AA/DD
C          RETURN
C          END
C          SUBROUTINE BESJ
C
C          PURPOSE
C          COMPUTE THE J BESSEL FUNCTION FOR A GIVEN ARGUMENT AND ORDER
C
C          USAGE
C          CALL BESJ(X,N,BJ,D,IER)
C
C          DESCRIPTION OF PARAMETERS
C          X -THE ARGUMENT OF THE J BESSEL FUNCTION DESIRED
C          N -THE ORDER OF THE J BESSEL FUNCTION DESIRED
C          BJ -THE RESULTANT J BESSEL FUNCTION
C          D -REQUIRED ACCURACY
C          IER-RESULTANT ERROR CODE WHERE
C          IER=0 NO ERROR
C          IER=1 N IS NEGATIVE
C          IER=2 X IS NEGATIVE OR ZERO
C          IER=3 REQUIRED ACCURACY NOT OBTAINED
C          IER=4 RANGE OF N COMPARED TO X NOT CORRECT (SEE REMARKS)
C
C          REMARKS
C          N MUST BE GREATER THAN OR EQUAL TO ZERO, BUT IT MUST BE
C          LESS THAN
C          20+10*X-X** 2/3 FOR X LESS THAN OR EQUAL TO 15
C          90+X/2 FOR X GREATER THAN 15
C
C          SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C          NONE
C
C          METHOD
C          RECURRENCE RELATION TECHNIQUE DESCRIBED BY H. GOLDSTEIN AND
C          R.M. THALER,"RECURRENCE TECHNIQUES FOR THE CALCULATION OF
C          BESSEL FUNCTIONS",M.T.A.C.V.13,PP.102-108 AND I.A. STEGUN
C          AND M. ABRAMOWITZ,"GENERATION OF BESSEL FUNCTIONS ON HIGH
C          SPEED COMPUTERS",M.T.A.C.V.11,1957,PP.255-257

```

```

C      SUBROUTINE BESJ(X,N,BJ,D,IER)
      I = 0
      IF(N.EQ.0.AND.X.EQ.0.)BJ = 1.0
      IER = 0
      IF(X)30,200,10
      10 IF(N)20,31,31
      20 IER = 1
      RETURN
      30 IER=2
      RETURN
      31 IF(X-15.)32,32,34
      32 NTEST=20.*10.-X** 2/3
      GO TO 36
      34 NTEST=90.*X/2.
      36 IF(N-NTEST)40,38,38
      38 IER=4
      RETURN
      40 BPREV = 0.0
      C      COMPUTE STARTING VALUE OF M
      C
      C      IF(X-5.)50,60,60
      50 MA=X+6.
      GO TO 70
      60 MA=1.4*X+60./X
      70 MB=N+FIX(X)/4+2
      MZERO=MAX(MA,MB)
      C      SET UPPER LIMIT OF M
      C
      C      MMAX=NTEST
      100 DO 150 M=MZERO,MMAX,3
      C
      C      SET F(M).F(M-1)
      C
      FM1=1.0E-28
      FM= 0
      ALPHA=.0
      IF(M-(M/2)*2)120,110,120
      110 JT=-1
      GO TO 130
      120 JT=1
      130 M2=M-2
      DO 160 K=1,M2
      MK=M-K
      BMK+2.=FLOAT(MK)*FM1/X-FM
      FM=FM1
      FM1=BMK
      IF(MK-N-1)150,140,150
      140 BJ=BMK
      150 JT=-JT
      S=1+JT
      160 ALPHA=ALPHA+BMK*S
      BMK+2.=FM1/X-FM
      IF(N)180,170,180
      170 BJ=BMK
      180 ALPHA=ALPHA+BMK
      BJ=BJ/ALPHA
      IF(ABS(BJ-BPREV)-ABS(D*BJ))200,200,190
      190 BPREV=BJ
      IER=3
      200 RETURN
      END
      C      SUBROUTINE BESK

```



```

28 P=GO
C      JRN
C      COMPUTE K1 USING POLYNOMIAL APPROXIMATION
29 G1=A*(1.2533141+.4699927*T(1)-.1465583*T(2)+.1280427*T(3)
30 2-.1735432*T(4)+.2847618*T(5)-.4594342*T(6)+.6283381*T(7)
31 3-.6632295*T(8)+.5050239*T(9)-.2581304*T(10)+.07860001*T(11)
32 4-.01082418*T(12))*C
33 IF(N-1)20,30,31
34 BK=G1
35 RETURN
36 FROM KO,K1 COMPUTE KN USING RECURRENCE RELATION
37 RETURN
38 DO 35 J=2,N
39 GJ=2.*(FLOAT(J)-1)*G1/X*GO
40 IF(GJ-1.0E76)33,33,32
41 IER=4
42 GO TO 34
43 GO=G1
44 G1=GJ
45 BK=GJ
46 RETURN
47 B=X/2
48 A=.5772157+ALOG(B)
49 C=B*B
50 IF(N-1)37,43,37
51 COMPUTE K0 USING SERIES EXPANSION
52 GO=-A
53 X2J=1.
54 FACT=1.
55 HJ=0
56 DO 40 J=1,6
57 RJ=1./FLOAT(J)
58 X2J=X2J*C
59 FACT=FACT*RJ*RJ
60 HJ=HJ+RJ
61 GO=GO+X2J*FACT*(HJ-A)
62 IF(N)43,42,43
63 BK=GO
64 RETURN
65 COMPUTE K1 USING SERIES EXPANSION
66 X2J=B
67 FACT=1.
68 HJ=1.
69 G1=1./X+X2J*(.5+A-HJ)
70 DO 50 J=2,8
71 X2J=X2J*C
72 RJ=1./FLOAT(J)
73 FACT=FACT*RJ*RJ
74 HJ=HJ+RJ
75 G1=G1+X2J*FACT*(.5*(A-HJ)+FLOAT(J))
76 IF(N-1)31,52,31
77 BK=G1
78 RETURN
79 END

```

Program Listing

- COAX -

(Reference: Vol. I, Section II.4)

```

PROGRAM COAX(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
PROGRAM COAX FINDS THE MODES OF A 3-MEDIUM DIELECTRIC WAVEGUIDE
OF ARBITRARY CROSS-SECTION. THE WEAKLY-GUIDING CONDITION IS
ASSUMED TO HOLD TRUE. THE INDICES OF REFRACTION OF THE THREE REGIONS
ARE SUCCESSIVELY LESS AS ONE PROGRESSES OUTWARD.

```

```

INTEGER S,SC,FLAG
REAL NL,NL1,N1,N2
DIMENSION PR(10),TABLE(51,3),INDEX(51)
DIMENSION STORE(200,28),INDX(12),GD(24,24),C(24)
DIMENSION ROOT(20,14),ROOM(20,4),IROOT(20,3)
COMMON/A/B,P2,P1,NOT,NOT2,QQ
COMMON/B/STORE,INOX,GD,C,CON1,CON2
DATA (PR(1),1=1,3)/10H CIRCLE,10H ELLIPSE,10H SQUARE /
DATA (PR(1),1=4,7)/10H S' ELLIPSE,10H RECTANGLE,6M SINE /
DATA (PR(1),1=8,9)/10H PLIPIDED,10H CUSPED /

```

# LIST OF VARIABLES WITH SINGLE ASSIGNED PURPOSES

```

ACC ACCURACY TO WITHIN WHICH THE ROOT-LOCATION PROCEDURE FINDS THE ROOTS
B NORMALIZED FREQUENCY (READ IN)
C VECTOR OF COEFFICIENTS IN THE FIELD EXPANSION (DESIRED RESULT)
CON1 FACTOR USED TO KEEP DETERMINANT FROM UNDERFLOWING WHEN QQ >> 1
CON2 FACTOR USED TO KEEP DET FROM OVERFLOWING
DET VALUE OF THE DETERMINANT OF THE MATRIX GD
DNDP D(RHO)/D(PHI) ON THE PERIMETER OF THE GUIDE
FLAG OVERFLOW INDICATOR (SEE SUBROUTINE CHECK) (INTEGER)
GD MATRIX OF COEFFICIENTS
ICON EXPONENT USED TO PRESET CON2 TO 10**(-ICON) (READ IN)
ID # OF FUNCTION EVALUATIONS DURING ROOT-LOCATION PROCEDURE
INDIAG DIAGNOSIS CODE (SEE COMMENTS) (READ IN)
INDEX ARRAY WITH TABLE INDICES NEAR WHICH ROOTS OCCUR; ALSO NOTES OVERFLOW
INX ARRAY FOR STORING VALUES OF INDICES USED IN FIELD EXPANSION
IROOT ARRAY FOR STORING INFORMATION USED IN PRINTING RESULTS AT THE END
JDEX COUNTER TO KEEP TRACK OF THE NUMBER OF SEPARATE CASES COMPLETED
L INDEX OF FIRST TERM IN FIELD EXPANSION (READ IN)
LLS VALUE OF L FROM PREVIOUS CASE (HELPS AVOID REPEATED CALCULATIONS)
NL,N2 SUPERELLIPSE INDICES (REAL) (READ IN)
NL VALUE OF N1 FROM PREVIOUS CASE (HELPS AVOID REPEATED CALCULATIONS)
NLL VALUE OF N2 FROM PREVIOUS CASE (HELPS AVOID REPEATED CALCULATIONS)
NOP # OF INTEGRATION POINTS (READ IN)
NOT # OF TERMS IN EACH SUBMATRIX (READ IN)
NOT2 DIMENSION OF DETERMINANT = 2*NOT
NP2 # OF VALUES OF P2 TAKEN TO CONSTRUCT TABLE (READ IN)
NR # OF ROOTS FOUND
PHI ANGLE IN (RHO, PHI) COORDINATE SYSTEM
PI
P1 LARGEST VALUE OF P2 USED IN TABLE (READ IN)
P2 SMALLEST VALUE OF P2 USED IN TABLE (READ IN)
PMIN ARRAY FOR STORING ALPHANUMERIC PROGRAM INFORMATION
PR PROPAGATION CONSTANT (DESIRED RESULT)
QQ RATIO OF NUMERICAL APERTURES (1-3 TO 1-2) (READ IN)
RHO ASPECT RATIOS (READ IN)
R1,R2 RADIUS IN (RHO, PHI) COORDINATE SYSTEM
RL VALUE OF R1 FROM PREVIOUS CASE (HELPS AVOID REPEATED CALCULATIONS)
RL VALUE OF R2 FROM PREVIOUS CASE (HELPS AVOID REPEATED CALCULATIONS)
ROOM ARRAY FOR STORING OUTPUT INFORMATION
ROOT ARRAY FOR STORING OUTPUT INFORMATION, INCLUDING VALUES OF P2
S INDEX OF LAST TERM IN FIELD EXPANSION (INTEGER)

```

```

C SC INDICATES SINE OR COSINE EXPANSION (INTEGER) (READ IN)
C SW ARRAY FOR STORING FUNCTIONS USED REPEATEDLY, WHICH D2 ONLY ON PHI
C SZ RATIO OF OUTER SEMIMINOR AXIS TO INNER (SZ.GT.1)
C SZL VALUE OF SZ FROM PREVIOUS CASE (HELPS AVOID REPEATED CALCULATIONS)
C TO TABLE INITIAL TABLE OF VALUES OF DETERMINANT FOR DIFFERENT VALUES OF P2
C T0 INITIAL CP TIME
C T1 LATER CP TIME
C .....
100 FORMAT(6I5)
101 FORMAT(////,16X,'PROGRAM COAX-//,6X,DATE:',A10,
1. TIME:',A10)
102 FORMAT(////,1X,12,'--TERM TRUNCATION FOR ALL CASES: NOT2 =,13)
103 FORMAT(' IDIAG = DIAGNOSIS CODE, FOR ALL CASES =,13)
104 FORMAT(' ACC = ACCURACY OF ROOT LOCATION = 1.0E-,12)
105 FORMAT(' NP2 = NUMBER OF INITIAL TABLE POINTS =,13)
106 FORMAT(//,10X,N,6X,52-,8X,DET',/)
107 FORMAT(//,F10.6,E14.7)
108 FORMAT(' NOP = NUMBER OF INTEGRATION POINTS =,13)
109 FORMAT(//,18X,CASE #,12,/,18X,L,14,/,18X,S,14,/,
116X,N1 =,F8.4,/,16X,R1 =,F8.4,/,
216X,N2 =,F8.4,/,16X,R2 =,F8.4,/,16X,SZ =,F8.4,/,
316X,B =,F8.4,/,16X,QQ =,F8.4,/)
110 FORMAT(5X,PMAX =,F6.4,10X,PMIN =,F6.4,/)
111 FORMAT(//)
112 FORMAT(//,1X,16,' EXPANSION: INNER CROSS-SECTION IS A,A10,/,
119X,OUTER CROSS-SECTION IS A,A10)
113 FORMAT(' $$$$$$ ROOT IS EXACT')
114 FORMAT(1X)
115 FORMAT(5X,'EXEC TIME FOR TABLE =,F7.3,' SEC')
116 FORMAT(//,' SUMMARY OF COAX RESULTS:',/)
15X,'SHAPE',19X,N,7X,R,6X,SZ/QQ,5X,B,6X,ROOTS (P2)...')
117 FORMAT(//,13,1X,R9,12,13,2X,AG,F8.4,F7.4,F8.3,F7.3,9F9.5)
118 FORMAT(1X,A3,':',9E14.5,/,6X,9E14.5,/,7X,9E14.5)
121 FORMAT(215,9F5.2,15)
122 FORMAT(//,' COEFFS ARE ',6(A1,A(,12,') =,E11.4,')/,12X,
16IAT,A(,12,') =,E11.4,')
123 FORMAT(' ////////////////////////////////////////')
124 FORMAT(16,' ROOT(S) IDENTIFIED FOR CASE #,13,/)
125 FORMAT(' CASE #,13,' ROOT #,12,': P2 =,F10.8, +/-,E9.3,')
113.' NEW FN EVALS: DET =,E13.5)
126 FORMAT(//,' EXEC TIME THIS CASE =,F7.3,' SEC',/)
127 FORMAT(//,13,1X,R9,12,13,2X,AG,F8.4,F7.4,F8.3,F9.5,7F10.8,/,58X,
1F10.8)
128 FORMAT(//,' $$$$$$ MORE THAN 8 ROOTS, LAST=,13,
1,' NOT LISTED IN SUMMARY',/)
129 FORMAT(4X,R9,13X,F8.4,F7.4,F8.3,/)
130 FORMAT(1X)
131 FORMAT(//,' $$$$$$ ERROR---NOP = 1 BUT CROSS-SECTION IS NOT A ',
1,CIRCLE $$$$$$,/)
132 FORMAT(12,12X,G(A1,C(,12,') =,E11.4,')/,/)
133 FORMAT(//,' $$$$$$ CASE #,12,': NOTE ADJUSTED VALUE OF SZ =,
1F6.3, $$$$$$)
134 FORMAT(' $$$$$$ CASE #,12,': SZ ADJUSTED TO 1.01, UNDERFLOW ',
1,LIKELY $$$$$$)
135 FORMAT(//,' CONT =,E9.3,/)
136 FORMAT(//,40X,'$$$$$ IF JOB BOMBS AT 1ST OR 2ND TABLE ENTRY DUE',
1,' TO OVERFLOW IN MATIN',/,40X,'$$$$$ (CHECK ARRAY DDD IN ',
2,' COMMON/M/ WITH A PMD), TRY OVER AGAIN WITH',/,40X,
3,'$$$$$ ICON >,12, TO PRESET CON2 TO A VALUE SMALLER THAN ',
4,'UNITY: 1.0E-ICON')
137 FORMAT(' CON2 IS INITIALLY 1.0E-,12,/)
C .....
C
C

```

# C INITIALIZE CONSTANTS FOR ENTIRE JOB

```

( 3.1415926535898
2 CALL DATE(PN)
  CALL TIME(W)
  RL = 0.0
  NL = 0.0
  RLL = 0.0
  NLL = 0.0
  LLL = -100
  LLS = -100
  SZL = -100.

```

```

NOT = NUMBER OF TERMS TAKEN BEFORE TRUNCATION
ACCURACY OF HALF-INTERVAL SEARCH IS 10**(-5)
NP2 = NUMBER OF INTERVALS IN THE INITIAL TABLE
NOP = NUMBER OF INTEGRATION POINTS IN THE RANGE (0,PI/2)
IDIAG = DIAGNOSIS CODE: 0 = NORMAL CONDITION (DEFAULT)
                        1 = INITIAL TABLE, FINAL MATRIX PRINTED
                        2 = SUMMARY PRINTED AT END (VALUES OF P2)
                        3 = DETAILS OF ROOT-LOCATION PRINTED
                        4 = UNDERFLOW, OVERFLOW CHECKS PERFORMED
DEF: 1.0E-3
DEF: 10
DEF: 50

```

NOTE: IDIAG = 4 SHOULD BE USED ONLY FOR LARGE VALUES OF NOT WHEN UNDERFLOW OR OVERFLOW IS ANTICIPATED, SINCE IT CAUSES COMPUTATIONS TO TAKE PLACE WHICH ARE UNNECESSARY IN NORMAL CIRCUMSTANCES.

- IF THE CROSS-SECTION IS CIRCULAR, THE PHI-INTEGRALS CAN BE DONE IN CLOSED FORM. TO SAVE TIME, ONE CAN SET NOP = 1, LEAVING ONLY ONE "INTEGRATION POINT". IN THIS CASE ADJUSTMENTS IN THE VALUES OF THE TRIG FUNCTIONS ARE MADE AT STATEMENT 87 SO THAT THE INTEGRALS WILL COME OUT RIGHT. NOTE THAT IF NOP = 1, NOT MUST ALSO EQUAL 1 OR ELSE THE OFF-DIAGONAL MATRIX ELEMENTS MAY BE NONZERO AND THEREFORE INCORRECT.

READ IN INFORMATION WHICH REMAINS FIXED FOR MANY CASES.  
RETURN TO STATEMENT 2 ONLY AFTER DUMPING RESULTS FOR THESE CASES.  
A BLANK CARD HERE TERMINATES PROGRAM EXECUTION.

```

READ(5,100)NOT,S,NP2,NOP,IDIAG
IF(NOT.EQ.0)STOP
WRITE(6,130)
WRITE(6,123)
WRITE(6,101)PN,W
IF(NOP.GT.12)NOT = 12
IF(S.LE.0)S = 3
IF(S.GT.12)S = 12
ACC = 10.0**(-5)
IF(NP2.LT.1)NP2 = 10
IF(NP2.GT.50)NP2 = 50
IF(NOP.GT.200)NOP = 200
IF(NOP.EQ.0)NOP = 50
IF(NOP.EQ.1)OT = 1
NOT2 = 2*NOT
M = -1
IF(IDIAG.GT.3)CALL CHECK(DEF,NOT2,M,NP2,PMIN,1,FLAG,CON2)

```

PRINT THE FIXED INFORMATION READ FROM THE FIRST DATA CARD

```

WRITE(6,102)NOT,NOT2
WRITE(6,104)S
WRITE(6,105)NP2
WRITE(6,108)NOP
WRITE(6,103)IDIAG

```



```

C THE K BESSEL FUNCTIONS, INDIVIDUALLY, MAY BE SMALL WITHOUT UNDERFLOWING,
C THEIR PRODUCTS (IN NLS, THE 2ND QUADRANT OF GD) MAY UNDERFLOW, OR
C (WHOLE DETERMINANT MAY UNDERFLOW, THEREFORE SET THE FIRST, SECOND, OR
C TO A LARGE VALUE AND MULTIPLY ALL BESSEL FUNCTIONS USED IN THE FIRST
C AND SECOND QUADRANTS BY THIS CONSTANT TO KEEP THE DETERMINANT WITHIN
C BOUNDS. IN OTHER WORDS, MULTIPLY THE FIRST NOT HOMOGENEOUS EQUATIONS
C BY CON1**2.
C
62 CON1 = 1.0
W = SZ*W
IF((NOT*W).LT.400.)GO TO 64
CON1 = EXP(W/2.0)
CON1 = ALOG10(CON1)
I = CON1
CON1 = 10.0**I
C
C .....
C DETERMINE S, THE INDEX OF THE LAST TERM IN THE FIELD EXPANSION
C
64 S = L + 2*(NOT-1)
IF(R1.NE.1.0.OR.R2.NE.1.0)GO TO 65
I = L/2
I = 2*I
IF(I.NE.L)GO TO 65
S = L + 4*(NOT-1)
C
C .....
C PRINT INPUT DATA
C
65 WRITE(6,111)
WRITE(6,123)
WRITE(6,111)
WRITE(6,109).DEX,L,S,N1,R1,N2,R2,SZ,B,QQ
IF(PMAX.NE.1.0.OR.PMIN.NE.0.0)WRITE(6,110)PMAX,PMIN
IF(IDIAG.GT.0.AND.ICON.NE.1.)WRITE(6,135)CON1
IF(IDIAG.GT.3.AND.ICON.NE.0)WRITE(6,137)ICON
C
C .....
C INITIALIZE STORAGE ARRAYS FOR THIS CASE
C
DO 70 I = 7,14
70 ROOT(JDEX,1) = 0.0
ROOT(JDEX,3) = N1
ROOT(JDEX,4) = R1
ROOT(JDEX,5) = SZ
ROOT(JDEX,6) = B
ROOM(JDEX,2) = N2
ROOM(JDEX,3) = R2
ROOM(JDEX,4) = QQ
ROOT(JDEX,1) = 0
ROOT(JDEX,2) = L
ROOT(JDEX,3) = S
C
C .....
C IDENTIFY SHAPES OF CROSS-SECTIONS
C
J = SC + 5
I = 4
IF (N1.GT.1.0) GO TO 90
IF (N1-.5) 94,95,96
94 I = 9
GO TO 92

```

```

98 I = 8
99 ( TO 92
96 ( (N1-NE.1.0) GO TO 92
1=2
IF (R1.EQ.1.0) I=1
90 IF (N1.LT.30.) GO TO 92
I=5
IF (R1.EQ.1.0) I=3
92 CONTINUE
W = PR(I)
IF (NOP.EQ.1.AND.I.NE.1) WRITE(6.131)
I=4
IF (N2.GT.1.0) GO TO 50
IF (N2-.5) 54.55.56
94 I=9
GO TO 52
95 I=8
GO TO 52
96 IF (N2.NE.1.0) GO TO 52
I=2
IF (R2.EQ.1.0) I=1
50 IF (N2.LT.30.) GO TO 52
I=5
IF (R2.EQ.1.0) I=3
92 CONTINUE
WRITE(6.112) PR(J), W, PR(I)
IF (NOP.EQ.1.AND.I.NE.1) WRITE(6.131)
ROOT(JDEX,2) = PR(J)
ROOT(JDEX,1) = W
ROOM(JDEX,1) = PR(I)
.....
C
C
C
PN = NOP
W = PI/(PN*2.0)
IF (RL.NE.R1.OR.RLL.NE.R2) GO TO 71
IF (NL.NE.N1.OR.NLL.NE.N2) GO TO 71
IF (SZ.NE.SZL) GO TO 71
GO TO 79
71 CONTINUE
RLL = R2
NLL = N2
RL = R1
NL = N1
SZL = SZ
C
C
C
CALCULATE AND STORE RHO(PHI) AND D(RHO)/D(PHI) FOR EACH BOUNDARY
DO 75 I = 1,NOP
PH = 2*(NOP-I) + 1
PHI = W*PN/2.0
CALL PERIM(N1,R1,PHI,RHO,DRDP)
STORE(1,1) = RHO
STORE(1,2) = DRDP
CALL PERIM(N2,R2,PHI,RHO,DRDP)
STORE(1,3) = RHO*SZ
STORE(1,4) = DRDP*SZ
75 STORE(1,4) = DRDP*SZ
C
C
C
CALCULATE AND STORE COS(M*PHI) AND SIN(M*PHI) WHERE M CAN BE L,.....,S.
VALUES OF M ARE STORED AS INDX.
79 CONTINUE
IF (L.EQ.LLL.AND.S.EQ.LLS) GO TO 89
LLL = L
LLS = S
INDX(1) = L

```

```

      IF(NOT.EQ.1)GO TO 86
      ( = (S - L)/(NOT - 1)
      85 K = 2,NOT
      86 LST = 2*NOT + 4
      IF(NOP.EQ.1)GO TO 87
      DO 80 I = 1,NOP
      PN = 2*(NOP - I) + 1
      PHI = W*PN/2.0
      DO 80 M = 5,LST,2
      K = M/2 - 1
      PN = INDEX(K)
      PN = PN*PHI
      STORE(I,M) = COS(PN)
      80 STORE(I,M+1) = SIN(PN)
      GO TO 89
C
C IF NOP = 1, ASSUME CIRCULAR CROSS-SECTIONS. SET TRIG FUNCTIONS EQUAL TO
C CONSTANTS CHOSEN SO THAT THE ANALYTICAL RESULTS FOR THE INTEGRALS ARE
C OBTAINED.
C
      87 STORE(1,5) = SORT(.5)
      STORE(1,6) = SORT(.5)
      IF(INDEX(1).NE.0)GO TO 89
      STORE(1,5) = 1.0
      .....
C
C CONSTRUCT THE TABLE OF DETERMINANT VALUES FOR PMIN < P2 < PMAX.
C FOR EACH VALUE OF P2, SUBROUTINE CHEBY CALCULATES THE MATRIX
C GD, WHOSE DETERMINANT IS SOUGHT. SUBROUTINE MATIN CALCULATES
C THE DETERMINANT.
C
      89 LST = NP2 + 1
      IF(IDIAG.GT.3)WRITE(6,136)ICON
      IF(IDIAG.GT.0)WRITE(6,106)
      PN = NP2
      PN = (PMAX - PMIN)/PN
      W = PN/100.
      P2 = PMAX
      IF(PMAX.EQ.1.0)P2 = 1.0 - W
      CON2 = 10.0*(-ICON)
      DO 190 M = 1,LST
      190 INDEX(M) = 0
C
      DO 200 M = 1,LST
      CALL CHEBY(NOP,SC)
      CALL MATIN(GD,NOT2,C,DET,0)
      TABLE(M,1) = P2
      TABLE(M,2) = DET
      TABLE(M,3) = CON2
      IF(IDIAG.GT.0)WRITE(6,107)M,P2,DET
      P2 = PMAX - M*PN
      IF(P2.LT.W)P2 = W
C
      CHECK FOR OVERFLOW, UNDERFLOW, IF DESIRED
C
      IF(IDIAG.LE.3)GO TO 200
      CALL CHECK(DET,NOT2,M,NP2,PMIN,1,FLAG,CON2)
      INDEX(M) = 1
      IF(FLAG.EQ.2)GO TO 205
      200 CONTINUE
      GO TO 208
      205 LST = M
      208 IF(IDIAG.GT.0)WRITE(6,114)
C

```

[illegible]

```

C COEFFICIENTS. THE FINAL COEFFICIENTS ARE THE COEFFICIENTS OF THE I AND M
C / SEL FUNCTIONS IN REGION 2. TO GET THE COEFFICIENTS OF  $x^j$ , j = J AND M
C / SEL FUNCTIONS, IT IS NECESSARY TO MULTIPLY BY COMPLEX CONJUGATES.
C
DO 255 K = 1,NOT
  J = INDX(K)
  PN = (-I,O)+*J
  C(K) = PNC(C(K))
255 C(K+NOT) = 2.O*PN-C(K+NOT)/PI
  PN = IH
  WRITE(6,122)((PN,INDX(K),C(K)),K=1,NOT)
  J = NOT + 1
  WRITE(6,132)((PN,INDX(K-NOT),C(K)),K=J,NOT2)
  WRITE(6,114)
  IF(DET.EQ.O.O)WRITE(6,113)
250 CONTINUE
C
CALL SECOND(T1)
T1 = T1 - T0
WRITE(6,126)T1
GO TO 1
.....
AFTER ALL CASES (20 OR LESS) HAVE BEEN COMPLETED, PRINT A TABLE
OF RESULTS GIVING THE ROOTS FOR EACH CASE.
.....
999 WRITE(6,123)
    WRITE(6,123)
    WRITE(6,123)
    WRITE(6,116)
    DO 900 I = 1,JDEX
      K = IROOT(I,1) + 6
      IF(IDJAG.LE.1)WRITE(6,117)I,I,ROOT(I,1),(IROOT(I,M),M=2,3),
        1(ROOT(I,J),J=2,K)
      IF(IDJAG.GE.2)WRITE(6,127)I,I,ROOT(I,1),(IROOT(I,M),M=2,3),
        1(ROOT(I,J),J=2,K)
    900 WRITE(6,129)(ROOM(I,M),M=1,4)
    GO TO 2
END
SUBROUTINE CHECK(DET,NOT2,M,NP2,PMIN,UFLAG,CON2)
SUBROUTINE CHECK CHECKS TABLE ENTRIES TO DETERMINE IF:
1) A ZERO ENTRY IS AN ACTUAL ROOT OR AN UNDERFLOW ERROR
2) OVERFLOW IS LIKELY TO OCCUR IN THE NEXT TABLE ENTRY
IN THE LATTER CASE, IT REDEFINES (IF POSSIBLE) THE FACTOR CON2 IN AN
ATTEMPT TO SCALE THE DETERMINANT (BY AN AMOUNT CON2**NOT2) DOWN TO
SMALLER VALUES AND THUS PREVENT OVERFLOW. OVERFLOW ACTUALLY OCCURS IN
INTERMEDIATE STEPS OF THE DETERMINANT DETERMINATION.
.....
NOTE THAT THE OVERFLOW-AVOIDANCE PROCEDURE IS NOT FOOLPROOF; THE ANTI-
CIPATION OF OVERFLOW RESULTS WHEN A POLYNOMIAL FIT TO POINTS COMPRISING
THE LOGARITHMS OF THE LARGEST INTERMEDIATE VALUES OF DETER IN SUBROU-
TIME MATIN FOR PREVIOUS TABLE ENTRIES, IS EXTRAPOLATED TO THE NEXT EN-
TRY AND EXCEEDS 320 AT THAT POINT. HOWEVER THE APPROXIMATION AFFORDED
BY THE POLYNOMIAL CAN IN SOME CIRCUMSTANCES BE AN UNDERESTIMATE OF THE
ACTUAL NEXT ENTRY AND THEREFORE FAIL TO ANTICIPATE AN ACTUAL OVERFLOW
SITUATION.
.....
INTEGER UF,FLAG

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```

C      DIMENSION DDD(24),PVT(24),F(4)
      MON/M/DDD,PVT

C      100 FORMAT(40X,'$$$$$ THIS TABLE ENTRY UNDERFLOWS. SIGN OF DET',
101   '1. WOULD BE ',A4,'-TIVE $$$$$$')
101   FORMAT(40X,'$$$$$ ANTICIPATE OVERFLOW ON NEXT TABLE ENTRY. RE',
102   '1-SET CON2 = 1.00E-14./40X,'$$$$$ 36X,'TOTAL FACTOR IS NOW ',
103   '2.1.00E-14)
102   FORMAT(40X,'$$$$$ 21X,'FO...FN =',A4,'(F6.1),F6.1)
103   FORMAT(40X,'$$$$$ ANTICIPATE UNAVOIDABLE OVERFLOW ON NEXT TABLE',
104   '1. ENTRY. EXIT LOOP.'/40X,'$$$$$ RESET LST TO',A4,'.')
104   FORMAT(40X,'$$$$$ THIS PROBABLY WON'T PROVIDE A SUFFICIENT',
105   '1. CUSHION WITH F3 =',F6.1)

C      IF(M.LT.0)GO TO 520
      DTT = ABS(DET)
      UF = 0

C      CHECK UNDERFLOW POSSIBILITY. WE ASSUME THAT NONE OF THE PIVOT ELEMENTS
C      (ARRAY PVT) USED IN MATIN UNDERFLOW, BUT THAT THE PRODUCT OF ALL PIVOT
C      ELEMENTS (THAT IS, THE DETERMINANT) MAY UNDERFLOW. (THE RUNNING PRODUCT
C      IS STORED IN ARRAY DDD.) THEREFORE PIVOT ELEMENTS 1 TO NOT2 ARE ALL DE-
C      FINED AND NONZERO WHEN THERE IS AN UNDERFLOW PROBLEM. WHEN THE DETERMI-
C      NANT OF THE MATRIX IS TRULY ZERO, THE LAST PIVOT ELEMENT IS NOT DEFINED-
C      SINCE THE EXIT TO STATEMENT 750 FROM STATEMENT 105 (IN MATIN) BYPASSES
C      THE DEFINITION OF PIVOT. (HERE, 'UNDEFINED' MEANS HAVING A VALUE EQUAL
C      TO RST.)

C      IF UNDERFLOW OCCURS, SET UF = 1 AND CHECK TO SEE WHAT SIGN THE DETERMI-
C      NANT WOULD HAVE HAD. MAKE ADJUSTMENTS IN MAIN.

      IF(DTT.NE.0.0)GO TO 300
      IF(PVT(NOT2).EQ.RST)GO TO 300
      UF = 1
      DO 200 I = 1,NOT2
      IF(DDD(I).NE.0.0)GO TO 200
      DDD(I) = DDD(I-1)*(PVT(I)/ABS(PVT(I)))
200   CONTINUE
      CD = 4*POST
      IF(DDD(NOT2).LT.0.0)CD = 4*HNEGA
      WRITE(6,100)CD

C      CHECK OVERFLOW POSSIBILITIES. OVERFLOW CAN OCCUR IN AN INTERMEDIATE STEP
C      OF MATIN. SO WE SEARCH FOR THE LARGEST VALUE THAT DDD TAKES ON FOR EACH
C      TABLE ENTRY (DMAX) AND TRY TO ANTICIPATE IF, FOR THE NEXT TABLE ENTRY,
C      THIS VALUE WILL BECOME TOO LARGE TO HANDLE. IF OVERFLOW SEEMS LIKELY,
C      CON2 IS DETERMINED TO BE THE SMALLEST POSITIVE NUMBER OF THE FORM 10**K
C      (K INTEGER) WHICH STILL WILL BE LARGE ENOUGH TO PREVENT UNDERFLOW WHERE-
C      EVER (INCLUDING SUBROUTINE RTT) IT MIGHT OCCUR.

C      300 IF(M.GT.NP2)GO TO 500
      DMAX = -RST
      DO 320 I = 1,NOT2
      CD = ABS(DDD(I))
      IF(CD.LT.DMAX)GO TO 320
      DMAX = I
      DMAX = CD
320   CONTINUE
      FF = ALOG10(DMAX)
      IF(M.GT.1)GO TO 330
      FLAG = 0
      DO 325 I = 1,4
      F(I) = FF
      FN = FF
      FAC = 1.0
      IF(PMIN.EQ.0..AND.M.EQ.NP2)FAC = 3.0
325   F(I) = FF

```

```

330 L = M
      ( L.GT.5) L = 5
      TO(360,340,350,370,380).L
C
C FORWARD-DIFFERENCE FORMULAE FOR EXTRAPOLATION TO NEXT TABLE ENTRY
C
340 FN = 2.0*FAC*FF - F(1)
      GO TO 380
350 FN = 3.0*FAC*FF - 3.0*F(2) + F(1)
      GO TO 380
360 L = 4
      DO 385 I = 1,3
385 F(I) = F(I+1)
370 FN = 4.0*FAC*FF - 6.0*F(3) + 4.0*F(2) - F(1)
380 F(L) = FF
      IF(FN.LT.320.)GO TO 500
C
C DEAL WITH OVERFLOW ANTICIPATED IN THE NEXT TABLE ENTRY BY REDEFINING
C CON2. CHECK CON2 TO SEE IF THIS HELPS. IF NOT, GIVE UP.
C
      IF(DTT.EQ.0)DTT = 1.0E-293
      CD = ALOG10(DTT)
      K = CD
      IF(K.GT.12)K = 12
      K = (280 + K)/NOT2
      IF(K.LE.0)GO TO 410
      CON2 = CON2*10.0**(-K)
      FLAG = 1
      CD = ALOG10(CON2) - .0001
      J = CD
      I = J*NOT2
      WRITE(6,101)C,I
      WRITE(6,102)(F(J),J=1,L).FN
      FN = FN - J*MAX*K
      IF(FN.GT.320.)GO TO 400
      DO 390 J = 1,L
390 F(J) = F(J) - J*MAX*K
      GO TO 500
C
C CAN'T DO MUCH ABOUT ANTICIPATED OVERFLOW. RESET LST IN MAIN.
C
400 WRITE(6,104)FN
410 FLAG = 2
      WRITE(6,103)M
      IF(K.LE.0)WRITE(6,102)(F(J),J=1,L).FN
C
C RESET ARRAYS ODD AND PVT
C
500 DO 510 I = 1,NOT2
510 PVT(I) = RST
      RETURN
C
520 RST = (1./3.)*1.0E+321
      GO TO 500
      END
      SUBROUTINE RTT(TABLE,ACC,PN,NOP,SC,M,ID,IDIAG)
C
C SUBROUTINE RTT PRECISELY LOCATES THE ROOT SPECIFIED BY M (USING
C ELEMENTS M AND M+1 IN THE ARRAYS TABLE(M,1) AND TABLE(M,2)). AND THEN
C DETERMINES THE FIELD EXPANSION COEFFICIENTS C(J) CORRESPONDING TO
C THAT ROOT. ACC IS THE DESIRED ACCURACY OF THE ROOT. AND PN IS THE
C DIFFERENCE IN THE ACTUAL BOUNDS OBTAINED. ID IS THE NUMBER OF FUN-
C CTION EVALUATIONS NECESSARY. SC, IDIAG, AND NOP ARE AS IN MAIN. GO, THE
C FINAL MATRIX AT THE ROOT P2, IS ALSO RETURNED. (C, GO, AND P2 ARE ALL

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```

C      IN COMMON BLOCKS.)
C      (
C      IER SC
C      DIMENSION STORE(200,28),INDX(12),GD(24,24),C(24),SINV(24,24)
C      DIMENSION TABLE(S1,3)
C      COMMON/A/B,P2,P1,NOT,NOT2,GD
C      COMMON/B/STORE,INDX,GD,C,CON1,CON2
C
C 700 FORMAT(/, 'SEARCH ID=7X, X1=,11X, X2=,10X, F1=,9X, F2=,11X, DX=,
19X, P2=,8X, F=')
C 701 FORMAT(/, ' $$$$ ERROR IN HINTS OR REG, IER =,13, $$$$')
C
C      IF(IDIAG.GE.3)WRITE(6,700)
C      CON2 = TABLE(M+1,3)
C      K = 0
C      ID = 0
C      IF(M.LT.1000)GO TO 230
C      P2 = TABLE(M-1000,1)
C      CON2 = TABLE(M,3)
C      CALL CHEBY(NOP,SC)
C      ID = 1
C      PN = 0.0
C      GO TO 248
C
C 230 G1 = TABLE(M+1,2)
C      G2 = TABLE(M,2)
C      X1 = TABLE(M+1,1)
C      X2 = TABLE(M,1)
C      DN = TABLE(M,3)
C      IF(DN.NE.CON2)G2 = G2*((CON2/DN)**NOT2)
C      DN = X2 - X1
C
C      IF G1 AND G2 ARE TWO OR MORE ORDERS OF MAGNITUDE DIFFERENT, USE
C      A HALF-INTERVAL SEARCH TO NARROW THE RANGE X1 TO X2
C
C 235 NW = -G1/G2
C      IF(DN.LT.100..AND.NW.GT..01)GO TO 240
C      IF(DN.LT..ACC)GO TO 240
C      DN = DN/3.0
C
C      CALL HINTS(X1,X2,G1,G2,DN,PN,NOP,SC,ID,IDIAG,IER)
C      IF(IER.GT.0)WRITE(6,701)IER
C
C      K = K + ID
C      IF(PN.EQ.0.0)GO TO 248
C      GO TO 235
C
C      MODIFIED REGULA-FALSI METHOD TO GET THE ROOT PRECISELY
C
C 240 CALL REG(X1,X2,G1,G2,ACC,PN,NOP,SC,ID,IDIAG,IER)
C      IF(IER.GT.0)WRITE(6,701)IER
C
C 248 ID = ID + K
C
C      CALCULATE THE COEFFICIENTS. SUBROUTINE LOAD COPIES (NOT2-1) OF THE
C      NOT2 ROWS OF GD INTO ARRAY SINV AND VECTOR C. SUBROUTINE MATIN SOLVES
C      THE (NOT2-1)X(NOT2-1) SYSTEM FOR THE COEFFICIENTS AND RETURNS THEM IN C.
C
C      CALL LOAD(GD,SINV,C,NOT2,M)
C      LST = NOT2 - 1
C      CALL MATIN(SINV,LST,C,DN,1)
C
C      STACK THE COEFFICIENTS IN THE PROPER ORDER IN VECTOR C, INCLUDING 1.000
C
C      IF(M.EQ.NOT2)GO TO 280
C      DO 270 J = M,LST

```

```

      K = NOT2 + M - J
270 ( ) = C(K-1)
280 (,4) = 1.0
      RETURN
      END
      SUBROUTINE LOAD(SS,SINV,C,NOT,1D)
      C
      C SUBROUTINE LOAD LOADS SS (NEE GO) INTO ARRAY SINV, ELIMINATING
      C THE ROW AND COLUMN CONTAINING THE DIAGONAL ELEMENT WHICH IS SMALLEST
      C IN MAGNITUDE. THE NEGATIVE OF THE COLUMN OF SS WHICH IS LEFT OUT IS
      C LOADED INTO ARRAY C. EXCEPT FOR THE ELEMENT IN THE ROW WHICH IS ELIM-
      C INATED. THUS, SINV IS (NOT-1) BY (NOT-1) IN DIMENSION, AND C IS A VEC-
      C TOR (NOT-1) IN LENGTH.
      C
      C REMEMBER, NOT IS WHAT IS CALLED NOT2 IN COAX. ALSO, ONLY THE FIRST
      C FOUR ELEMENTS OF EACH HALF OF THE DIAGONAL ARE SEARCHED FOR THE
      C SMALLEST VALUE.
      C
      C DIMENSION SS(24,24),SINV(24,24),C(24)
      C G = 1.0E+321
      C 1D = 1
      C K = NOT/2
      C IF(K.GT.4)K = 4
      C DO 200 J = 1,K
      C H = ABS(SS(J,J))
      C IF(H.GT.G)GO TO 200
      C G = H
      C 1D = J
      C 200 CONTINUE
      C
      C JJ = NOT/2 + 1
      C K = JJ + 3
      C IF(K.GT.NOT)K = NOT
      C DO 210 J = JJ,K
      C H = ABS(SS(J,J))
      C IF(H.GT.G)GO TO 210
      C G = H
      C 1D = J
      C 210 CONTINUE
      C
      C JJ = 0
      C DO 300 J = 1,NOT
      C IF(J.EQ.1D)GO TO 300
      C JJ = JJ + 1
      C C(JJ) = -SS(J,1D)
      C KK = 0
      C DO 299 K = 1,NOT
      C IF(K.EQ.1D)GO TO 299
      C KK = KK + 1
      C SINV(JJ,KK) = SS(J,K)
      C 299 CONTINUE
      C 300 CONTINUE
      C RETURN
      C END
      C SUBROUTINE HINTS(X1,X2,G1,G2,ACC,PN,NOP,SC,1D,1DIAG,1ER)
      C
      C SUBROUTINE HINTS PERFORMS A HALF-INTERVAL SEARCH TO LOCATE A ROOT
      C OF THE EQUATION DET(RIS) = 0 TO WITHIN A DESIRED ACCURACY ACC.
      C 1D IS THE NUMBER OF BISECTIONS, AND PN IS THE ACTUAL MAXIMUM ERROR.
      C
      C INTEGER SC
      C DIMENSION STORE(200,28),INDX(12),GD(24,24),C(24)
      C COMMON/A/B,P2,P1,NOT,NOT2,QQ
      C COMMON/B/STORE,INDX,GD,C,CON1,CON2
      C
      C 1D = 0

```

```

100 IER = 0
C
100 ( = ABS(X2 - X1)
IF(PN.LT.ACC)RETURN
ID = ID + 1
IF(ID.GT.25)GO TO 230
P2 = (X1 + X2)/2.0
CALL CHEBY(NOP,SC)
CALL MATIN(GD,NOT2,C,G,0)
C
100 IF(IDIAG.GE.3)WRITE(6,700)ID,X1,X2,G1,G2,PN,P2,G
700 FORMAT(' MINTS:',I3,2F13.10,3E11.4,F13.10,E11.4)
C
100 IF(G*(G1/ABS(G1)))120,210,130
120 X2 = P2
G2 = G
GO TO 100
130 X1 = P2
G1 = G
GO TO 100
C
C IF THE DETERMINANT IS EXACTLY ZERO, THE MATRIX (WHICH WILL HAVE BEEN
C ALTERED IN MATIN) IS RECOMPUTED.
C
210 PN = 0.0
CALL CHEBY(NOP,SC)
ID = ID + 1
RETURN
C
C ERROR RETURN: TOO MANY ITERATIONS
C
230 IER = 6
RETURN
END
SUBROUTINE REG(X1,X2,F1,F2,ACC,PN,NOP,SC,ID,IDIAG,IER)
C
C SUBROUTINE REG SOLVES THE EQUATION DET(RLS) = 0 FOR P2 TO WITHIN
C A DESIRED ACCURACY ACC BY THE MODIFIED REGULA-FALSI METHOD. ID IS
C THE NUMBER OF FUNCTION EVALUATIONS AND PN GIVES THE ACTUAL MAXIMUM ERROR.
C
C INTEGER SC
C DIMENSION STORE(200,28),INDX(12),GD(24,24),C(24)
C COMMON/A,B,P2,P1,NOT,NOT2,OO
C COMMON/B/STORE,INDX,GD,C,CON1,CON2
C
C ID = 0
C IER = 0
C G = F1
100 PN = ABS(X1 - X2)
C
C F0 = G/ABS(G)
P2 = (F2*X1 - F1*X2)/(F2 - F1)
CALL CHEBY(NOP,SC)
ID = ID + 1
IF(PN.LT.ACC)RETURN
IF(ID.GT.25)GO TO 230
CALL MATIN(GD,NOT2,C,G,0)
C
100 IF(IDIAG.GT.2)WRITE(6,700)ID,X1,X2,F1,F2,PN,P2,G
700 FORMAT(' REC: ',I3,2F13.10,3E11.4,F13.10,E11.4)
C
C IF(G*(F1/ABS(F1)))150,190,160
150 X2 = P2
F2 = G
IF((F0*G).GT.0.0)F1 = F1/2.0
GO TO 100

```

```

180 X1 = P2
   ( .G
      ( (FO-G).GT.0.0)F2 = F2/2.0
      GO TO 100
C
C IF THE DETERMINANT IS EXACTLY ZERO, THE MATRIX (WHICH WILL HAVE
C BEEN ALTERED IN MATIN) IS RECOMPUTED.
C
190 PN = 0.0
   CALL CHEBY(NOP,SC)
   ID = ID + 1
   RETURN
C
C ERROR RETURN: TOO MANY ITERATIONS
C
230 IER = 5
   RETURN
   END
C
C SUBROUTINE CHEBY(NOP,SC)
C
C INTEGER S,SC
C REAL MLS,NLS
C DIMENSION STORE(200,28),INDX(12),GD(24,24),C(24)
C DIMENSION GDM(12,12),GDM1(12,12),GDP(12,12),GDO(12,12)
C DIMENSION BIST1(49),BIST2(49),BJST(49),BKST1(49),BKST2(49)
C DIMENSION BKST3(49)
C COMMON/A,B,P2,P1,NOT,NOT2,QQ
C COMMON/R/STORE,INDX,GD,C,CON1,CON2
C
C CHEBYSHEV INTEGRATION OF THE EXTERNAL FUNCTION G(PHI)
C
NMAX = INDX(NOT) + 1
IF(NMAX.EQ.1)NMAX = 2
DR = 0.0
NN = 0
PTS = NOP
W = PI/(PTS+2.0)
P = SORT(P2)
OMP = SORT(1.0 - P2)
DO 50 I = 1,NOT
DO 50 J = 1,NOT
GDM(I,J) = 0.0
GDM1(I,J) = 0.0
GDP(I,J) = 0.0
GDO(I,J) = 0.0
50 GDO(I,J) = 0.0
C
C LOOP DO 100 DOES INTEGRATION USING NOP POINTS IN THE RANGE (0,PI/2)
C
DO 100 JJJ = 1,NOP
OBTAIN RHO(PHI), COS(PHI), AND SIN(PHI)
RHI = STORE(JJJ,1)
RH2 = STORE(JJJ,3)
DR1 = STORE(JJJ,2)
DR2 = STORE(JJJ,4)
C
C AK1R1=K1*RHO1 FOR SURFACE 1-2
C AK2R1=K2*RHO1 FOR SURFACE 1-2
C AK3R2=K3*RHO2 FOR SURFACE 2-3
C AK1R1 = PI*B*RHI*OMP
C AK2R1 = PI*B*RHI*P
C AK3R2 = PI*B*RH2*P

```

```

C      PI=8*RM2*SQR(00 - 1.0 + P2)
C      OBTAIN THE BESSEL FUNCTIONS WHICH WILL BE REQUIRED
C
C      IS(AK2R1) IS STORED IN VECTOR BIST1:  BIST1(1) = 10, BIST1(2) = 11, ETC.
C      IS(AK2R2) IS STORED IN VECTOR BIST2
C      JL(AK1R1) IS STORED IN VECTOR BJST
C      JS(AK2R1) IS STORED IN VECTOR BKST1
C      KS(AK2R2) IS STORED IN VECTOR BKST2
C      KL(AK3R2) IS STORED IN VECTOR BKST3
C
C      CALL BES1(AK2R1,0R,1,NMAX,BIST1,NZ)
C      CALL BES1(AK2R2,0R,1,NMAX,BIST2,NZ)
C      CALL BESJ(AK1R1,0R,NMAX,BJST,NZ)
C      CALL BESK(AK2R1,NH,1,NMAX,BKST1,NZ)
C      CALL BESK(AK2R2,NH,1,NMAX,BKST2,NZ)
C      CALL BESK(AK3R2,NH,1,NMAX,BKST3,NZ)
C
C      LOOP DO 100 I... CORRESPONDS TO L
C      LOOP DO 100 J... CORRESPONDS TO S
C
C      DO 100 I = 1,NOT
C      L = INDX(I)
C      XL = L
C
C      OBTAIN COS(L*PHI) AND SIN(L*PHI)
C
C      CL = STORE(JJJ,2*1+3)
C      SL = STORE(JJJ,2*1+4)
C      LL = L - 1
C      IF(L.EQ.0)LL = 1
C
C      RETRIEVE THE BESSEL FUNCTIONS OF ORDER L AND L-1
C
C      BK3 = CON1*BKST3(LL+1)
C      BK3M = CON1*BKST3(LL+1)
C      BJ = BJST(LL+1)
C      BJM = BJST(LL+1)
C      IF(L.EQ.0)BJM=-BJM
C
C      DO 100 J = 1,NOT
C      S = INDX(J)
C      XS = S
C
C      OBTAIN COS(S*PHI) AND SIN(S*PHI)
C
C      CS = STORE(JJJ,2*J+3)
C      SS = STORE(JJJ,2*J+4)
C
C      RETRIEVE THE BESSEL FUNCTIONS OF ORDER S AND S-1
C
C      LL = S - 1
C      IF(S.EQ.0)LL = 1
C      B12 = CON1*BIST2(S+1)
C      B12M = CON1*BIST2(LL+1)
C      B11 = BIST1(S+1)
C      B11M = BIST1(LL+1)
C      B12M = CON1*BKST2(S+1)
C      B12M = CON1*BKST2(LL+1)
C      BK1 = BKST1(S+1)
C      BK1M = BKST1(LL+1)
C
C      TM1 = AK2R2*B12M*BK3 + AK3R2*B12*BK3M
C      TM2 = B12*BK3
C      TN1 = -AK2R2*BK2M*BK3 + AK3R2*BK2*BK3M
C      TN2 = BK2*BK3

```

```

      101 = AK1R1*BJ*BJ*BI1 - AK2R1*BJ*BJ*BI1M
      102 = -BJ*BI1
      103 = AK1R1*BJ*BJ*BK1 + AK2R1*BJ*BJ*BK1M
      104 = -BJ*BK1
C
      IF(SC.EQ.2) GO TO 250
      ZETA1 = 0.0
      ZETA2 = 0.0
      IF (L.EQ.5) GO TO 80
      ZETA1 = (XL - XS)*CL*CS + (XS*CL*SS - XL*SL*CS)*DR1/RH1
      ZETA2 = (XL - XS)*CL*CS + (XS*CL*SS - XL*SL*CS)*DR2/RH2
      80 MLS = TM1*CL*CS + TM2*ZETA2
      NLS = TM1*CL*CS + TM2*ZETA2
      PLS = TP1*CL*CS + TP2*ZETA1
      QLS = TQ1*CL*CS + TQ2*ZETA1
      GO TO 91
C
      250 ZETA1 = 0.0
      ZETA2 = 0.0
      IF(L.EQ.5) GO TO 81
      ZETA1 = (XL - XS)*SL*SS + (XL*CL*SS - XS*SL*CS)*DR1/RH1
      ZETA2 = (XL - XS)*SL*SS + (XL*CL*SS - XS*SL*CS)*DR2/RH2
      81 MLS = TM1*SL*SS + TM2*ZETA2
      NLS = TM1*SL*SS + TM2*ZETA2
      PLS = TP1*SL*SS + TP2*ZETA1
      QLS = TQ1*SL*SS + TQ2*ZETA1
C
      91 GDM(I,J) = GDM(I,J) + MLS
      GDN(I,J) = GDN(I,J) + NLS
      GDP(I,J) = GDP(I,J) + PLS
      GDO(I,J) = GDO(I,J) + QLS
      100 CONTINUE
C
      C LOAD THE FOUR SUBMATRICES INTO GD
C
      DO 105 J = 1,NOT2
      DO 105 I = 1,NOT2
      IF(I.LE.NOT.AND.J.LE.NOT)GD(I,J) = GDM(I,J)
      IF(I.LE.NOT.AND.J.GT.NOT)GD(I,J) = GDN(I,J)
      IF(I.GT.NOT.AND.J.LE.NOT)GD(I,J) = GDP(I,J)
      IF(I.GT.NOT.AND.J.GT.NOT)GD(I,J) = GDO(I,J)
      GD(I,J) = GD(I,J)*W*CON2
      105 CONTINUE
C
      RETURN
      END
      SUBROUTINE PERIM(N,R,PHI,RHO,DRDP)
C
      EVALUATE RHO(PHI) AND D(RHO)/D(PHI) ON THE PERIMETER
C
      REAL N
C
      IF(R.NE.1.0.OR.N.NE.1.0)GO TO 100
      RHO = 1.0
      DRDP = 0.0
      RETURN
C
      100 CONTINUE
      CP = COS(PHI)
      SP = SIN(PHI)
      PN = 2.0*N
      DD = (CP/R)*PN + SP*PN
      PN = 1.0/PN
      RHO = 1.0/(DD*PN)
C
      PN = 2.0*N - 2.0

```

```

      S = SP*OPN - ((CP/R)*OPN)/(R*OP)
      P = -RHO*SP*CP*AA/DO
      RETURN
END
SUBROUTINE MATIN(A,N,B,DETER,M)
  DIMENSION A(24,24),B(24,1),INDEX(24,3)
  DIMENSION DDD(24),PVT(24)
  COMMON/M/DDO,PVT
  DETER = 1.0
  DO 20 J = 1,N
    INDEX(J,3) = 0
  DO 550 I = 1,N
    SEARCH FOR PIVOT ELEMENT
    AMAX = 0.
    DO 105 J = 1,N
      IF (INDEX(J,3)-1) 60,105,60
    DO 100 K = 1,N
      IF (INDEX(K,3)-1) 80,100,740
    80 IF(AMAX-ABS(A(J,K))) 85,100,100
    85 IROW = J
    ICOL = K
    AMAX = ABS(A(J,K))
  100 CONTINUE
  105 CONTINUE
  IF(AMAX.EQ.0.0)GO TO 750
  INDEX(ICOL,3) = INDEX(ICOL,3) + 1
  INDEX(I,1) = IROW
  INDEX(I,2) = ICOL
  INTERCHANGE ROWS TO PUT PIVOT ELEM.IN DIAG.
  S = 1.0
  IF(IROW-ICOL) 140,310,140
  140 DETER = -DETER
  F = -1.0
  DO 200 L = 1,N
    SWAP = A(IROW,L)
    A(IROW,L) = A(ICOL,L)
    A(ICOL,L) = SWAP
  IF(M) 310,310,210
  210 DO 250 L = 1,M
    SWAP = B(IROW,L)
    B(IROW,L) = B(ICOL,L)
    B(ICOL,L) = SWAP
  250 B(ICOL,L) = SWAP
  DIVIDE PIVOT ROW BY PIVOT ELEMENT
  310 PIVOT = A(ICOL,ICOL)
  DETER = DETER*PIVOT
  DDD(I) = DETER
  PVT(I) = PIVOT*S
  A(ICOL,ICOL) = 1.0
  DO 350 L = 1,N
    A(ICOL,L) = A(ICOL,L)/PIVOT
  IF(M) 380,380,360
  360 DO 370 L = 1,M
    B(ICOL,L) = B(ICOL,L)/PIVOT
  370 B(ICOL,L) = B(ICOL,L)/PIVOT
  REDUCE NON PIVOTS ROWS
  DO 550 LI = 1,N
    IF(LI-ICOL) 400,550,40

```

```

400 I A(L1, ICOLUM)
      I, ICOLUM) = 0.
DU 450 L = 1, N
450 A(L1, L) = A(L1, L) - A(ICOLUM, L) * I
      IF(M) 550, 550, 460
460 DO 500 L = 1, M
500 B(L1, L) = B(L1, L) - B(ICOLUM, L) * I
550 CONTINUE
C
C INTERCHANGE COLUMNS
C
DO 710 I = 1, N
  L = N + 1 - I
  IF (INDEX(L, 1) - INDEX(L, 2)) 630, 710, 630
630 JROW = INDEX(L, 1)
  JCOLUM = INDEX(L, 2)
  DO 705 K = 1, N
    SWAP = A(K, JROW)
    A(K, JROW) = A(K, JCOLUM)
    A(K, JCOLUM) = SWAP
705 CONTINUE
710 CONTINUE
740 RETURN
C
C 750 DETER = 0.0
  RETURN
  END

```

Program Listing

- HICOAX -

(Reference: Vol. I, Section II.4)

```

PROGRAM HICQAK(INPUT,OUTPUT,TAPES=INPUT,TAPE6=OUTPUT)

PROGRAM HICQAX FINDS THE MODES OF A DIELECTRIC WAVEGUIDE HAVING THE SHAPE
OF A RING OF ARBITRARY CROSS-SECTION. THE WEAKLY-GUIDING CONDITION IS
ASSUMED TO HOLD TRUE. THE INDEX OF REFRACTION OF THE RING IS GREATER
THAN THAT OF THE SURROUNDING REGIONS, INSIDE AND OUTSIDE. THE INDEX
IN EACH OF THESE IS ASSUMED TO BE THE SAME.

INTEGER S,SZ
REAL NL,NLL,N1,N2
DIMENSION PR(10),TABLE(51,2),INDEX(20)
DIMENSION STORE(200,28),INDX(1,2),GD(24,24),C(24)
DIMENSION ROOT(20,14),ROOM(20,3),IROOT(20,3)
COMMON/A/B,P2,P1,NOT,NOT2
COMMON/B/STORE,INDX,GD,C,SZ
DATA (PR(1),1,-.3)/10H CIRCLE,10H ELLIPSE,10H SQUARE,10H RING /
DATA (PR(1),1,-4.7)/10H S' ELLIPSE,10H RECTANGLE,10H OSCINE,6H SINE /
DATA (PR(1),1,-8.9)/10H S' PLIPED,10H CUSPED,10H RING /

*****
LIST OF VARIABLES WITH SINGLE ASSIGNED PURPOSES

ACC ACCURACY TO WITHIN WHICH THE ROOT-LOCATION PROCEDURE FINDS THE ROOTS
B NORMALIZED FREQUENCY (READ IN)
C VECTOR OF COEFFICIENTS IN THE FIELD EXPANSION (DESIRED RESULT)
DET VALUE OF THE DETERMINANT OF THE MATRIX GD
DROP D(RHO)/D(PHI) ON THE PERIMETER OF THE GUIDE
GD MATRIX OF COEFFICIENTS
ID # OF FUNCTION EVALUATIONS DURING ROOT-LOCATION PROCEDURE
IDTAG DIAGNOSIS CODE (SEE COMMENTS) (READ IN)
INDEX ARRAY FOR STORING TABLE INDICES NEAR WHICH ROOTS OCCUR
INDX ARRAY FOR STORING VALUES OF INDICES USED IN FIELD EXPANSION
IROOT ARRAY FOR STORING INFORMATION USED IN PRINTING RESULTS AT THE ROOT
JDEX COUNTER TO KEEP TRACK OF THE NUMBER OF SEPARATE CASES COMPLETED
L INDEX OF 1ST TERM IN FIELD EXPANSION (READ IN)
LLL VALUE OF L FROM PREVIOUS CASE (HELPS AVOID REPEATED CALCULATIONS)
LLS VALUE OF S FROM PREVIOUS CASE (HELPS AVOID REPEATED CALCULATIONS)
NL,N2 SUPERELLIPSE INDICES (REAL) (READ IN)
NL VALUE OF N1 FROM PREVIOUS CASE (HELPS AVOID REPEATED CALCULATIONS)
NLL VALUE OF N2 FROM PREVIOUS CASE (HELPS AVOID REPEATED CALCULATIONS)
NOP # OF INTEGRATION POINTS (READ IN)
NOT # OF TERMS IN EACH SUBMATRIX (READ IN)
NOT2 DIMENSION OF DETERMINANT = 2*NOT
NP2 # OF VALUES OF P2 TAKEN TO CONSTRUCT TABLE (READ IN)
NR # OF ROOTS FOUND
PHI ANGLE IN (RHO, PHI) COORDINATE SYSTEM
PI P1
PMAX LARGEST VALUE OF P2 USED IN TABLE (READ IN)
PMIN SMALLEST VALUE OF P2 USED IN TABLE (READ IN)
PRIN ARRAY FOR STORING ALPHANUMERIC PROGRAM INFORMATION
P2 PROPAGATION CONSTANT (DESIRED RESULT)
R1,R2 ASPECT RATIOS (READ IN)
RHO RADIUS IN (RHO, PHI) COORDINATE SYSTEM
RH0 VALUE OF R1 FROM PREVIOUS CASE (HELPS AVOID REPEATED CALCULATIONS)
RLL VALUE OF R2 FROM PREVIOUS CASE (HELPS AVOID REPEATED CALCULATIONS)
ROOM ARRAY FOR STORING OUTPUT INFORMATION
ROOT ARRAY FOR STORING OUTPUT INFORMATION, INCLUDING VALUES OF P2
S INDEX OF LAST TERM IN FIELD EXPANSION (INTEGER)
SC INDICATES SINE OR COSINE EXPANSION (INTEGER) (READ IN)
STORE ARRAY FOR STORING FUNCTIONS USED REPEATEDLY, WHICH DEPEND ONLY ON PHI
SZ RATIO OF INNER SEMIMAJOR AXIS TO OUTER (0.1E-52.LT.1)
SZL VALUE OF SZ FROM PREVIOUS CASE (HELPS AVOID REPEATED CALCULATIONS)

```



```

N,N = NUMBER OF INTERVALS IN THE INITIAL TABLE      DEF: 10
{ = NUMBER OF INTERGRATION POINTS IN THE RANGE (0,PI/2) { DEF: 50
1,LAG = DIAGNOSIS CODE: 0 = NORMAL CONDITION (DEFAULT)
1 = INITIAL TABLE, FINAL MATRIX PRINTED
2 = SUMMARY PRINTED AT END (VALUES OF P2)
3 = DETAILS OF ROOT-LOCATION PRINTED

* IF THE CROSS-SECTION IS CIRCULAR, THE PHI-INTEGRALS CAN BE DONE IN
* CLOSED FORM. TO SAVE TIME, ONE CAN SET NOP = 1, LEAVING ONLY ONE "IN-
* TEGRATION POINT". IN THIS CASE ADJUSTMENTS IN THE VALUES OF THE TRIG
* FUNCTIONS ARE MADE AT STATEMENT 87 SO THAT THE INTEGRALS WILL COME OUT
* RIGHT. NOTE THAT IF NOP = 1, NOT MUST ALSO EQUAL 1 OR ELSE THE OFF-
* DIAGONAL MATRIX ELEMENTS MAY BE NONZERO AND THEREFORE INCORRECT.

READ IN INFORMATION WHICH REMAINS FIXED FOR MANY CASES.
RETURN TO STATEMENT 2 ONLY AFTER DUMPING RESULTS FOR THESE CASES.
A BLANK CARD HERE TERMINATES PROGRAM EXECUTION.

READ(5,100)NOT,$,NP2,NOP,IDIAG
IF(NOT.EQ.0)STOP
WRITE(6,130)
WRITE(6,123)
WRITE(6,101)PN,W
IF(NOT.GT.12)NOT = 12
IF($.LE.0)$ = 3
IF($.GT.12)$ = 12
ACC = 10.0*(1-$)
IF(NP2.LT.1)NP2 = 10
IF(NP2.GT.50)NP2 = 50
IF(NOP.GT.200)NOP = 200
IF(NOP.EQ.0)NOP = 50
IF(NOP.EQ.1)NOT = 1
NOT2 = 2*NOT

PRINT THE FIXED INFORMATION READ FROM THE FIRST DATA CARD

WRITE(6,102)NOT,NOT2
WRITE(6,104)$
WRITE(6,105)NP2
WRITE(6,108)NOP
WRITE(6,103)IDIAG

*****
SC = 1 IF COSINE TERMS ARE TO BE USED; 2 IF SINE TERMS ARE DESIRED
L = INDEX OF FIRST TERM IN FIELD EXPANSION
N1,N2 = INDICES OF OVOIDS (1 = ELLIPSE, >30 = SQUARE) DEF: 1
N1,N2 = INDICES OF OVOIDS (1 = ELLIPSE, >30 = SQUARE) DEF: 1
R1,R2 = ASPECT RATIOS DEF: 0
R1,R2 = ASPECT RATIOS DEF: 0
SZ = RATIO OF SEMINOR AXES, B1/B2 (0 < SZ < 1) DEF: 1
B = ASSUMED VALUE OF THE BINDING CONSTANT DEF: 1
PMAX, PMIN = RANGE OF VALUES OF P2 TO BE SEARCHED DEF: 1, 0

ONE DATA CARD CORRESPONDS TO ONE INDIVIDUAL CASE.

RETURN TO STATEMENT 1 IMMEDIATELY AFTER EACH CASE IS COMPLETE. A BLANK
CARD READ AT STATEMENT 1 CAUSES RESULTS FOR ALL CASES TO BE DUMPED
(PRINTED) AND CONTROL WILL THEN GO TO STATEMENT 2. A BLANK CARD THERE
TERMINATES THE JOB EXECUTION, SO TWO BLANK CARDS SHOULD FOLLOW THE LAST
DATA CARD.

JDEX = 0
1 READ(5,121)$C,L,N1,R1,N2,R2,SZ,B,PMAX,PMIN
IF(R1.LE.0.0)GO TO 999
IF(JDEX.GE.20)GO TO 999
JDEX = JDEX + 1

```

```

C ALL SECOND(TO)
( PMAE.O.O.O)PMAE = 1.0
I INT.LE.O.O)N1 = 1.0
IF(N2.LE.O.O)N2 = 1.0
IF(R2.LE.O.O)R2 = 1.0
IF(SC.LE.O.O)SC = 1
IF(SZ.GT.2)SC = 2
IF(B.LE.O.O)B = 1.0
IF(SZ.LT.O.O.OR.SZ.GE.1.0)SZ = 0.0
.....
C C C C C
IF SZ = 0, THE INNER "CROSS-SECTION" MUST BE A CIRCLE
.....
IF(SZ.GT.O.O)GO TO 64
N1 = 1.0
R1 = 1.0
.....
C C C C C
DETERMINE S, THE INDEX OF THE LAST TERM IN THE FIELD EXPANSION
.....
64 S = L + 2*(NOT-1)
IF(R1.NE.1.0.OR.R2.NE.1.0)GO TO 85
I = L/2
I = 2*I
IF(I.NE.L)GO TO 65
S = L + 4*(NOT-1)
.....
C C C C C
PRINT INPUT DATA
.....
65 WRITE(6,111)
WRITE(6,123)
WRITE(6,111)
WRITE(6,109)JDEX,L,S,N1,R1,N2,R2,SZ,B
IF(PMAE.NE.1.0.OR.PMIN.NE.0.0)WRITE(6,110)PMAE,PMIN
.....
C C C C C
INITIALIZE STORAGE ARRAYS FOR THIS CASE
DO 70 I = 7,14
70 ROOT(JDEX,I) = 0.0
ROOT(JDEX,3) = N1
ROOT(JDEX,4) = R1
ROOT(JDEX,5) = SZ
ROOT(JDEX,6) = B
ROOT(JDEX,2) = N2
ROOT(JDEX,3) = R2
IROOT(JDEX,1) = 0
IROOT(JDEX,2) = L
IROOT(JDEX,3) = S
.....
C C C C C
IDENTIFY SHAPES OF CROSS-SECTIONS
.....
J = SC + 5
I = 4
IF (N1.GT.1.0) GO TO 90
IF (N1-.5) 94,95,96
94 I = 9
GO TO 92
95 I = 8

```

```

96 ( TO 92
    I=2 (N1.NE.1.0) GO TO 92
    IF (R1.EQ.1.0) I=1
90 IF (N1.LT.30.) GO TO 92
    I=5
    IF (R1.EQ.1.0) I=3
92 CONTINUE
    W = PR(I)
    IF(NOP.EQ.1.AND.I.NE.1)WRITE(6,131)
    I=4
    IF (N2.GT.1.0) GO TO 50
    IF (N2-.5) 54,55,56
54 I=9
    GO TO 52
55 I=8
    GO TO 52
56 IF (N2.NE.1.0) GO TO 52
    I=2
    IF (R2.EQ.1.0) I=1
50 IF (N2.LT.30.) GO TO 52
    I=5
    IF (R2.EQ.1.0) I=3
52 CONTINUE
    WRITE(6,112)PR(J),W,PR(I)
    IF(NOP.EQ.1.AND.I.NE.1)WRITE(6,131)
    ROOT(JDEX,2) = PR(J)
    ROOT(JDEX,1) = W
    ROOM(JDEX,1) = PR(I)

```

C  
C  
C

```

PN = NOP
W = PI/(PN+2.0)
IF(RL.NE.R1.OR.RLL.NE.R2)GO TO 71
IF(NL.NE.N1.OR.NLL.NE.N2)GO TO 71
IF(SZ.NE.SZL)GO TO 71
GO TO 79
71 CONTINUE
RLL = R2
NLL = N2
RL = R1
NL = N1
SZL = SZ

```

C  
C  
C

CALCULATE AND STORE RHO(PHI) AND D(RHO)/D(PHI) FOR EACH BOUNDARY

```

DO 75 I = 1,NOP
PN = 2*(NOP - I) + 1
PHI = W*PN/2.0
CALL PERIM(N1,R1,PHI,RHO,DRDP)
STORE(I,1) = RHO+SZ
STORE(I,2) = DRDP+SZ
CALL PERIM(N2,R2,PHI,RHO,DRDP)
STORE(I,3) = RHO
75 STORE(I,4) = DRDP

```

C  
C  
C  
C

CALCULATE AND STORE COS(M\*PHI) AND SIN(M\*PHI) WHERE M CAN BE L,.....S.  
VALUES OF M ARE STORED AS INDX.

```

79 CONTINUE
IF(L.EQ.LLL.AND.S.EQ.LLS)GO TO 89
LLL = L
LLS = S
INDX(1) = L
IF(NOT.EQ.1)GO TO 86

```

```

      T = (S - L)/(NOT - 1)
      95 K = 2, NOT
      96 INDEX(K) = L + (K - 1)*LST
      IF(NOP.EQ.1)GO TO 87
      DO 80 I = 1, NOP
      PN = 2*(NOP - 1) + 1
      PHI = M*PN/2.0
      DO 80 M = 5, LST, 2
      K = M/2 - 1
      PN = INDEX(K)
      PN = PN*PHI
      STORE(I, M) = COS(PN)
      80 STORE(I, M+1) = SIN(PN)
      GO TO 89
C
C IF NOP = 1, ASSUME CIRCULAR CROSS-SECTIONS. SET TRIG FUNCTIONS EQUAL TO
C CONSTANTS CHOSEN SO THAT THE ANALYTICAL RESULTS FOR THE INTEGRALS ARE
C OBTAINED.
C
      87 STORE(1, 5) = SORT(.5)
      STORE(1, 6) = SORT(.5)
      IF(INDEX(1).NE.0)GO TO 89
      STORE(1, 5) = 1.0
C
C .....
C
C CONSTRUCT THE TABLE OF DETERMINANT VALUES FOR PMIN < P2 < PMAX.
C FOR EACH VALUE OF P2. SUBROUTINE CHEBY CALCULATES THE MATRIX
C GD, WHOSE DETERMINANT IS SOUGHT. SUBROUTINE MATINV CALCULATES
C THE DETERMINANT.
C
      89 LST = NP2 + 1
      IF(IDIAG.GT.0)WRITE(6,106)
      PN = NP2
      PN = (PMAX - PMIN)/PN
      M = PN/100.
      P2 = PMAX
      IF(PMAX.EQ.1.0)P2 = 1.0 - M
      DO 200 M = 1, LST
      CALL CHEBY(NOP, SC)
      CALL MATINV(GD, NOT2, C, DET, 0)
      TABLE(M, 1) = P2
      TABLE(M, 2) = DET
      IF(IDIAG.GT.0)WRITE(6,107)M, P2, DET
      P2 = PMAX - M*PN
      200 IF(P2.LT.M)P2 = M
      IF(IDIAG.GT.0)WRITE(6,114)
C
      CALL SECOND(T1)
      T1 = T1 - TO
      WRITE(6,115)T1
C
C .....
C
C MAKE TENTATIVE IDENTIFICATION OF ROOTS OF EQUATION DET(RLS) = 0
C
      NR = 0
      IF(TABLE(1, 2).NE.0.0)GO TO 210
      NR = 1
      INDEX(NR) = 1001
      210 DO 220 M = 2, LST
      PN = ABS(TABLE(M, 2))
      IF(PN.GT.0.0)GO TO 215
      NR = NR + 1
      INDEX(NR) = M + 1000

```

```

GO TO 220
215 ( = (TABLE(M,2)/PN)*TABLE(M-1,2)
    PN,GE,0,0)GO TO 220
    NR = NR + 1
    INDEX(NR) = M - 1
220 IF(NR,GE,20)GO TO 221
221 WRITE(6,124)NR,JDEX
    IF(NR,EQ,0)GO TO 1
    .....
C C LOCATE THE ROOTS OF DET(RLS) PRECISELY.
C C LOOP DO 250 IS COMPLETED ONCE FOR EACH ROOT (I) IDENTIFIED ABOVE
C C
C C IF THERE ARE MORE THAN 8 ROOTS, MAKE NOTE OF IT. ONLY THE FIRST 8
C C ROOTS WILL BE LISTED IN THE SUMMARY AT THE END. (IF THERE ARE MORE
C C THAN 20 ROOTS, THE LAST ONES WILL HAVE BEEN MISSED COMPLETELY.)
C C
C K = NR
C IF(NR,LE,8)GO TO 251
C K = K - 8
C WRITE(6,128)K
C K = 8
251 IROOT(JDEX,1) = K
C
C DO 250 I = 1,NR
C M = INDEX(I)
C CALL RTT1(TABLE,ACC,PN,NOP,SC,M,IO,IDIAG)
C
C RTT1 RETURNS A FINAL MATRIX, GD, A SET OF COEFFICIENTS, C, AND THE
C ROOT, P2. SEE THE SUBROUTINE FOR A DESCRIPTION OF PARAMETERS.
C
C PRINT THE MATRIX, IF DESIRED; STORE ROOT; CALCULATE A FINAL VALUE
C FOR THE DETERMINANT. ALSO PRINT THE COEFFICIENTS.
C
C IF(I,LE,8)ROOT(JDEX,I+6) = P2
C IF(IDIAG,LT,1)GO TO 253
C W = 3HGD
C WRITE(6,114)
C DO 252 J = 1,NOT2
252 WRITE(6,118)W,(GD(J,K),K=1,NOT2)
253 WRITE(6,114)
C CALL MATINV(GD,NOT2,C,DET,0)
C WRITE(6,125)JDEX,I,P2,PN,IO,DET
C PN = 1H
C WRITE(6,122)((PN,INDEX(K),C(K)),K=1,NOT)
C J = NOT + 1
C WRITE(6,132)((PN,INDEX(K-NOT),C(K)),K=J,NOT2)
C WRITE(6,114)
C IF(DET,EQ,0.0)WRITE(6,113)
250 CONTINUE
C
C CALL SECOND(T1)
C T1 = T1 - TO
C WRITE(6,126)T1
C GO TO 1
C
C .....
C C AFTER ALL CASES (20 OR LESS) HAVE BEEN COMPLETED, PRINT A TABLE
C C OF RESULTS GIVING THE ROOTS FOR EACH CASE.
C C
C 999 WRITE(6,123)
C WRITE(6,123)
C WRITE(6,123)

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```

C 248 ( = ID + K
C
C CALCULATE THE COEFFICIENTS. SUBROUTINE LOAD COPIES (NOT2-1) OF THE
C NOT2 ROWS OF GO INTO ARRAY SINV AND VECTOR C. SUBROUTINE MATINV SOLVES
C THE (NOT2-1)X(NOT2-1) SYSTEM FOR THE COEFFICIENTS AND RETURNS THEM IN C.
C
C CALL LOAD(GO,SINV,C,NOT2,M)
C LST = NOT2 - 1
C CALL MATINV(SINV,LST,C,DN,1)
C
C STACK THE COEFFICIENTS IN THE PROPER ORDER IN VECTOR C, INCLUDING 1.000
C
C IF(M.EQ.NOT2)GO TO 280
C DO 270 J = M,LST
C K = NOT2 + M - J
C 270 C(K) = C(K-1)
C 280 C(M) = 1.0
C RETURN
C END
C SUBROUTINE LOAD(SS,SINV,C,NOT2,ID)
C
C SUBROUTINE LOAD LOADS SS (SEE GO) INTO ARRAY SINV, ELIMINATING
C THE ROW AND COLUMN CONTAINING THE DIAGONAL ELEMENT WHICH IS SMALLEST
C IN MAGNITUDE. THE NEGATIVE OF THE COLUMN OF SS WHICH IS LEFT OUT IS
C LOADED INTO ARRAY C, EXCEPT FOR THE ELEMENT IN THE ROW WHICH IS ELIM-
C INATED. THUS, SINV IS (NOT2-1) BY (NOT2-1) IN DIMENSION, AND C IS A VEC-
C TOR (NOT2-1) IN LENGTH.
C
C NOTE THAT ONLY THE FIRST THREE DIAGONAL ELEMENTS ARE SEARCHED
C FOR THE SMALLEST ONE.
C
C DIMENSION SS(24,24),SINV(24,24),C(24)
C G = 1.0E+321
C ID = 1
C K = NOT2
C IF(K.GT.3)K = 3
C DO 200 J = 1,K
C H = ABS(SS(J,J))
C IF(H.GT.G)GO TO 200
C G = H
C ID = J
C 200 CONTINUE
C
C JJ = 0
C DO 300 J = 1,NOT2
C IF(J.EQ.ID)GO TO 300
C JJ = JJ + 1
C C(JJ) = -SS(J,ID)
C KK = 0
C DO 299 K = 1,NOT2
C IF(K.EQ.ID)GO TO 299
C KK = KK + 1
C SINV(JJ,KK) = SS(J,K)
C 299 CONTINUE
C 300 CONTINUE
C RETURN
C END
C SUBROUTINE HINTS(X1,X2,G1,G2,ACC,PN,NOP,SC,ID,IDIAG,IER)
C
C SUBROUTINE HINTS PERFORMS A HALF-INTERVAL SEARCH TO LOCATE A ROOT
C OF THE EQUATION DET(RLS) = 0 TO WITHIN A DESIRED ACCURACY ACC.
C ID IS THE NUMBER OF BISECTIONS, AND PN IS THE ACTUAL MAXIMUM ERROR.
C
C INTEGER SC
C DIMENSION STORE(200,28),INDX(12),GO(24,24),C(24)

```

```

C      (  COMMON/A,B,P2,P1,NOT,NOT2
        /COMMON/B/STORE,INDX,GD,C,SZ
        ID = 0
        IER = 0
        100 PN = ABS(X2 - X1)
C
        IF(PN.LT.ACC)RETURN
        ID = ID + 1
        IF(ID.GT.25)GO TO 230
        P2 = (X1 + X2)/2.0
        CALL CHEBY(NOP,SC)
        CALL MATINV(GD,NOT2,C,G,0)
C
        IF(IDIAG.GE.3)WRITE(6,700)ID,X1,X2,G1,G2,PN,P2,G
        700 FORMAT('  HINTS:',I3.2F13.10,3E11.4,F13.10,E11.4)
C
        IF(G*(G1/ABS(G1)))120,210,130
        120 X2 = P2
        G2 = G
        GO TO 100
        130 X1 = P2
        G1 = G
        GO TO 100
C
C      IF THE DETERMINANT IS EXACTLY ZERO, THE MATRIX (WHICH WILL HAVE BEEN
C      ALTERED IN MATINV) IS RECOMPUTED.
C
        210 PN = 0.0
        CALL CHEBY(NOP,SC)
        ID = ID + 1
        RETURN
C
C      ERROR RETURN: TOO MANY ITERATIONS
C
        230 IER = 6
        RETURN
        END
C
        SUBROUTINE REG(X1,X2,F1,F2,ACC,PN,NOP,SC,ID,IDIAG,IER)
C
C      SUBROUTINE REG SOLVES THE EQUATION DET(RLS) = 0 FOR P2 TO WITHIN
C      A DESIRED ACCURACY ACC BY THE MODIFIED REGULA-FALSI METHOD. ID IS
C      THE NUMBER OF FUNCTION EVALUATIONS AND PN GIVES THE ACTUAL MAXIMUM ERROR.
C
        INTEGER SC
        DIMENSION STORE(200,28),INDX(12),GD(24,24),C(24)
        COMMON/A,B,P2,P1,NOT,NOT2
        /COMMON/B/STORE,INDX,GD,C,SZ
C
        ID = 0
        IER = 0
        G = F1
        100 PN = ABS(X1 - X2)
C
        F0 = G/ABS(G)
        P2 = (F2*X1 - F1*X2)/(F2 - F1)
        CALL CHEBY(NOP,SC)
        ID = ID + 1
        IF(PN.LT.ACC)RETURN
        IF(ID.GT.25)GO TO 230
        CALL MATINV(GD,NOT2,C,G,0)
C
        IF(IDIAG.GT.2)WRITE(6,700)ID,X1,X2,F1,F2,PN,P2,G
        700 FORMAT('  REG: ',I3.2F13.10,3E11.4,F13.10,E11.4)
C
        IF(G*(F1/ABS(F1)))150,190,160

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```

150  F = P2
    G = G
    A = ((F0*G).GT.0.0)F1 = F1/2.0
    GO TO 100
160  X1 = P2
    F1 = G
    IF((F0*G).GT.0.0)F2 = F2/2.0
    GO TO 100
C
C IF THE DETERMINANT IS EXACTLY ZERO, THE MATRIX (WHICH WILL HAVE
C BEEN ALTERED IN MATINV) IS RECOMPUTED.
C
190  PN = 0.0
    CALL CHEBY(NOP,SC)
    ID = ID + 1
    RETURN
C
C ERROR RETURN: TOO MANY ITERATIONS
C
230  IER = 5
    RETURN
    END
C
C SUBROUTINE CHEBY(NOP,SC)
C
C INTEGER S,SC
C REAL MLS,NLS
C DIMENSION STORE(200,28),INDX(12),GD(24,24),C(24)
C DIMENSION GDM(12,12),GDM(12,12),GDP(12,12),GDD(12,12)
C DIMENSION BIST1(49)
C DIMENSION BKST3(49),BYST1(49),BYST2(49),BYST2(49)
C COMMON/A/B,P2,P1,NOT,NOT2
C COMMON/B/STORE,INDX,GD,C,SZ
C
C CHEBYSHEV INTEGRATION OF THE MATRIX ELEMENTS OF RLS
C
C
C NMAX = INDX(NOT) + 1
C IF(NMAX.EQ.1)NMAX = 2
C OR = 0.0
C NN = 0
C PTS = NOP
C W = PI/(PTS*2.0)
C P = SORT(P2)
C OMP = SORT(1.0 - P2)
C DO 50 I = 1,NOT
C DO 50 J = 1,NOT
C GDM(I,J) = 0.0
C GDM(I,I) = 0.0
C GDP(I,I) = 0.0
C GDD(I,I) = 0.0
50  GDD(I,J) = 0.0
C
C LOOP DO 100 DOES INTEGRATION USING NOP POINTS IN THE RANGE (0,PI/2)
C
C DO 100 JJJ = 1,NOP
C
C OBTAIN RHO(PHI), COS(PHI), AND SIN(PHI)
C
C RH1 = STORE(JJJ,1)
C RH2 = STORE(JJJ,3)
C DR1 = STORE(JJJ,2)
C DR2 = STORE(JJJ,4)
C
C AK1R1 = K1*RHO1 FOR SURFACE 1-2
C AK2R1 = K2*RHO1 FOR SURFACE 1-2
C AK2R2 = K2*RHO2 FOR SURFACE 2-3
C AK3R2 = K3*RHO2 FOR SURFACE 2-3

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C      R1 = PI*B*RH1*SQR(1.0 - P2)
C      AK3R2 = PI*B*RH2*SQR(1.0 - P2)
C      AK2R2 = PI*B*RH2*SQR(1.0 - P2)
C      OBTAIN THE BESSEL FUNCTIONS WHICH WILL BE REQUIRED
C
C      IL(AK1R1) IS STORED IN B1ST1
C      JS(AK2R1) IS STORED IN B1ST1
C      JS(AK2R2) IS STORED IN B1ST1
C      KL(AK3R2) IS STORED IN B1ST1
C      YS(AK2R1) IS STORED IN B1ST1
C      YS(AK2R2) IS STORED IN B1ST1
C
C      CALL BESJ(AK1R1,OR,1,NMAX,B1ST1,NZ)
C      CALL BESJ(AK2R1,OR,NMAX,B1ST1,NZ)
C      CALL BESJ(AK2R2,OR,NMAX,B1ST1,NZ)
C      CALL BESJ(AK3R2,NN,1,NMAX,BKST3,NZ)
C      IF(SZ.GT.0.0)CALL BESYN(AK2R1,NN,NMAX,B1ST1)
C      CALL BESYN(AK2R2,NN,NMAX,B1ST2)
C
C      IF SZ = 0, THE Y BESSEL FUNCTIONS MUST BE DEFINED. ALTHOUGH USED IN
C      HUSB AND GLSB, THE LATTER ARE REDEFINED LATER.
C
C      IF(SZ.GT.0.0)GO TO 70
C      DO 60 I = 1,NMAX
C      60 B1ST1(I) = 0.0
C
C      LOOP DO 100 I... CORRESPONDS TO L
C      LOOP DO 100 J... CORRESPONDS TO S
C
C      70 DO 100 I = 1,NOT
C      L = INDX(I)
C      XL = L
C
C      OBTAIN COS(L*PHI) AND SIN(L*PHI)
C
C      CL = STORE(JJJ,2*I+3)
C      SL = STORE(JJJ,2*I+4)
C      LL = L - 1
C      IF(L.EQ.0)LL = 1
C
C      RETRIEVE THE BESSEL FUNCTIONS OF ORDER L AND L-1
C
C      BK3 = BKST3(LL+1)
C      BK3M = BKST3(LL+1)
C      B1 = B1ST1(LL+1)
C      B1M = B1ST1(LL+1)
C
C      DO 100 J = 1,NOT
C      S = INDX(J)
C      XS = S
C
C      OBTAIN COS(S*PHI) AND SIN(S*PHI)
C
C      CS = STORE(JJJ,2*J+3)
C      SS = STORE(JJJ,2*J+4)
C
C      RETRIEVE THE BESSEL FUNCTIONS OF ORDER S AND S-1
C
C      LL = S - 1
C      IF(S.EQ.0)LL = 1
C      B1 = B1ST1(S+1)
C      B1M = B1ST1(LL+1)
C      B1J2 = B1ST2(S+1)

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      BU2M= BUST2(LL+1)
      (  = BUST1(S+1)
      )  = BUST1(LL+1)
      BY2 = BUST2(S+1)
      BY2M= BUST2(LL+1)
      IF(S.GT.0)GO TO 80
      BU1M = -BU1M
      BU2M = -BU2M
      BY1M = -BY1M
      BY2M = -BY2M
C
C 80 CONTINUE
      FLS = 2.0*B-RH2*(BK3*BU2M*OMP + P*BU2*BK3M)
      FLSB = 2.0*B-RH2*(BK3*BY2M*OMP + P*BY2*BK3M)
      MLS = PI*B-RH1*(BI*BU1M*OMP - P*BI*BUI)
      MLSB = PI*B-RH1*(BI*BY1M*OMP - P*BI*BY1)
      ELS = 0.0
      ELSB = 0.0
      GLS = 0.0
      GLSB = 0.0
C
C  ELS, ELSB, GLS, AND GLSB ARE NOT ACTUALLY ZERO IF L = S, BUT THEY
C  ARE MULTIPLIED BY ETA, WHICH IS ZERO IN THAT CASE
C
      IF(L.EQ.S)GO TO 85
      FLS = FLS + (XL - XS)*2.0*BK3*BU2/PI
      FLSB = FLSB + (XL - XS)*2.0*BK3*BY2/PI
      ELS = 2.0*BK3*BU2*DR2/(PI*RM2)
      ELSB = 2.0*BK3*BY2*DR2/(PI*RM2)
      IF(SZ.EQ.0)GO TO 85
      MLS = MLS + (XL - XS)*BI*BU1
      MLSB = MLSB + (XL - XS)*BI*BY1
      GLS = BI*BU1*DR1/RH1
      GLSB = BI*BY1*DR1/RH1
C
C 85 CONTINUE
      IF(SZ.GT.0)GO TO 87
      GLSB = 0.0
      MLSB = 0.0
      IF(L.NE.S)GO TO 87
      MLSB = 2.0*PI
      IF(L.EQ.0)GO TO 87
      MLSB = 2.0*((P/OMP)*L)/PI
C
C 87 CONTINUE
      IF(SC.EQ.2)GO TO 90
      ETA = XS*CL*SS - XL*SL*CS
      MLS = FLS*CL*CS + ELS*ETA
      MLS = FLSB*CL*CS + ELSB*ETA
      PLS = MLS*CL*CS + GLS*ETA
      QLS = MLSB*CL*CS + GLSB*ETA
      GO TO 91
C
C 90 CONTINUE
      ETA = XS*SL*CS - XL*CL*SS
      MLS = FLS*SL*SS - ELS*ETA
      MLS = FLSB*SL*SS - ELSB*ETA
      PLS = MLS*SL*SS - GLS*ETA
      QLS = MLSB*SL*SS - GLSB*ETA
C
C 91 GDM(I,J) = GDM(I,J) + MLS
      GDM(I,J) = GDM(I,J) + MLS
      GDP(I,J) = GDP(I,J) + PLS
      GDQ(I,J) = GDQ(I,J) + QLS
C
C 100 CONTINUE

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```

C      LOAD THE FOUR SUBMATRICES INTO GO
C      (
      DO 105 J = 1,NOT2
      DO 105 I = 1,NOT2
      IF(I.LE.NOT.AND.J.LE.NOT)GD(I,J) = GOM(I,J)
      IF(I.LE.NOT.AND.J.GT.NOT)GD(I,J) = GOM(I,J-NOT)
      IF(I.GT.NOT.AND.J.LE.NOT)GD(I,J) = GOM(I-NOT,J)
      IF(I.GT.NOT.AND.J.GT.NOT)GD(I,J) = GOM(I-NOT,J-NOT)
      GD(I,J) = GD(I,J)*W
105 CONTINUE
C      RETURN
C      END
C      SUBROUTINE PERIM(N,R,PHI,RHO,DRDP)
C      EVALUATE RHO(PHI) AND D(RHO)/D(PHI) ON THE PERIMETER
C      REAL N
C      IF(R.NE.1.0.OR.N.NE.1.0)GO TO 100
C      RHO = 1.0
C      DRDP = 0.0
C      RETURN
C      100 CONTINUE
C      CP = COS(PHI)
C      SP = SIN(PHI)
C      PN = 2.0*N
C      DD = (CP/R)*PN + SP*PN
C      PN = 1.0/PN
C      RHO = 1.0/(DD*PN)
C      PN = 2.0*N - 2.0
C      AA = SP*PN - ((CP/R)*PN)/(R*R)
C      DRDP = -RHO*SP*CP*AA/DD
C      RETURN
C      END
C      SUBROUTINE MATINV(A,N,B,DETER,M)
C      DIMENSION A(24,24),B(24,1),INDEX(24,3)
C      10 DETER = 1.0
C      15 DO 20 J = 1,N
C      20 INDEX(J,3) = 0
C      30 DO 550 I = 1,N
C      SEARCH FOR PIVOT ELEMENT
C      40 AMAX = 0.
C      45 DO 105 J = 1,N
C      IF (INDEX(J,3)-1) 60,105,60
C      60 DO 100 K = 1,N
C      IF (INDEX(K,3)-1) 80,100,740
C      80 IF (AMAX-ABS(A(J,K))) 85,100,100
C      85 IROW = J
C      90 ICOL = K
C      AMAX = ABS(A(J,K))
C      100 CONTINUE
C      105 CONTINUE
C      IF (AMAX.EQ.0.0)GO TO 750
C      INDEX(ICOL,3) = INDEX(ICOL,3) + 1
C      260 INDEX(I,1) = IROW
C      270 INDEX(I,2) = ICOL
C      INTERCHANGE ROWS TO PUT PIVOT ELEM.IN DIAG.
C      130 IF (IROW-ICOL) 140,310,140

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140 DETER = -DETER
150 DO 200 L = 1,N
160   P = A(IROW,L)
170   A(IROW,L) = A(ICOLUMN,L)
200   A(ICOLUMN,L) = SWAP
      IF(M) 310,310,210
210 DO 250 L = 1,M
220   SWAP = B(IROW,L)
230   B(IROW,L) = B(ICOLUMN,L)
250   B(ICOLUMN,L) = SWAP
C
C   DIVIDE PIVOT ROW BY PIVOT ELEMENT
C
310 PIVOT = A(ICOLUMN,ICOLUMN)
   DETER = DETER*PIVOT
330 A(ICOLUMN,ICOLUMN) = 1.0
340 DO 350 L = 1,N
350   A(ICOLUMN,L) = A(ICOLUMN,L)/PIVOT
355 IF (M) 380,380,360
360 DO 370 L = 1,M
370   B(ICOLUMN,L) = B(ICOLUMN,L)/PIVOT
C
C   REDUCE NON PIVOTS ROWS
C
380 DO 550 L1 = 1,N
390 IF (L1-ICOLUMN) 400,550,400
400   T = A(L1,ICOLUMN)
420   A(L1,ICOLUMN) = 0.
430 DO 450 L = 1,N
450   A(L1,L) = A(L1,L)-A(ICOLUMN,L)*T
455 IF (M) 550,550,460
460 DO 500 L = 1,M
500   B(L1,L) = B(L1,L)-B(ICOLUMN,L)*T
550 CONTINUE
C
C   INTERCHANGE COLUMNS
C
600 DO 710 I = 1,N
610   L = N + 1 - I
620 IF (INDEX(L,1)-INDEX(L,2)) 630,710,630
630   JROW = INDEX(L,1)
640   JCOLUMN = INDEX(L,2)
650 DO 705 K = 1,N
660   SWAP = A(K,JROW)
670   A(K,JROW) = A(K,JCOLUMN)
700   A(K,JCOLUMN) = SWAP
705 CONTINUE
710 CONTINUE
740 RETURN
C
750 DETER = 0.0
   RETURN
   END

```

Program Listing

- VAR -

(Reference: Vol. I, Section II.5)



```

C      A = INNER-CORE RADIUS
C      (
C      = NORMALIZED FREQUENCY
C      PS1 = P2 OF UNPERTURBED INNER-CORE MODE
C      PS2 = P2 OF UNPERTURBED OUTER-CORE MODE
C      IDIAG = DIAGNOSTIC CODE
C
1 READ(5,101)L,A,C,B,PS1,PS2,IDIAG
  WRITE(6,120)
  IF(A.EQ.0.0)GO TO 900
  IF(JDEX.GE.40)GO TO 900
  JDEX = JDEX + 1
C
C      STORE INPUT VALUES FOR USE IN TABLE AT THE END
C
C      STORE(JDEX,1) = A
C      STORE(JDEX,2) = C
C      STORE(JDEX,3) = B
C      STORE(JDEX,4) = PS1
C      SSTORE(JDEX,1) = PS2
C      ISS(JDEX) = L
C
C      AL1 = PI*B*SQRT(1.0 - PS1)
C      AL2 = PI*B*SQRT(1.0 - PS2)
C      BT1 = PI*B*SQRT(PS1)
C      BT2 = PI*B*SQRT(PS2)
C      EPS = 1.0
C      IF(L.EQ.0)EPS = 2.0
C      WRITE(6,102)L,A,C,B
C      WRITE(,103)PS1,PS2,AL1,AL2,BT1,BT2
C
C      CALCULATE AND STORE THE BESSEL FUNCTIONS WHICH WILL BE NEEDED DURING
C      THE REMAINING PARTS OF THE PROGRAM
C
ORD = 0.0
NOR = 0
ARG = AL1*A
CALL BESJ(ARG,ORD,NUM,BJ1A,NZ)
ARG = BT1*A
CALL BESKN(ARG,NOR,1,NUM,BK1A,NZ)
ARG = BT1*C
CALL BESKN(ARG,NOR,1,NUM,BK1C,NZ)
ARG = BT1*D
CALL BESKN(ARG,NOR,1,NUM,BK1D,NZ)
C
ARG = AL2*C
CALL BESJ(ARG,ORD,NUM,BJ2C,NZ)
CALL BESYN(ARG,NOR,NUM,BY2C)
ARG = AL2*D
CALL BESJ(ARG,ORD,NUM,BJ2D,NZ)
CALL BESYN(ARG,NOR,NUM,BY2D)
ARG = BT2*A
CALL BESJ(ARG,ORD,1,NUM,BI2A,NZ)
ARG = BT2*C
CALL BESJ(ARG,ORD,1,NUM,BI2C,NZ)
CALL BESKN(ARG,NOR,1,NUM,BK2C,NZ)
ARG = BT2*D
CALL BESKN(ARG,NOR,1,NUM,BK2D,NZ)
C
C      STACK THE BESSEL FUNCTIONS (ORDERS -1, 0, 1, 2,...) IN POSITIONS 1, 2, 3,...
C      OF THE STORAGE VECTORS SO THAT, FOR EXAMPLE, JL(K1A) = BJ1A(L+2) FOR ALL L
C
DO 180 I = 1,NUM
  J = NUM - I + 1
  BJ1A(J+1) = BJ1A(J)

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      R=1A(J+1) = BK1A(J)
      C(J+1) = BK1C(J)
      L=1D(J+1) = BK1D(J)
C
      BJ2C(J+1) = BJ2C(J)
      BJ2D(J+1) = BJ2D(J)
      BY2C(J+1) = BY2C(J)
      BY2D(J+1) = BY2D(J)
      B12A(J+1) = B12A(J)
      B12C(J+1) = B12C(J)
      BK2C(J+1) = BK2C(J)
      BK2D(J+1) = BK2D(J)
180 C
      BJ1A(1) = -BJ1A(3)
      BK1A(1) = BK1A(3)
      BK1C(1) = BK1C(3)
      BK1D(1) = BK1D(3)
C
      BJ2C(1) = -BJ2C(3)
      BJ2D(1) = -BJ2D(3)
      BY2C(1) = -BY2C(3)
      BY2D(1) = -BY2D(3)
      B12A(1) = B12A(3)
      B12C(1) = B12C(3)
      BK2C(1) = BK2C(3)
      BK2D(1) = BK2D(3)
C
C PRINT THE BESSEL FUNCTIONS OF ORDER L
C
      IF(IDIAG.LE.1)GO TO 190
      WRITE(6,116)
      WRITE(6,105)BJ1A(L+2)
      WRITE(6,105)BK1A(L+2)
      WRITE(6,105)BK1C(L+2)
      WRITE(6,105)BK1D(L+2)
C
      WRITE(6,105)BJ2C(L+2)
      WRITE(6,105)BJ2D(L+2)
      WRITE(6,105)BY2C(L+2)
      WRITE(6,105)BY2D(L+2)
      WRITE(6,105)B12A(L+2)
      WRITE(6,105)B12C(L+2)
      WRITE(6,105)BK2C(L+2)
      WRITE(6,105)BK2D(L+2)
C
C CALCULATE THE 4X4 MATRIX NECESSARY TO EVALUATE THE EXPANSION COEFFICIENTS
C FOR THE OUTER-CORE MODE
C
190 CMAT(1,1) = BJ2C(L+2)
      CMAT(1,2) = BY2C(L+2)
      CMAT(1,3) = -B12C(L+2)
      CMAT(1,4) = 0.0
C
      CMAT(2,1) = AL2*(BJ2C(L+1) - BJ2C(L+3))/2.0
      CMAT(2,2) = AL2*(BY2C(L+1) - BY2C(L+3))/2.0
      CMAT(2,3) = -B12*(B12C(L+1) + B12C(L+3))/2.0
      CMAT(2,4) = 0.0
C
      CMAT(3,1) = BJ2D(L+2)
      CMAT(3,2) = BY2D(L+2)
      CMAT(3,3) = 0.0
      CMAT(3,4) = -BK2D(L+2)
C
      CMAT(4,1) = AL2*(BJ2D(L+1) - BJ2D(L+3))/2.0
      CMAT(4,2) = AL2*(BY2D(L+1) - BY2D(L+3))/2.0
      CMAT(4,3) = 0.0

```

```

C      CMAT(4,4) = B12*(BK2D(L+1) + BK2D(L+3))/2.0
      (
      , , (DIAG.GE.1)WRITE(6,104)
      DO 200 I = 1,4
      IF (DIAG.GE.1)WRITE(6,105)(CMAT(I,J),J=1,4)
      DO 200 J = 1,4
      200 AMAT(I,J) = CMAT(I,J)
C
C      CALL MATINV(CMAT,4,W,DET,0)
C
      IF (DIAG.GE.1)WRITE(6,108)DET
      DO 210 I = 1,3
      W(I) = -AMAT(I,1)
      DO 210 J = 1,3
      210 SINV(I,J) = AMAT(I,J+1)
      CALL MATINV(SINV,3,W,DET,1)
      DO 220 I = 1,3
      220 W(5-I) = W(4-I)
      W(1) = 1.0
C
C      NORMALIZE THE INNER-CORE MODE
C
      I = 1
      CL = -PI*EPS*A*BJ1A(L+3)*BJ1A(L+1)/(2.0*PS1)
      IF (CL.GT.0.0)GO TO 230
      WRITE(6,107)I,CL
      CL = ABS(CL)
      230 CL = 1.0/SQRT(CL)
      GAM = BJ1A(L+2)/BK1A(L+2)
C
C      NORMALIZE THE OUTER-CORE MODE
C
      S1 = B12C(L+2)*2 - B12C(L+1)*B12C(L+3)
      S1 = S1*(W(3)+C)*2/2.0
      XX = BJ2D(L+2)*2 - BJ2D(L+1)*BJ2D(L+3)
      YY = 2.0*BJ2D(L+2)*BY2D(L+2) - BJ2D(L+1)*BY2D(L+3) - BJ2D(L+3)*
      1BY2D(L+1)
      ZZ = BY2D(L+2)*2 - BY2D(L+1)*BY2D(L+3)
      S2 = (XX + W(2)*YY + W(2)*W(2)*ZZ)/2.0
      XX = BJ2C(L+2)*2 - BJ2C(L+1)*BJ2C(L+3)
      YY = 2.0*BJ2C(L+2)*BY2C(L+2) - BJ2C(L+1)*BY2C(L+3) - BJ2C(L+3)*
      1BY2C(L+1)
      ZZ = BY2C(L+2)*2 - BY2C(L+1)*BY2C(L+3)
      S2 = S2 - C*C*(XX + W(2)*YY + W(2)*W(2)*ZZ)/2.0
      S3 = BK2D(L+2)*2 - BK2D(L+1)*BK2D(L+3)
      S3 = -S3*W(4)*2/2.0
      AL = (S1 + S2 + S3)*PI*EPS
      IF (AL.GT.0.0)GO TO 240
      I = 2
      WRITE(6,107)I,AL
      AL = ABS(AL)
      240 AL = 1.0/SQRT(AL)
C
C      CHECK FOR "PROPER" SIGN OF AL
C
      ARG = AL2*(C+D)/2.0
      CALL BESJ(ARG,ORD,1,ST,NZ)
      XX = ST(1)
      CALL BESYN(ARG,NOR,1,ST)
      ARG = XX + W(2)*ST(1)
      IF (ARG.LT.0.0)AL = -AL
C
C      PRINT THE COEFFICIENTS FOR THE TWO MODES
C
      IF (DIAG.EQ.0)GO TO 251
      ARG = SHINER

```

```

WRITE(6,106)ARG
      = 1.0
      = GAM*CL
WRITE(6,105)ARG,CL
WRITE(6,105)GAM,XX
ARG = SHOUTER
WRITE(6,106)ARG
DO 250 I = 1,4
  XX = AL*W(I)
250 WRITE(6,105)W(I),XX
C
C EVALUATE THE INTEGRALS FOR J1, J2, K1, K2
C
251 J1 = BK1D(L+2)*2 - BK1D(L+1)*BK1D(L+3)
  J1 = J1 - C*C*(BK1C(L+2)*2 - BK1C(L+1)*BK1C(L+3))
  J1 = -EPS*(PI/2.0)*J1*(GAM*CL)*2
C
J2 = B12A(L+2)*2 - B12A(L+1)*B12A(L+3)
J2 = -((A*W(3)*AL)*2)*EPS*J2*PI/2.0
C
K1 = B12*B1A(L+2)*B12A(L+1) - AL1*B1A(L+1)*B12A(L+2)
K1 = -PI*EPS*A*CL*AL*W(3)*K1/(AL1*2 + B12*2)
C
XX = B11*B1D(L+2)*BK1D(L+1) + AL2*B1D(L+1)*BK1D(L+2)
YY = B11*B1D(L+2)*BK1C(L+1) + AL2*B1D(L+1)*BK1C(L+2)
K2 = AL*(XX - C*YY)
XX = B11*B1D(L+2)*BK1D(L+1) + AL2*B1D(L+1)*BK1D(L+2)
YY = B11*B1D(L+2)*BK1C(L+1) + AL2*B1D(L+1)*BK1C(L+2)
K2 = K2 + W(2)*AL*(XX - C*YY)
K2 = GAM*CL*PI*EPS*K2/(AL2*2 + B11*2)
C
C EVALUATE THE INTEGRAL FOR DELTA
C
S1 = -K1
XX = 2.0*B12C(L+2)*BK1C(L+2) + B12C(L+1)*BK1C(L+3) + B12C(L+3)*
  BK1C(L+1)
YY = 2.0*B12A(L+2)*BK1A(L+2) + B12A(L+1)*BK1A(L+3) + B12A(L+3)*
  BK1A(L+1)
S2 = C*C*XX - A*A*YY
S2 = GAM*CL*W(3)*AL*PI*EPS*S2/4.0
S3 = -K2
IF(PS1.NE.PS2)GO TO 350
GO TO 360
350 S4 = B12*B1D(L+2)*BK2D(L+1) - B11*B1D(L+1)*BK2D(L+2)
  S4 = S4/(B11*2 - B12*2)
360 S4 = -GAM*CL*W(4)*AL*PI*EPS*S4
DEL = S1 + S2 + S3 + S4
C
C PRINT THE RESULTS FOR J1, J2, K1, K2, DELTA
C
WRITE(6,109)
XX = THJ
YY = THK
I = 1
J = 2
WRITE(6,110)XX,I,J1
WRITE(6,110)XX,J,J2
WRITE(6,110)YY,I,K1
WRITE(6,110)YY,J,K2
WRITE(6,111)DEL
C
C CALCULATE THE MATRIX ELEMENTS H(I,J):
C
H(1,1) = S1
H(1,2) = S2
H(2,1) = S3
H(2,2) = S4
C
C

```

```

XX = (P1*B)**2
( = (U1 - PS1)*XX
S3 = (K1 - DEL*PS2)*XX
S4 = (K2 - DEL*PS1)*XX
S4 = (J2 - PS2)*XX
WRITE(6,115)
WRITE(6,105)S1,S2
WRITE(6,105)S3,S4
MB = (S2 + S3)/2.0
C
CGAM = (PS1 - PS2)/(K1 + K2)
C
ARG = 4.0*(MB - DEL*S1)*(MB - DEL*S4)
NZ = 0
YY = (S1 - S4)**2
S5 = ABS(ARG)/YY
IF(S5.LT.1.0E-12)NZ = 1
ARG = YY + ARG
ARG = SORT(ARG)
C
E1 = -(S1 + S4 - 2.0*MB*DEL + ARG)/(2.0*XX*(1.0 - DEL*DEL))
E2 = -(S1 + S4 - 2.0*MB*DEL - ARG)/(2.0*XX*(1.0 - DEL*DEL))
IF(E1.LE.E2)GO TO 375
YY = E1
E1 = E2
E2 = YY
375 CONTINUE
C
STORE(JDEX,5) = E1
STORE(JDEX,2) = E2
I = 1
WRITE(6,109)
WRITE(6,112)I,E1
I = 2
WRITE(6,112)I,E2
C
C CALCULATE THE EXPANSION COEFFICIENTS, C AND R, AND CHECK ORTHOGONALITY
C
R1 = -(S1 + XX*E1)/(MB + XX*E1*DEL)
R2 = -(S1 + XX*E2)/(MB + XX*E2*DEL)
C
NZ IS A FLAG INDICATING POSSIBLE ROUND OFF ERROR IN DETERMINING
R1 OR R2. USE APPROXIMATE FORMULAE.
C
IF(NZ.EQ.0.AND.R2.NE.0.0)GO TO 390
WRITE(6,132)R1,R2
DG = DEL*CGAM
S5 = 2.0*CGAM/(1.0 - DG)
S6 = (1.0 + DG/2.0 + 0.5/DG)
S6 = -DEL*S6/(1.0 + DG)
IF(ABS(R1).GT.1.6)GO TO 385
R1 = S6
R2 = S5
GO TO 390
385 R1 = S5
R2 = S6
C
390 C1 = 1.0 + 2.0*DEL*R1 + R1*R1
C2 = 1.0 + 2.0*DEL*R2 + R2*R2
C1 = 1.0/SORT(C1)
C2 = 1.0/SORT(C2)
WRITE(6,109)
I = 1
WRITE(6,121)I,C1,I,R1
I = 2

```

```

WRITE(6,121)I,C2,I,R2
( = 1.0 + DEL*(R1 + R2) + R1*R2
  , LITE(6,122)XX
  ARG = SORT(1.0 + CGAM*CGAM)
  XX = CGAM + ARG
  YY = CGAM - ARG
  WRITE(6,125)CGAM,XX,YY
  STORE(JDEX,6) = R1
  SSTORE(JDEX,3) = R2

C CALCULATE THE POWER ASSOCIATED WITH EACH CORE FOR EACH INDIVIDUAL COMPOSIT
C
C
C INNER-CORE POWER
S1 = BU1A(L+2)*.02 - BU1A(L+1)*BU1A(L+3)
S1 = S1*(A*CL)*.02
T1 = S1*PI*EPS/2.0
S2 = BT2*BU1A(L+2)*B12A(L+1) - AL1*BU1A(L+1)*B12A(L+2)
S2 = 4.0*CL*W(3)*AL*A*S2/(AL1*.02 + BT2*.02)
T2 = S2*PI*EPS/4.0
S3 = S2*R2
S2 = S2*R1
S4 = B12A(L+2)*.02 - B12A(L+1)*B12A(L+3)
S4 = S4*(W(3)*AL*A)*.02
T3 = S4*PI*EPS/2.0
S5 = S4*R2*R2
S4 = S4*R1*R1
PC1 = (C1+C1*PI*EPS/2.0)*(S1 + S2 + S4)
PC2 = (C2+C2*PI*EPS/2.0)*(S1 + S3 + S5)
STORE(JDEX,7) = PC1
SSTORE(JDEX,4) = PC2

C OUTER-CORE POWER
C
C
S1 = BK1D(L+2)*.02 - BK1D(L+1)*BK1D(L+3)
XX = BK1C(L+2)*.02 - BK1C(L+1)*BK1C(L+3)
S1 = S1 - C*C*XX
S2 = BU2D(L+2)*.02 - BU2D(L+1)*BU2D(L+3)
XX = BU2C(L+2)*.02 - BU2C(L+1)*BU2C(L+3)
S2 = S2 - C*C*XX
S3 = 2.0*BU2D(L+2)*BY2D(L+2) - BU2D(L+1)*BY2D(L+3) -
1 BU2D(L+3)*BY2D(L+1)
XX = 2.0*BU2C(L+2)*BY2C(L+2) - BU2C(L+1)*BY2C(L+3) -
1 BU2C(L+3)*BY2C(L+1)
S3 = S3 - C*C*XX
S4 = BY2D(L+2)*.02 - BY2D(L+1)*BY2D(L+3)
XX = BY2C(L+2)*.02 - BY2C(L+1)*BY2C(L+3)
S4 = S4 - C*C*XX
S5 = BT1*BU2D(L+2)*BK1D(L+1) + AL2*BU2D(L+1)*BK1D(L+2)
XX = BT1*BU2C(L+2)*BK1C(L+1) + AL2*BU2C(L+1)*BK1C(L+2)
S5 = S5 - C*C*XX
S6 = BT1*BY2D(L+2)*BK1D(L+1) + AL2*BY2D(L+1)*BK1D(L+2)
XX = BT1*BY2C(L+2)*BK1C(L+1) + AL2*BY2C(L+1)*BK1C(L+2)
S6 = S6 - C*C*XX
SUM = 0.0
XX = C1
YY = R1
I = 0
400 I = I + 1
PR2 = SUM

```

```

1*(I,GT,2)GO TO 420
      SUM = SUM
      SUM = S1*(PI*EPS/2.0)*(CL*GAM*XX)**2
      SUM = SUM + S2*(PI*EPS/2.0)*(AL*YY*XX)**2
      SUM = SUM + S3*(PI*EPS*W(2)/2.0)*(AL*YY*XX)**2
      SUM = SUM + S4*(PI*EPS/2.0)*(AL*W(2)*YY*XX)**2
      SUM = SUM - 2.0*S5*XX*XX*PI*EPS*YY*GAM*CL*AL/(AL2**2 + BT1**2)
      SUM = SUM - 2.0*S6*XX*XX*PI*EPS*YY*GAM*CL*AL*W(2)/(AL2**2+BT1**2)
      XX = C2
      YY = R2
      GO TO 400

C C C
      WRITE AND SAVE THE RESULTS OF THE POWER CALCULATION

420 STORE(JDEX,8) = PRI
      SSTORE(JDEX,5) = PR2
      XX = (PRI - PC1)/(PR1 + PC1)
      YY = (PR2 - PC2)/(PR2 + PC2)
      S1 = PRI/PC1
      S2 = PR2/PC2
      STORE(JDEX,9) = S1
      SSTORE(JDEX,6) = S2
      STORE(JDEX,10) = XX
      SSTORE(JDEX,7) = YY
      WRITE(6,123)
      WRITE(6,124)PC1,PR1,S1,XX
      WRITE(6,124)PC2,PR2,S2,YY

C C C C
      CALCULATE THE BEAT-LENGTH AND MAXIMUM POWER TRANSFER FOR TWO INTERFERING
      COMPOSITE MODES RESULTING FROM INITIAL CONDITION OF INNER-CORE MODE

      BTL = 1.0/(E2 - E1)
      STORE(JDEX,11) = BTL
      WRITE(6,126)BTL
      PT1 = 4.0*(C1**4)*(R1/R2)*(1.0 + DEL*R1)**2
      STORE(JDEX,12) = PT1

C C C C
      T1, T2, AND T3 (CALCULATED PREVIOUSLY) ARE THE INTEGRALS OF 1*1, 1*2, AND
      2*2 OVER THE INNER CORE.

      PT2 = T1 + R1*R2*T3 + (R1 + R2)*T2
      PT2 = PT1*PT2
      STORE(JDEX,13) = PT2
      WRITE(6,127)PT1
      WRITE(6,127)PT2

C C C C
      CALCULATE THE EXPANSION COEFFICIENTS, D1 AND D2, FOR EXCITATION BY A
      UNIFORM SPOT ON THE AXIS. SPOT SIZES, SPSZ, ARE .05, .10, .15, ... ETC. UP
      TO 420

      IF(L,GT,0)GO TO 450
      IF(JDEX,GT,10)GO TO 450
      NOS = -1
      DO 425 I = 1,10
      NOS = NOS + 1
      IF(SPSZ(I).GT.A)GO TO 426
      425 CONTINUE
      426 WRITE(6,128)
      ORD = 1.0
      DO 430 I = 1,NOS
      AP = SPSZ(I)
      ARG = ALL*AP
      CALL BESJ(ARG,ORD,1,ST,NZ)
      SUM = CL*ST(1)/ALL
      ARG = BT2*AP
      CALL BESI(ARG,ORD,1,1,ST,NZ)
      ARG = SUM + R1*AL*W(3)*ST(1)/BT2

```

```

S*W = SUM + R2*AL*W(3)*ST(1)/BT2
( = 2.0*SQRT(P1)
L = D2*C1*ARG
D2 = D2*C2*SUN
DRAY(JDEX,1,1) = D1
DRAY(JDEX,2,1) = D2
XX = D1*D1*PC1
YY = D1*D1*PR1
J = 1
WRITE(6,129)AP,J,D1,XX,YY
J = 2
XX = D2*D2*PC2
YY = D2*D2*PR2
430 WRITE(6,129)AP,J,D2,XX,YY
C
C CALCULATE THE FIELDS OF THE TWO UNPERTURBED MODES AND THE TWO SEPARATE
C COMPOSITE MODES.
C
450 ORD = L
WRITE(6,113)
NO = 25
IF((DIAG.GT.1)NO = 61
DR = 1.20/(NO - 1)
DO 480 I = 1,NO
RH = (1 - I)*DR
IF(RH.GT.A)GO TO 455
ARG = AL1*RH
CALL BESJ(ARG,ORD,1,1,ST,NZ)
F1 = ST(1)*CL
ARG = BT2*RH
CALL BESJ(ARG,ORD,1,1,ST,NZ)
F2 = ST(1)*AL*W(3)
GO TO 470
455 ARG = BT1*RH
CALL BESKN(ARG,L,1,1,ST,NZ)
F1 = CL*GAM*ST(1)
IF(RH.GT.C)GO TO 460
ARG = BT2*RH
CALL BESJ(ARG,ORD,1,1,ST,NZ)
F2 = AL*W(3)*ST(1)
GO TO 470
460 IF(RH.GT.1.0)GO TO 465
ARG = AL2*RH
CALL BESJ(ARG,ORD,1,1,ST,NZ)
F2 = AL*W(1)*ST(1)
CALL BESYN(ARG,L,1,ST)
F2 = F2 + AL*W(2)*ST(1)
GO TO 470
465 ARG = BT2*RH
CALL BESKN(ARG,L,1,1,ST,NZ)
F2 = AL*W(4)*ST(1)
470 CONTINUE
FP = C1*(F1 + R1*F2)
FM = C2*(F1 + R2*F2)
480 WRITE(6,114)RH,F1,F2,FP,FM
GO TO 1
C
900 CONTINUE
IF(JDEX.EQ.0)GO TO 999
WRITE(6,117)
DO 910 I = 1,JDEX
WRITE(6,118)ISS(I),(STORE(I,J),J=1,13)
910 WRITE(6,119)SSSTOR(I,J),J=1,7)
C
C CALCULATE THE TOTAL CORE-TO-RING CROSSTALK, XT, FOR THESE MODES
C

```



```

DETER = DETER*PIVOT
330 C(COLUMN,ICOLUMN) = 1.0
340 L = 1,N
350 A(ICOLUMN,L) = A(ICOLUMN,L)/PIVOT
355 IF (N) 380,380,360
360 DO 370 L = 1,M
370 B(ICOLUMN,L) = B(ICOLUMN,L)/PIVOT
C
C REDUCE NON PIVOTS ROWS
C
380 DO 550 L1 = 1,N
390 IF(L1-ICOLUMN) 400,550,400
400 T = A(L1,ICOLUMN)
420 A(L1,ICOLUMN) = 0.
430 DO 450 L = 1,N
450 A(L1,L) = A(L1,L)-A(ICOLUMN,L)*T
455 IF (N) 550,550,460
460 DO 500 L = 1,M
500 B(L1,L) = B(L1,L)-B(ICOLUMN,L)*T
550 CONTINUE
C
C INTERCHANGE COLUMNS
C
600 DO 710 I = 1,N
610 L = N + 1 - I
620 IF(INDEX(L,1)-INDEX(L,2)) 630,710,630
630 JROW = INDEX(L,1)
640 JCOLUMN = INDEX(L,2)
650 DO 705 K = 1,N
660 SWAP = A(K,JROW)
670 A(K,JROW) = A(K,JCOLUMN)
700 A(K,JCOLUMN) = SWAP
705 CONTINUE
710 CONTINUE
740 RETURN
C
750 DETER = 0.0
RETURN
END

```

Program Listing

- BESSELFUNCT-

(BESSELFUNCT routines are used by all of  
the preceding programs.)

```

C SUBROUTINE ERCHK(NCHARS,NARRAY)
C
C SANDIA MATHEMATICAL PROGRAM LIBRARY
C APPLIED MATHEMATICS DIVISION 2642
C SANDIA LABORATORIES
C ALBUQUERQUE, NEW MEXICO 87115
C
C SIMPLIFIED VERSION FOR STAND-ALONE USE. APRIL 1977
C
C ABSTRACT
C THE ROUTINES ERCHK, ERXSET, AND ERGET TOGETHER PROVIDE
C A UNIFORM METHOD WITH SEVERAL OPTIONS FOR THE PROCESSING
C OF DIAGNOSTICS AND WARNING MESSAGES WHICH ORIGINATE
C IN THE MATHEMATICAL PROGRAM LIBRARY ROUTINES.
C ERCHK IS THE CENTRAL ROUTINE, WHICH ACTUALLY PROCESSES
C MESSAGES.
C
C DESCRIPTION OF ARGUMENTS
C NCHARS - NUMBER OF CHARACTERS IN HOLLERITH MESSAGE.
C IF NCHARS IS NEGATED, ERCHK WILL UNCONDITIONALLY
C PRINT THE MESSAGE AND STOP EXECUTION. OTHERWISE,
C THE BEHAVIOR OF ERCHK MAY BE CONTROLLED BY
C AN APPROPRIATE CALL TO ERXSET.
C NARRAY - NAME OF ARRAY OR VARIABLE CONTAINING THE MESSAGE,
C OR ELSE A LITERAL HOLLERITH CONSTANT CONTAINING
C THE MESSAGE. BY CONVENTION, ALL MESSAGES SHOULD
C BEGIN WITH *IN SUBNAM, ..., WHERE SUBNAM IS THE
C NAME OF THE ROUTINE CALLING ERCHK.
C
C EXAMPLES
C 1. TO ALLOW CONTROL BY CALLING ERXSET, USE
C 1. CALL ERCHK(30,30*IN QUAD, INVALID VALUE OF ERR.)
C 2. TO UNCONDITIONALLY PRINT A MESSAGE AND STOP EXECUTION, USE
C 1. CALL ERCHK(-30,30*IN QUAD, INVALID VALUE OF ERR.)
C
C ERCHK USES SUBROUTINES ERGET, ERRPRT, ERXSET, ERSTGT
C COMPILER DECKS ERCHK
C
C DIMENSION NARRAY(14)
C
C CALL ERGET(NF,NT)
C IF ERCHK WAS CALLED WITH NEGATIVE CHARACTER COUNT, SET FATAL FLAG
C IF (NCHARS.LT.0) NF = -1
C IF MESSAGES ARE TO BE SUPPRESSED, RETURN
C IF (NF.EQ.0) RETURN
C IF CHARACTER COUNT IS INVALID, STOP
C IF (NCHARS.EQ.0) PRINT 'S
C 1. FORMAT(/31H ERCHK WAS CALLED INCORRECTLY.)
C IF (NCHARS.EQ.0) STOP
C PRINT MESSAGE
C CALL ERRPRT((ABS(NCHARS),NARRAY)
C IF LAST MESSAGE, SAY SO
C IF (NF.EQ.1) PRINT 10
C 10 FORMAT (30H ERCHK MESSAGE LIMIT REACHED.)
C PRINT TRACE-BACK IF ASKED TO
C IF ((NT.GT.0).OR.(NF.LT.0)) CALL SYSTEM ROUTINE FOR TRACEBACK
C DECREMENT MESSAGE COUNT
C IF (NF.GT.0) NF = NF-1
C CALL ERXSET(NF,NT)
C IF ALL IS WELL, RETURN
C IF (NF.GE.0) RETURN

```

ERCK 10  
 ERCK 20  
 ERCK 30  
 ERCK 40  
 ERCK 50  
 ERCK 60  
 ERCK 70  
 ERCK 80  
 ERCK 90  
 ERCK 100  
 ERCK 110  
 ERCK 120  
 ERCK 130  
 ERCK 140  
 ERCK 150  
 ERCK 160  
 ERCK 170  
 ERCK 180  
 ERCK 190  
 ERCK 200  
 ERCK 210  
 ERCK 220  
 ERCK 230  
 ERCK 240  
 ERCK 250  
 ERCK 260  
 ERCK 270  
 ERCK 280  
 ERCK 290  
 ERCK 300  
 ERCK 310  
 ERCK 320  
 ERCK 330  
 ERCK 340  
 ERCK 350

ERCK 370  
 ERCK 400

ERCK 450  
 ERCK 460

ERCK 630

ERCK 640

```

C      IF THIS MESSAGE IS SUPPRESSABLE BY AN ERXSET CALL.
C      THEN EXPLAIN ERXSET USAGE.
      IF (CHARS.GT.0) PRINT 15
15  FORMAT (1/13H ** NOTE **
1/53H TO MAKE THE ERROR MESSAGE PRINTED ABOVE BE NONFATAL.
2/39H OR TO SUPPRESS THE MESSAGE COMPLETELY,
3/37H INSERT AN APPROPRIATE CALL TO ERXSET
4/30H AT THE START OF YOUR PROGRAM.
5/62H FOR EXAMPLE: TO PRINT UP TO 10 NONFATAL WARNING MESSAGES, USE
6/27H CALL ERXSET(10,0) )
      PRINT 20
20  FORMAT (1/28H PROGRAM ABORT DUE TO ERROR.)
      STOP
      END
      SUBROUTINE ONECHK(NCHARS,NARRAY)
      C
      C      ABSTRACT
      C      ONECHK IS A COMPANION ROUTINE OF ERRCHK. IT IS CALLED
      C      JUST LIKE ERRCHK, AND MESSAGES FROM IT MAY BE SUPPRESSED
      C      BY AN APPROPRIATE CALL TO ERXSET. IT DIFFERS FROM ERRCHK
      C      IN THAT EACH CALL TO ONECHK WILL PRODUCE NO MORE THAN ONE
      C      PRINTED MESSAGE, REGARDLESS OF HOW MANY TIMES THAT CALL IS
      C      EXECUTED, AND ONECHK NEVER TERMINATES EXECUTION.
      C      ITS PURPOSE IS TO PROVIDE ONE-TIME-ONLY INFORMATIVE
      C      DIAGNOSTICS.
      C
      C      DESCRIPTION OF ARGUMENTS
      C      NCHARS - NUMBER OF CHARACTERS IN THE MESSAGE.
      C      IF NEGATED, THE MESSAGE WILL BE PRINTED (ONCE) EVEN
      C      IF NFATAL HAS BEEN SET TO 0 (SEE ERXSET).
      C      NARRAY - SAME AS IN ERRCHK
      C
      C      ONECHK USES SUBROUTINES ERRGET, ERRPRT, ERXSET, ERSTGT
      C      TO COMPILE DECKS OF ERRCHK
      C
      C      DIMENSION NARRAY(14)
      C      DATA NFLAG/4H,5,0/
      C      IF (NARRAY(1).EQ.NFLAG) RETURN
      C      CALL ERRGET(NF,NT)
      C      IF ((NF.EQ.0).AND.(NCHARS.GT.0)) RETURN
      C      CALL ERRPRT (59,59H THE FOLLOWING INFORMATIVE DIAGNOSTIC WILL APPEAR
      C      ONLY ONCE.)
      C      CALL ERRPRT(14BS(NCHARS),NARRAY)
      C      IF (NF.GT.0) NF = NF-1
      C      CALL ERXSET(NF,NT)
      C      NARRAY(1) = NFLAG
      C      END
      C      SUBROUTINE ERRPRT(NCHARS,NARRAY)
      C
      C      UTILITY ROUTINE TO SIMPLY PRINT THE HOLLERITH MESSAGE IN NARRAY,
      C      WHOSE LENGTH IS NCHARS CHARACTERS.
      C
      C      DIMENSION NARRAY(14)
      C
      C      NOTE - NCH MUST BE THE NUMBER OF HOLLERITH CHARACTERS STORED
      C      PER WORD. IF NCH IS CHANGED, FORMAT 1 MUST ALSO BE
      C      CHANGED CORRESPONDINGLY.
      C
      C      NCH = 10
      C      FOR LINE PRINTERS, USE
      C      1  FORMAT (1X,13A10)
      C      FOR DATA TERMINALS, USE
      C      1  FORMAT (1X,7A10)
      C      NWORDS = (NCHARS+NCH-1)/NCH

```

ERCK 660  
ERCK 670

ERCK 690  
ERCK 700  
USEERCK 710  
ERCK 720

ERCK 780  
ERCK 790  
ERCK1350  
ERCK1360  
ERCK1370  
ERCK1380  
ERCK1390  
ERCK1400  
ERCK1410  
ERCK1420  
ERCK1430  
ERCK1440  
ERCK1450  
ERCK1460  
ERCK1470  
ERCK1480  
ERCK1490  
ERCK1510  
ERCK1520

ERCK1530

ERCK1640

ERCK 410  
ERCK 430  
ERCK 440

```

PRINT 1.(NARRAY(I),I=1,NWORDS)
RQ N
ENL
SUBROUTINE ERXSET(NFATAL,NTRACE)

ABSTRACT
  ERXSET IS A COMPANION ROUTINE TO SUBROUTINE ERRCHK.
  ERXSET ASSIGNS THE VALUES OF NFATAL AND NTRACE RESPECTIVELY
  TO NF AND NT IN COMMON BLOCK MBLK0 THEREBY SPECIFYING THE
  STATE OF THE OPTIONS WHICH CONTROL THE EXECUTION OF ERRCHK.

DESCRIPTION OF ARGUMENTS
  BOTH ARGUMENTS ARE INPUT ARGUMENTS OF DATA TYPE INTEGER.
  NFATAL - IS A FATAL-ERROR / MESSAGE-LIMIT FLAG. A NEGATIVE
  VALUE DENOTES THAT DETECTED DIFFICULTIES ARE TO BE
  TREATED AS FATAL ERRORS. NONNEGATIVE MEANS NONFATAL.
  A NONNEGATIVE VALUE IS THE MAXIMUM NUMBER OF NONFATAL
  WARNING MESSAGES WHICH WILL BE PRINTED BY ERRCHK,
  AFTER WHICH NONFATAL MESSAGES WILL NOT BE PRINTED.
  (DEFAULT VALUE IS -1.)
  NTRACE - .GE.1 WILL CAUSE A TRACE-BACK TO BE GIVEN,
  IF THIS FEATURE IS IMPLEMENTED ON THIS SYSTEM.
  .LE.0 WILL SUPPRESS ANY TRACE BACK, EXCEPT FOR
  CASES WHEN EXECUTION IS TERMINATED.
  (DEFAULT VALUE IS 0.)

*NOTE* --- SOME CALLS TO ERRCHK WILL CAUSE UNCONDITIONAL
TERMINATION OF EXECUTION. ERXSET HAS NO EFFECT ON SUCH CALLS.

EXAMPLES
  1. TO PRINT UP TO 100 MESSAGES AS NONFATAL WARNINGS USE
     CALL ERXSET(100,0)
  2. TO SUPPRESS ALL MATHLIB WARNING MESSAGES USE
     CALL ERXSET(0,0)

ERXSET USES SUBROUTINES ERSTGT
  AND ERSET DECKS ERRCHK

CALL ERSTGT(0,NFATAL,NTRACE)
RETURN
END
SUBROUTINE ERGET(NFATAL,NTRACE)

ABSTRACT
  ERGET IS A COMPANION ROUTINE TO SUBROUTINE ERRCHK.
  ERGET ASSIGNS TO FATAL AND NTRACE RESPECTIVELY THE VALUES
  OF NF AND NT IN COMMON BLOCK MBLK0 THEREBY ASCERTAINING THE
  STATE OF THE OPTIONS WHICH CONTROL THE EXECUTION OF ERRCHK.

DESCRIPTION OF ARGUMENTS
  BOTH ARGUMENTS ARE OUTPUT ARGUMENTS OF DATA TYPE INTEGER.
  NFATAL - CURRENT VALUE OF NF (SEE DESCRIPTION OF ERXSET.)
  NTRACE - CURRENT VALUE OF NT (SEE DESCRIPTION OF ERXSET.)

CALL ERSTGT(1,NFATAL,NTRACE)
RETURN
END
SUBROUTINE ERSTGT(K,NFATAL,NTRACE)

THIS ROUTINE IS A SLAVE TO ERGET AND ERSET WHICH KEEPS
THE FLAGS AS LOCAL VARIABLES.

*** IF LOCAL VARIABLES ARE NOT NORMALLY RETAINED BETWEEN
CALLS ON THIS SYSTEM, THE VARIABLES LNF AND LNT CAN BE

```

PLACED IN A COMMON BLOCK AND PRESET TO THE FOLLOWING  
VAY 5 IN THE MAIN PROGRAM.

```
DATA LNF/-1/.LNT/0/
IF (K.LE.0) LNF = NFATAL
IF (K.LE.0) LNT = NTRACE
IF (K.GT.0) NFATAL = LNF
IF (K.GT.0) NTRACE = LNT
RETURN
END
FUNCTION BESK01(X,NU,KODE,NZ)
```

WRITTEN BY D.E. AMOS AND S.L. DANIEL, FEBRUARY, 1974.

REFERENCE SAND-75-0149

# ABSTRACT

BESK01 COMPUTES BESSEL FUNCTIONS  $K/SUB(NU)/(X)$ ,  $NU=0$  OR  $1$   
OR SCALED BESSEL FUNCTIONS  $EXP(X)*K/SUB(NU)/(X)$ ,  
 $NU=0$  OR  $1$  FOR  $X.GT.0$ . CHEBYSHEV EXPANSIONS, PROPERLY SCALED  
FOR SMALL AND LARGE  $X$ , ARE USED ON INTERVALS  $0.LT.X.LE.2$ ,  
 $2.LT.X.LE.5$ , AND  $X.GT.5$ . THE UNDERFLOW TEST IS MADE ON  
 $X.LE.ELIM$  WITH  $ELIM=667$ . BESK01 CALLS FUNCTION BES101.

# DESCRIPTION OF ARGUMENTS

## INPUT

X -  $0.LT.X.LE.667$ . FOR  $KODE=1$ ,  $X.GT.0$  FOR  $KODE=2$   
NU - ORDER DESIRED,  $NU=0$  OR  $1$   
KODE - A PARAMETER TO INDICATE THE SCALING OPTION  
KODE=1 RETURNS  $ANS=K/SUB(NU)/(X)$ ,  $NU=0$  OR  $1$   
KODE=2 RETURNS  $ANS=EXP(X)*K/SUB(NU)/(X)$ ,  $NU=0$  OR  $1$

## OUTPUT

BESK01 - K BESSEL FUNCTION OF ORDER NU AT X SCALED ACCORDING  
TO KODE  
NZ - UNDERFLOW INDICATOR  
NZ=0, NORMAL RETURN, COMPUTATION COMPLETED  
NZ.NE.0, ANS SET TO ZERO DUE TO UNDERFLOW WITH  
KODE=1 AND  $X.GT.667$

## ERROR CONDITIONS

IMPROPER INPUT ARGUMENTS - A FATAL ERROR  
UNDERFLOW WITH  $KODE=1$  - A NON-FATAL ERROR(NZ.NE.0)

BESK01 USES SUBROUTINES BES101, ERRCHK, ERRGET, ERRPRT,  
ERRSET, ERSTGT  
COMPILE DECKS BESK01, BES101, ERRCHK

DIMENSION AKO(15),BKO(21),CKO(14),AK1(15),BK1(22),CK1(14)

DATA ELIM / 667. /

DATA N1,N2,N3,N4/15,21,14,22/

DATA M1,M2,M3,M4/13,19,12,20/

DATA EMLT/-1.15931515658412E-1/

DATA AKO /

1 2.20263147944787E-01, 2.94143920648988E-02, 4.89766239821850E-01, 6.85071478635282E-01,

2 3.65095673281410E-04, 3.96552105234993E-05, 2.13571244350405E-06,

3 1.74123741003729E-07, 7.26649791536202E-09, 4.74231032016171E-10,

4 1.61749721604899E-11, 8.8011420783973E-13, 2.53991582114493E-14,

5 1.18507018244790E-15/

DATA BKO /

1-3.32975981412374E-03, 6.70802900717429E-04, 1.36272865384502E-04,

2 2.78660969643180E-05, 5.74300240638451E-06, 1.1894225477576E-06,

3-2.47573075834394E-07, 5.17613906627857E-08, 1.0865350852187E-08,

4 2.28699509247299E-09, 4.83794289746110E-10, 1.02556286108303E-10,

```

5-2.17950101893024E-11, 4.64499519650372E-12, -9.92020238192219E-13,
6 2.30748247184E-13, -4.55250947249326E-14, 9.77948196627E-15,
7-2.428853630363E-15/
DATA CKO / 1.23878593820170E+00, -1.41756153741679E-02,
1 3.37809034043106E-04, -1.35201655758935E-05, 7.92815326397551E-07,
2-5.64584651140153E-08, 4.75381608069133E-09, -4.56892133422380E-10,
3 4.89340910679015E-11, -5.74009881252931E-12, 7.27938173682122E-13,
4-9.88017112659449E-14, 1.32381749918418E-14, -2.16443591571975E-15/
DATA AK1 / 4.62486240842478E-01, 6.05181915156673E-01,
1 1.53253828510709E-01, 3.35117014379959E-02, 3.36879962666501E-03,
2 4.41042696291495E-04, 2.83189625853098E-05, 2.64548793222978E-06,
3 1.25637058646502E-07, 9.12550891350504E-09, 3.44783302431681E-10,
4 2.04890221867815E-11, 6.43563932230815E-13, 3.23636474094122E-14,
5 8.70458673285249E-16/
DATA BK1 / 1.38930840300441E+00, -5.89753920597669E-02,
1 1.20249909184072E-02, -2.55387533195591E-03, 5.45258387639594E-04,
2-1.16936329705121E-04, 2.51746904352026E-05, -5.43777151950107E-06,
3 1.17797109704500E-06, -2.55830051061306E-07, 5.568564833337E-08,
4-1.21451129231835E-08, 2.65359402057287E-09, -5.80715571337876E-10,
5 1.27268495790715E-10, -2.79285412641216E-11, 6.13612949711377E-12,
6-1.34963109251970E-12, 2.97146532041432E-13, -6.54827012807628E-14,
7 1.44428209792469E-14, -3.18800891899413E-15/
DATA CK1 / 1.29837185686353E+00, 4.4449150892785E-02,
1-5.87146375780839E-04, 2.02849692864934E-05, -1.05926607637730E-06,
2 7.16046166444819E-08, -5.82233926413546E-09, 5.45714706062071E-10,
3-5.73482672655767E-11, 6.62769161077494E-12, -8.30462152737046E-13,
4 1.11605017034540E-13, -1.59497634434667E-14, 2.40744099380385E-15/
C
BESK01 = -XIND
IF (MODE.LT.1.OR.MODE.GT.2) GO TO 90
IF (NU.LT.0.OR.NU.GT.1) GO TO 91
IF (X.LE.0.) GO TO 92
IK=NU+1
NZ=0
GO TO (100,200),IK
C
K/SUB(0)/(X) BESSEL FUNCTION
C
100 CONTINUE
IF (X.GT.2.) GO TO 110
TX=X-1.
TT=T+T
J=N1
F1=AK0(J)
F2=0.
DO 105 I=1,M1
J=J-1
TEMP1=F1
F1=TT-F1-F2*AK0(J)
F2=TEMP1
105 CONTINUE
BNS=BESJ01(X,0,1)
ANS=-(EMLT+ALOG(X))*BNS*(T+F1-F2*AK0(1))
GO TO (106,107), MODE
107 BESK01=ANS*EXP(X)
RETURN
108 BESK01=ANS
RETURN
110 IF (X.GT.5.) GO TO 120
T=(X+X-7.)/3.
TT=T+T
J=N2
F1=BK0(J)
F2=0.
DO 115 I=1,M2
J=J-1

```

```

TEMP=F1
F1=(F1-F2+BK0(J)
F2=TEMP1
115 CONTINUE
ANS=(T*F1-F2+BK0(1))/SORT(X)
GO TO (116,106),KODE
116 BESK01=ANS*EXP(-X)
RETURN
120 CONTINUE
T=10./X-1.
TT=T+T
J=N3
F1=CK0(J)
F2=0.
DO 125 I=1,M3
J=J-1
TEMP1=F1
F1=TT*F1-F2+CK0(J)
F2=TEMP1
125 CONTINUE
ANS=(T*F1-F2+CK0(1))/SORT(X)
GO TO (126,106),KODE
126 IF(X.GT.ELIM) GO TO 93
GO TO 116
C
C K/SUB(1)/(X) BESSEL FUNCTION
C
200 CONTINUE
IF(X.GT.2.) GO TO 210
T=X-1.
TT=T+T
J=N1
F1=AK1(J)
F2=0.
DO 205 I=1,M1
J=J-1
TEMP1=F1
F1=TT*F1-F2+AK1(J)
F2=TEMP1
205 CONTINUE
BNS=BESIO1(X,1,1)
ANS=((EMLT+ALOG(X))*BNS+1./X-(T*F1-F2+AK1(1)))
GO TO (106,107),KODE
210 IF(X.GT.5.) GO TO 220
T=(X+X-7.)/3.
TT=T+T
J=N4
F1=BK1(J)
F2=0.
DO 215 I=1,M4
J=J-1
TEMP1=F1
F1=TT*F1-F2+BK1(J)
F2=TEMP1
215 CONTINUE
ANS=(T*F1-F2+BK1(1))/SORT(X)
GO TO (116,106),KODE
220 CONTINUE
T=10./X-1.
TT=T+T
J=N3
F1=CK1(J)
F2=0.
DO 225 I=1,M3
J=J-1
TEMP1=F1

```

```

      F1=F1-F2+CK1(J)
      F2= F1
225 CONTINUE
      ANS=(F1-F2+CK1(1))/SORT(X)
      GO TO (226,106) ,MODE
226 IF(X.GT.ELIM) GO TO 93
      GO TO 116
90 CONTINUE
      CALL ERRCHK(44,44HIN BESK01, SCALING OPTION, MODE, NOT 1 OR 2.)
      RETURN
91 CONTINUE
      CALL ERRCHK(33,33HIN BESK01, ORDER, NU, NOT 0 OR 1.)
      RETURN
92 CONTINUE
      CALL ERRCHK(40,40HIN BESK01, X LESS THAN OR EQUAL TO ZERO.)
      RETURN
93 CONTINUE
      BESK01=0.
      NZ=1
      RETURN
      END
      FUNCTION BESY01(X,NU,ANSJ)

```

WRITTEN BY D.E. AMOS AND S.L. DANIEL, FEBRUARY, 1974

REFERENCE SAND-75-0148

# ABSTRACT

BESY01 COMPUTES BESSEL FUNCTIONS  $J_{\text{SUB}(NU)}/(X)$  AND  $Y_{\text{SUB}(NU)}/(X)$ ,  $NU=0$  OR  $1$  FOR  $X.GT.0$ . RATIONAL CHEBYSHEV APPROXIMATIONS, ASYMPTOTICALLY SCALED FOR SMALL AND LARGE  $X$ , ARE USED ON INTERVALS  $0.LE.X.LE.8$  AND  $X.GT.8$ . THE COST IN RETURNING  $J_{\text{SUB}(NU)}/(X)$  IS MINIMAL SINCE THIS FUNCTION IS NEEDED IN THE ASYMPTOTIC FORM FOR  $0.LT.X.LE.8$  AND ONLY REQUIRES A REARRANGEMENT OF 4 FACTORS NEEDED FOR  $X.GT.8$ . BESY01 CALLS FUNCTION BESJ01.

# DESCRIPTION OF ARGUMENTS

## INPUT

X - X.GT.0.  
NU - ORDER DESIRED,  $NU=0$  OR  $1$

## OUTPUT

BESY01 - Y BESSEL FUNCTION OF ORDER NU AT X  
ANSJ - J BESSEL FUNCTION OF ORDER NU AT X

## ERROR CONDITIONS

IMPROPER INPUT ARGUMENTS - A FATAL ERROR

BESY01 USES SUBROUTINES BESJ01, ERRCHK, ERRGET, ERRPRT,

ERRSET, ERSTGT

COMPILE DECKS BESY01, BESJ01, ERRCHK

DIMENSION P0(8),Q0(8),P1(7),Q1(8),P0N(4),P0D(5),Q0N(4),Q0D(5),  
1 P1N(4),P1D(5),Q1N(4),Q1D(5)

DATA N1,N2,N3,N4,M1,M2,M3,M4/B,7.5,4,6,5,3,2/  
DATA P104,TFPI,TOPI / 7.85398163397448E-01, 2.35619449019234E+00,  
1 6.36619772367581E-01/

DATA P0 /-1.01860971224170E+16, 2.43804485256950E+16,  
1-1.90205266308270E+15, 4.77091764845963E+13,-5.28348803896724E+11,  
2 2.86910828729839E+09,-7.53147545781676E+06, 7.70593364872924E+03/

```

DAT. 00 / 1.38014963863775E+17, 1.78633877994688E+15,
1 752268723807E+13, 5.21829549438144E+10, 1.71651744157E+08,
2 4.467794501471E+05, 8.18358066417824E+02, 1.000000000000E+00,
DATA P1 /-1.06525580073146E+16, 2.80179513474828E+15,
1-1.19784970206113E+14, 1.91351887731600E+12, -1.38006319639808E+10,
2 4.57445909094871E+07, -5.70683926136436E+04/
DATA Q1 / 5.43339594207058E+16, 7.71123767247204E+14,
1 5.61619655823693E+12, 2.77405419245635E+10, 1.02955286238916E+08,
2 2.97764175120939E+05, 8.58392393694713E+02, 1.000000000000E+00/
DATA PON / 5.43387249648474E+05, 3.99030011721535E+05,
1 6.08286198525507E+04, 1.57972732370604E+03/
DATA POD / 8.81034922021421E+05, 5.70858162215322E+05,
1 7.67689810942388E+04, 2.05200863309931E+03, 1.000000000000E+00/
DATA PIN /-4.21538738930469E+05, -2.97874854285400E+05,
1-4.26857048542501E+04, -9.89621122975118E+02/
DATA PID /-5.28320460927703E+05, -3.72363362370422E+05,
1-5.28353786226566E+04, -1.15530681849500E+03, 1.000000000000E+00/
DATA QON /-4.92207223828469E+02, -4.39316681986953E+02,
1-8.58904583816188E+01, -3.12667145228239E+00/
DATA QOD / 3.94809774152373E+04, 3.55999748740455E+04,
1 7.19786937649468E+03, 3.02885800333886E+02, 1.000000000000E+00/
DATA QIN / 3.68932567438047E+03, 3.37134265139743E+03,
1 6.32384402139958E+02, 2.17125499052971E+01/
DATA QID / 1.03990279374871E+05, 9.05851244162508E+04,
1 1.72767909550058E+04, 6.40089571203976E+02, 1.000000000000E+00/

```

```

C BESY01 = -X*IND
IF(NU.LT.0.OR.NU.GT.1) GO TO 91
IF(X.LE.0.) GO TO 92
JS=NU+1
IF(X.GT.8.) GO TO 200
X2=X*X
X.LE.8
GO TO (10,20),JS

```

```

C Y/SUB(0)/(X) AND Y/SUB(0)/(X) BESSEL FUNCTION

```

```

10 CONTINUE
SUMP=P0(N1)*X2+P0(N2)
SUMQ=Q0(N1)*X2+Q0(N2)
J=M1
DO 15 I=1,M1
SUMP=SUMP+X2*P0(J)
SUMQ=SUMQ+X2*Q0(J)
J=J-1

```

```

15 CONTINUE
ANS=SUMP/SUMQ
ANSJ=BESJ01(X,0)
BESY01=TOPI*ALOG(X)*ANSJ+ANS
RETURN

```

```

C Y/SUB(1)/(X) AND Y/SUB(1)/(X) BESSEL FUNCTION

```

```

20 CONTINUE
SUMP=P1(N2)*X2+P1(M1)
SUMQ=Q1(N1)*X2+Q1(N2)
J=M1
DO 25 I=1,M2
SUMQ=SUMQ+X2*Q1(J)
J=J-1
SUMP=SUMP+X2*P1(J)

```

```

25 CONTINUE
SUMQ=SUMQ+X2*Q1(J)
ANS=X*(SUMP/SUMQ)
ANSJ=BESJ01(X,1)
BESY01=TOPI*(ALOG(X)*ANSJ-1./X)+ANS
RETURN

```

```

C      X,GT,.8
C 200 CD/ NUC
      Z=b,X
      Z2=Z*Z
      GO TO (30,40),JS
C      Y/SUB(0)/(X) AND J/SUB(0)/(X) BESSEL FUNCTION
C
C 30 CONTINUE
      SUMPN=PON(N4)*Z2+PON(M3)
      SUMQN=QON(N4)*Z2+QON(M3)
      SUMPD=POD(M2)*Z2+POD(N4)
      SUMQD=QOD(M2)*Z2+QOD(N4)
      J=M3
      DO 35 I=1,M4
      SUMPD=SUMPD+Z2+POD(J)
      SUMQD=SUMQD+Z2+QOD(J)
      J=J+1
      SUMPN=SUMPN+Z2+PON(J)
      SUMQN=SUMQN+Z2+QON(J)
C 35 CONTINUE
      SUMPD=SUMPD+Z2+POD(J)
      SUMQD=SUMQD+Z2+QOD(J)
      XV=X-PI/4
      GO TO 50
C      Y/SUB(1)/(X) AND J/SUB(1)/(X) BESSEL FUNCTION
C
C 40 CONTINUE
      SUMPN=PIN(N4)*Z2+PIN(M3)
      SUMQN=QIN(N4)*Z2+QIN(M3)
      SUMPD=PID(M2)*Z2+PID(N4)
      SUMQD=QID(M2)*Z2+QID(N4)
      J=M3
      DO 45 I=1,M4
      SUMPD=SUMPD+Z2+PID(J)
      SUMQD=SUMQD+Z2+QID(J)
      J=J+1
      SUMPN=SUMPN+Z2+PIN(J)
      SUMQN=SUMQN+Z2+QIN(J)
C 45 CONTINUE
      SUMPD=SUMPD+Z2+PID(J)
      SUMQD=SUMQD+Z2+QID(J)
      XV=X-1/2*PI
C 50 CONTINUE
      CXV=COS(XV)
      SXV=SIN(XV)
      PV=SUMPN/SUMPD
      QV=(SUMQN/SUMQD)*Z
      RRTX=1./SQRT(X)
      ANSJ=(PV*CXV-OV*SV)*RRTX
      BESY01=(QV*CXV+PV*SV)*RRTX
      RETURN
C 91 CONTINUE
      CALL ERRCBK(33,33HIN BESY01, ORDER, NU, NOT 0 OR 1.)
      RETURN
C 92 CONTINUE
      CALL ERRCBK(40,40HIN BESY01, X LESS THAN OR EQUAL TO ZERO.)
      RETURN
      END
      FUNCTION BESJ01(X,NU)
C
C WRITTEN BY D.E. AMOS AND S.L. DANIEL, FEBRUARY, 1974
C
C REFERENCE SAND-75-0148
C

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      GO TO (10,20),JS
      C      J/SUB(0)/(X) BESSEL FUNCTION
      C
      10 CONTINUE
        SUMP=P0(N1)*X2+P0(N2)
        SUMQ=Q0(N1)*X2+Q0(N2)
        J=M1
        DO 15 I=1,M1
          SUMP=SUMP+X2+P0(J)
          SUMQ=SUMQ+X2+Q0(J)
          J=J+1
        15 CONTINUE
        BESJ01=SUMP/SUMQ
        RETURN
      C
      C      J/SUB(1)/(X) BESSEL FUNCTION
      C
      20 CONTINUE
        SUMP=P1(N2)*X2+P1(M1)
        SUMQ=Q1(N1)*X2+Q1(N2)
        J=M1
        DO 25 I=1,M2
          SUMP=SUMP+X2+Q1(J)
          SUMQ=SUMQ+X2+P1(J)
          J=J+1
        25 CONTINUE
        SUMP=SUMP+X2+Q1(J)
        BESJ01=X*(SUMP/SUMQ)
        RETURN
      C
      C      K.G1.B
      200 CONTINUE
        Z=8./AX
        Z2=Z*Z
        GO TO (30,40),JS
      C
      C      J/SUB(0)/(X) BESSEL FUNCTION
      C
      30 CONTINUE
        SUMPN=P0N(N4)*Z2+P0N(M3)
        SUMQN=Q0N(N4)*Z2+Q0N(M3)
        SUMPD=P0D(M2)*Z2+P0D(N4)
        SUMQD=Q0D(M2)*Z2+Q0D(N4)
        J=M3
        DO 35 I=1,M4
          SUMPD=SUMPD+Z2+P0D(J)
          SUMQD=SUMQD+Z2+Q0D(J)
          J=J+1
        35 CONTINUE
        SUMPN=SUMPN+Z2+P0N(J)
        SUMQN=SUMQN+Z2+Q0N(J)
        SUMPD=SUMPD+Z2+P0D(J)
        SUMQD=SUMQD+Z2+Q0D(J)
        XV=AX-PI/4
        BESJ01=((SUMPN/SUMPD)*COS(XV)-Z*(SUMQN/SUMQD)*SIN(XV))/SQRT(AX)
        RETURN
      C
      C      J/SUB(1)/(X) BESSEL FUNCTION
      C
      40 CONTINUE
        SUMPN=PIN(N4)*Z2+PIN(M3)
        SUMQN=Q1N(N4)*Z2+Q1N(M3)
        SUMPD=PID(M2)*Z2+PID(N4)
        SUMQD=Q1D(M2)*Z2+Q1D(N4)
        J=M3
        DO 45 I=1,M4

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SUMPO=SUMPO+Z2*PID(J)
SU( J )=SUMQD+Z2*Q1D(J)
J=J+1
SUMPN=SUMPN+Z2*PIN(J)
SUMON=SUMON+Z2*QIN(J)
45 CONTINUE
SUMPD=SUMPD+Z2*PID(J)
SUMQD=SUMQD+Z2*Q1D(J)
XV=AX-TFPI
ANS=((SUMPN/SUMPD)*COS(XV)-Z*(SUMON/SUMQD)*SIN(XV))/SQRT(AX)
BESJ01=ANS*SIGN(1...X)
RETURN
91 CONTINUE
CALL ERRCHK(33.33HIN BESJ01, ORDER, NU, NOT 0 OR 1.)
RETURN
END
FUNCTION BESJ01(X,NU,MODE)
WRITTEN BY D.E. AMOS AND S.L. DANIEL, FEBRUARY, 1974.
REFERENCE SAND-75-0149
ABSTRACT
BESJ01 COMPUTES BESSEL FUNCTIONS  $I_{\nu}(X)$  AND  $J_{\nu}(X)$  FOR  $\nu=0$  OR  $1$ 
OR SCALED BESSEL FUNCTIONS  $\exp(-\text{ABS}(X)) * I_{\nu}(\text{SUB}(NU)/(X))$  AND  $J_{\nu}(\text{SUB}(NU)/(X))$ 
FOR SMALL AND LARGE  $\text{ABS}(X)$ . ARE USED ON INTERVALS  $0 \leq X \leq 4$ ,
 $4 \leq X \leq 8$ , AND  $X \geq 8$ . THE SIGN IS FIXED ACCORDING TO THE
EVENNESS OR ODDNESS OF THE FUNCTION. THE OVERFLOW TEST IS MADE
ON  $\text{ABS}(X) \cdot \text{LE} \cdot \text{ELIM}$  WITH  $\text{ELIM}=667$ .
DESCRIPTION OF ARGUMENTS
INPUT
X -  $\text{ABS}(X) \cdot \text{LE} \cdot 667$ . FOR  $\text{MODE}=1$ , UNRESTRICTED FOR  $\text{MODE}=2$ 
NU - ORDER DESIRED,  $\text{NU}=0$  OR  $1$ 
MODE - A PARAMETER TO INDICATE THE SCALING OPTION
       $\text{MODE}=1$  RETURNS  $\text{ANS} = I_{\nu}(\text{SUB}(NU)/(X))$ ,  $\text{NU}=0$  OR  $1$ 
       $\text{MODE}=2$  RETURNS  $\text{ANS} = \exp(-X) * I_{\nu}(\text{SUB}(NU)/(X))$ ,  $\text{NU}=0$  OR  $1$ 
OUTPUT
BESJ01 -  $I_{\nu}$  BESSEL FUNCTION OF ORDER NU AT X SCALED ACCORDING
      TO MODE
ERROR CONDITIONS
IMPROPER INPUT ARGUMENTS - A FATAL ERROR
OVERFLOW FOR  $\text{MODE}=1$  - A FATAL ERROR
BESJ01 USES SUBROUTINES ERRCHK, ERRGET, ERRPRT, ERRSET, ERSTGT
COMPILER DECKS BESJ01, ERRCHK
DIMENSION A10(18),B10(18),C10(19),A11(18),B11(18),C11(19)
DATA ELIM / 667. /
DATA M1,M2/18,19/
DATA M1,M2/16,17/
DATA A10
1 1.93239151962331E+00, 5.12072928951984E-01, 1.19244431087252E-01,
2 2.04767667749564E-02, 3.29580433784138E-03, 4.17924020345661E-04,
3 5.13658909026708E-05, 5.16036527683877E-06, 5.12886756184602E-07,
4 4.26546791785515E-08, 3.5582325462485E-09, 2.52433036418873E-10,
5 1.81419096716532E-11, 1.12207674472741E-12, 7.08316386761487E-14,
6 3.88308114508344E-15/
DATA B10
1 4.27652251957361E-03, -8.56652186435765E-04, 1.08431306444222E-04,

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CC

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2-1 1385649051770E-05, 3.18457963070919E-06, -5.48113448889 E-07,
3 4.22416818253E-08, -1.5082504887134E-08, 2.35026532228 E-09,
4-3.4904194258885E-10, 4.91824029507119E-11, -6.5670859174462E-12,
5 8.30939305382694E-13, -9.97106489731714E-14, 1.13614195790750E-14,
6-1.2311094739334E-15/
DATA C10 / 4.02245205507054E-01, 3.36911647825569E-03,
1 6.88975834691682E-05, 2.89137052083476E-06, 2.04891858946905E-07,
2 2.2666889049816E-08, 3.39623202570920E-09, 4.94060238821134E-10,
3 1.18891471071382E-11, -3.14991652807488E-11, -1.32158118398615E-11,
4-1.79417853277908E-12, 7.18012446526379E-13, 3.85277837074653E-13,
5 1.54008635387642E-14, 4.15056915510500E-14, -9.55484713964049E-15,
6 3.81168204982708E-15, 1.77256039316563E-15/
DATA A11 / 1.11729135052299E+00, 8.95198877808018E-01,
1 3.36971231909350E-01, 7.24336830339257E-02, 1.53300913475520E-02,
2 2.26237207566113E-03, 3.36089476871761E-04, 3.78329005043256E-05,
3 4.3436708173434E-06, 3.95545350342659E-07, 3.70599583606154E-08,
4 2.83473074207374E-09, 2.24494698516683E-10, 1.4806707975197E-11,
5 1.01588206538587E-12, 5.89015292859435E-14, 3.56565058329046E-15,
6 1.84456468936977E-16/
DATA B11 / 1.5423684772347E-01, -2.20987741718608E-02,
1 2.19839683870564E-03, -2.0705598880429E-04, 1.22621848833534E-05,
2 1.43239687327040E-06, -7.91717404214453E-07, 2.15368543162913E-07,
3-4.70664774819712E-08, 9.04215441510569E-09, -1.57608499172581E-09,
4 2.52964136818147E-10, -3.77065425810734E-11, 5.25015808110040E-12,
5-6.85889299579377E-13, 8.43830741608493E-14, -9.80745080362827E-15,
6 1.07990041545005E-15/
DATA C11 / 3.89288117509140E-01, -9.76109749136147E-03,
1-1.10588938762624E-04, -3.88256480887769E-06, -2.51233623787023E-07,
2-2.6314688468956E-08, -3.8353803858299E-09, -5.58974346220626E-10,
3-1.8974958125807E-11, 3.25260358287477E-11, 1.41258074355730E-11,
4 2.03562854596022E-12, -7.19855176877244E-13, -4.08355112068096E-13,
5-2.10154176162218E-14, 4.27244003763443E-14, 1.04202768179381E-14,
6-3.81440283617941E-15, -1.88035459733009E-15/
C
BES101 = -X*IND
IF (MODE, LT, 1, OR, MODE, GT, 2) GO TO 90
IF (NU, LT, 0, OR, NU, GT, 1) GO TO 91
AX=ABS(X)
IX=NU+1
GO TO (100,200),IX
C
1/SUB(O)/(X) BESSEL FUNCTION
C
100 CONTINUE
IF (AX, GT, 4.) GO TO 110
TT=AX-2.
T=TT*.5
J=N1
F1=A10(J)
F2=0.
DO 105 I=1,M1
J=J-1
TEMP1=F1
F1=TT*F1-F2+A10(J)
F2=TEMP1
105 CONTINUE
ANS=T*F1-F2+A10(1)
GO TO (106,107), MODE
107 BES101=ANS*EXP(-AX)
RETURN
108 BES101=ANS
RETURN
110 IF (AX, GT, 8.) GO TO 120
TT=AX-6.
T=TT*.5
J=N1

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```

F1=1.0(U)
F2
DO 115 I=1,M1
J=J-1
TEMP1=F1
F1=TT*F1-F2+B10(U)
F2=TEMP1
115 CONTINUE
ANS=T*F1-F2+B10(1)
GO TO (116,108),MODE
116 BES101=ANS*EXP(AX)
120 RETURN
120 CONTINUE
T=16./AX-1.
TT=T*T
J=N2
F1=C10(U)
F2=0.
DO 125 I=1,M2
J=J-1
TEMP1=F1
F1=TT*F1-F2+C10(U)
F2=TEMP1
125 CONTINUE
ANS=(T*F1-F2+C10(1))/SORT(AX)
GO TO (126,106),MODE
126 IF(AX.GT.ELIM) GO TO 92
BES101=ANS*EXP(AX)
RETURN
C
C I/SUB(1)/(X) BESSEL FUNCTION
C
200 CONTINUE
IF(AX.GT.4.) GO TO 210
TT=AX-2.
T=TT*.5
J=N1
F1=A11(U)
F2=0.
DO 205 I=1,M1
J=J-1
TEMP1=F1
F1=TT*F1-F2+A11(U)
F2=TEMP1
205 CONTINUE
ANS=X*{(T*F1-F2+A11(1))}
GO TO (106,107),MODE
210 IF(AX.GT.8.) GO TO 220
TT=AX-6.
T=TT*.5
J=N1
F1=B11(U)
F2=0.
DO 215 I=1,M1
J=J-1
TEMP1=F1
F1=TT*F1-F2+B11(U)
F2=TEMP1
215 CONTINUE
ANS=T*F1-F2+B11(1)
GO TO (216,217), MODE
216 ANS=ANS*EXP(AX)
217 BES101=SIGN(ANS,X)
RETURN
220 CONTINUE
T=16./AX-1.

```

```

      TT=TT+T
      JH)  I(J)
      F1)
      F2=0.
      DO 225 I=1,M2
      J=J+1
      TEMPI=F1
      F1=TT+F1-F2+CI1(J)
      F2=TEMPI
225  CONTINUE
      ANS=((T*F1-F2+CI1(J))/SQRT(AX)
      GO TO (226,217),MODE
226  IF(AX.GT.ELIM) GO TO 92
      GO TO 216
90  CONTINUE
      CALL ERRCHK(44,44)IN BES101, SCALING OPTION, MODE, NOT 1 OR 2.)
      RETURN
91  CONTINUE
      CALL ERRCHK(33,33)IN BES101, ORDER, NU, NOT 0 OR 1.)
      RETURN
92  CONTINUE
      CALL ERRCHK(49,49)IN BES101, OVERFLOW, ABS(X) TOO LARGE FOR MODE=1
      1.)
      RETURN
      END
      SUBROUTINE BESJ(X,ALPHA,N,Y,NZ)

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WRITTEN BY D.E. AMOS, S.L. DANIEL AND M.K. WESTON, JANUARY, 1975.

REFERENCE SAND-75-0147

ABSTRACT  
 BESJ COMPUTES AN N MEMBER SEQUENCE OF J BESSEL FUNCTIONS  
 $J_{\text{SUB}(\text{ALPHA}+K-1)/(\text{X})}$ ,  $K=1, \dots, N$  FOR NON-NEGATIVE ALPHA AND X.  
 A COMBINATION OF THE POWER SERIES, THE ASYMPTOTIC EXPANSION  
 FOR X TO INFINITY AND THE UNIFORM ASYMPTOTIC EXPANSION FOR  
 NU TO INFINITY ARE APPLIED OVER SUBDIVISIONS OF THE (NU,X)  
 PLANE. FOR VALUES OF (NU,X) NOT COVERED BY ONE OF THESE  
 FORMULAE, THE ORDER IS INCREMENTED OR DECREMENTED BY INTEGER  
 VALUES INTO A REGION WHERE ONE OF THE FORMULAE APPLY. BACKWARD  
 RECURSION IS APPLIED TO REDUCE ORDERS BY INTEGER VALUES EXCEPT  
 WHERE THE ENTIRE SEQUENCE LIES IN THE OSCILLATORY REGION. IN  
 THIS CASE FORWARD RECURSION IS STABLE AND VALUES FROM THE  
 ASYMPTOTIC EXPANSION FOR X TO INFINITY START THE RECURSION  
 WHEN IT IS EFFICIENT TO DO SO. LEADING TERMS OF THE SERIES AND  
 UNIFORM EXPANSION ARE TESTED FOR UNDERFLOW. IF A SEQUENCE IS  
 REQUESTED AND THE LAST MEMBER WOULD UNDERFLOW, THE RESULT IS  
 SET TO ZERO AND THE NEXT LOWER ORDER TRIED, ETC., UNTIL A  
 MEMBER COMES ON SCALE OR ALL MEMBERS ARE SET TO ZERO. OVERFLOW  
 CANNOT OCCUR. BESJ CALLS SUBROUTINE JAIRY AND FUNCTION GAMLN.

# DESCRIPTION OF ARGUMENTS

```

INPUT
X      - X,GE.0
ALPHA  - ORDER OF FIRST MEMBER OF THE SEQUENCE, ALPHA,GE.0
N      - NUMBER OF MEMBERS IN THE SEQUENCE, N,GE.1

OUTPUT
Y      - A VECTOR WHOSE FIRST N COMPONENTS CONTAIN
        VALUES FOR  $J_{\text{SUB}(\text{ALPHA}+K-1)/(\text{X})}$ ,  $K=1, \dots, N$ 
NZ     - NUMBER OF COMPONENTS OF Y SET TO ZERO DUE TO
        UNDERFLOW.
        NZ=0, NORMAL RETURN, COMPUTATION COMPLETED
        NZ.NE.0, LAST NZ COMPONENTS OF Y SET TO ZERO,
        Y(K)=0.,  $K=N-NZ+1, \dots, N$ .

```



7-9. 18182415432400E+01, 4.25349987453885E+01, -7.36879435947963E+00,  
 8.2.108001708984E-01, 5(0.), 2.125701300397E+02,  
 9-7. 3252468141182E+02, 1.0599045252800E+03, -6.99579627376E+02,  
 A. 2.18190511744212E+02, -2.64914304869515E+01, 5.72501420974731E-01,  
 B. 4(0.), 1.9145766231841E+03, 8.06172218173731E+03,  
 C-1. 3586550064341E+04, 1.16553333368645E+04, -5.3056497961340E+03,  
 D. 1.20090291321635E+03, -1.0809019786395E+02, 1.72772750258446E+00,  
 E. 3(0.), 2.0204291330661E+04, -9.69805983866375E+04,  
 F. 1.92547001232532E+05, -2.03400177280416E+05, 1.22200464983017E+05,  
 G-4. 11926549688976E+04, 7.10951430248936E+03, -4.93915304773088E+02,  
 H. 6.07404200127348E+00, 2(0.)

C

DATA C2  
 -2. 4291918790551E+05, 1.31176361466298E+06,  
 1-2. 99801591853811E+06, 3.76327129765640E+06, -2.81356322658653E+06,  
 2. 1.2683652732162E+06, -3.1645172484564E+05, 4.52187689813627E+04,  
 3-2. 4598304818121E+03, 2.4380529695561E+01, 1(0.),  
 4. 2.8446985307204E+06, -1.9706819118322E+07, 5.0952602426646E+07,  
 5-7. 41051482115327E+07, 6.63445122747290E+07, -3.75671766607634E+07,  
 6. 1.32887671664218E+07, -2.78561812808645E+06, 3.08186404612662E+05,  
 7-1. 38860897537170E+04, 1.10017140269247E+02/

C

DATA ALFA1  
 -4. 4444444444444E-03, -9.22077922077922E-04,  
 1-8. 84892884932885E-05, 1.65927687832450E-04, 2.46691372741793E-04,  
 2. 2.6595559346255E-04, 2.61824297061501E-04, 2.48730437344656E-04,  
 3. 2.32721040083232E-04, 2.16362485712365E-04, 2.00738658762752E-04,  
 4. 1.86267636637545E-04, 1.73660775917875E-04, 1.81031705929016E-04,  
 5. 1.50274774160908E-04, 1.40503497391270E-04, 1.31686816545923E-04,  
 6. 1.23667445558253E-04, 1.16405271474738E-04, 1.09798298372713E-04,  
 7. 1.03772410422993E-04, 9.82626078369363E-05, 9.32120517249503E-05,  
 8. 8.93735541354589E-04, 2.32241745189222E-04, -1.41986273556691E-05,  
 9. 6.1644931672043E-04, -1.50803558053049E-04, -1.55121924918096E-04,  
 B-1. 4680975664666E-04, 1.33815503667491E-04, -1.19744975684254E-04,  
 C-1. 05184319207974E-04, -9.3759549891194E-05, -8.26923045588193E-05,  
 D-7. 2937348155221E-05, -6.44042357210165E-05, -5.69611566009369E-05,  
 E-5. 47310443030562E-05, -4.4813486800888E-05, 3.8668872717599E-05,  
 F-3. 55400532972042E-05, -3.17414256609022E-05, -2.83996793904175E-05,  
 G-2. 54522720634871E-05, -2.28459297164725E-05, -2.05352753106481E-05,  
 H-1. 84816217627666E-05, -1.66519330021394E-05/

C

DATA ALFA2  
 -3. 54211971457744E-04, -1.56161263945159E-04,  
 1. 3.04465503594936E-05, 1.30198655773243E-04, 1.67471106699712E-04,  
 2. 1.7022587683533E-04, 1.56501427608595E-04, 1.36339170977445E-04,  
 3. 1.14866692029835E-04, 9.45869093034688E-05, 7.64498419250898E-05,  
 4. 6.07570334965197E-05, 4.74394299290505E-05, 3.62757512005344E-05,  
 5. 2.69939714979232E-05, 1.93210938247939E-05, 1.30056674793963E-05,  
 6. 7.8262086674497E-06, 3.59257485819352E-06, 1.44040049814252E-07,  
 7-2. 65396769637993E-06, -4.91346867098486E-06, -6.72739296091248E-06,  
 8-8. 17269379678658E-06, -9.31304715093561E-06, -1.02011418798016E-05,  
 9. 3.78194199201773E-04, 2.02471952761816E-04, -6.37938506318862E-05,  
 A-2. 38598230603006E-04, -3.10916256027362E-04, -3.13680115247576E-04,  
 B-2. 78950273791323E-04, -2.28564082619141E-04, -1.75245280340847E-04,  
 C-1. 25544063060690E-04, -8.22982872820208E-05, -4.62860730588116E-05,  
 D-1. 72334302366962E-05, 5.60890482304603E-06, 2.31395443148287E-05,  
 E. 3.62642745856794E-05, 4.58006124490189E-05, 5.2435529459114E-05,  
 F. 5.68396208545815E-05, 5.94349820393104E-05, 6.064786527578422E-05,  
 G. 6.08023907788436E-05, 6.01577894539460E-05, 5.89199657344698E-05,  
 H. 5.72515823777593E-05, 5.52804375589653E-05/

C

DATA BETA1  
 1. 79988721413553E-02, 5.59964911064388E-03,  
 1. 2.88501402231133E-03, 1.8009606761054E-03, 1.24753110589199E-03,  
 2. 9.22878876572938E-04, 7.14430421727287E-04, 5.71787281789705E-04,  
 3. 4.694310077606482E-04, 3.932328354662917E-04, 3.34818889318298E-04,  
 4. 2.8952148495752E-04, 2.52211815549573E-04, 2.2260580798883E-04,  
 5. 1.97541838035053E-04, 1.76838655019718E-04, 1.59316595661821E-04,  
 6. 1.44347930197334E-04, 1.31448068119965E-04, 1.202454444949303E-04,

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7 1.449144504599E-04, 1.01828770740567E-04, 9.41998224204 E-05,
8 1.130545753834E-05, 8.13466262162801E-05, 7.59002269646 E-05,
9 -1.49282953213429E-03, -8.78204709546389E-04, -5.02916549572035E-04,
A -2.94822138512746E-04, -1.75463996970783E-04, -1.0400855040816E-04,
B -5.96141953046458E-05, -3.12038929076098E-05, -1.26089735980230E-05,
C -2.42892608575730E-07, 8.05996165414274E-06, 1.36507009262147E-05,
D 1.73964125472926E-05, 1.98672978842134E-05, 2.14463263790823E-05,
E 2.2395465932457E-05, 2.2896783814713E-05, 2.3078538981178E-05,
F 2.30321976860909E-05, 2.28236073720349E-05, 2.25005881105292E-05,
G 2.20981015361991E-05, 2.16418427448104E-05, 2.11507649256221E-05,
H 2.06388749782171E-05, 2.01165241997082E-05,
DATA BETA2 / 5.2213076721293E-04, 4.47932581552385E-04,
1 2.79520853992021E-04, 1.52468156198447E-04, 6.93271105657044E-05,
2 1.76258683069991E-05, -1.3574496343289E-05, -3.17972413350427E-05,
3 -4.18861861696693E-05, -4.6904889379141E-05, -4.87665437413787E-05,
4 -4.87010031186735E-05, -4.74755620890087E-05, -4.55813058138628E-05,
5 -4.3330964511268E-05, -4.09230193157750E-05, -3.84822638603221E-05,
6 -3.6085716735411E-05, -3.3779306123367E-05, -3.15888560722110E-05,
7 -2.95269561730807E-05, -2.7597891482336E-05, -2.58006174666864E-05,
8 -2.41308356761280E-05, -2.25823509518346E-05, -2.11479656768913E-05,
9 -4.74817796559960E-04, -4.77864567147321E-04, -3.20390228067038E-04,
A -1.61105016119962E-04, -4.25778101285435E-05, 3.44571294294968E-05,
B 7.97092684075675E-05, 1.03138236708272E-04, 1.12466775262204E-04,
C 1.13103642108481E-04, 1.08651634848774E-04, 1.01437951597662E-04,
D 9.29298396593364E-05, 8.40293133016090E-05, 7.52727991349134E-05,
E 6.68632521975731E-05, 5.92564547323195E-05, 5.22169308826976E-05,
F 4.58539485165361E-05, 4.0148513891487E-05, 3.50481730031328E-05,
G 3.05157993034347E-05, 2.64956119250516E-05, 2.293636333690998E-05,
H 1.97893056664022E-05, 1.70091984636413E-05,

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C

```

DATA BETA3 / 7.36465810572578E-04, 8.72790805146194E-04,
1 6.22614862373135E-04, 2.85998154194304E-04, 3.84737672879366E-06,
2 -1.87960603636972E-04, -2.97603646594555E-04, -3.45998126832265E-04,
3 -3.53382470916038E-04, -3.3571563575049E-04, -3.04321124789040E-04,
4 -2.66722723047613E-04, -2.27654214122820E-04, -1.89922611854562E-04,
5 -1.55058918599094E-04, -1.2378240761874E-04, -9.62926147717644E-05,
6 -7.2517832771425E-05, -5.22070028895634E-05, -3.50347750511901E-05,
7 -2.0648976103552E-05, -8.70106096849767E-06, 1.136986866675100E-06,
8 9.16426474122779E-08, 1.58477785428873E-05, 2.08223629482467E-05,

```

C

```

DATA GAMA / 6.29980524947437E-01, 2.51984209378975E-01,
1 1.54790300415656E-01, 1.10713062416159E-01, 8.57309395527395E-02,
2 6.97161316958684E-02, 5.86085671893714E-02, 5.04698873536311E-02,
3 4.42600580689155E-02, 3.93720661543510E-02, 3.54283195924455E-02,
4 3.21818857502098E-02, 2.84645240791158E-02, 2.71581677112934E-02,
5 2.5176827273862E-02, 2.34570755306079E-02, 2.19508390134907E-02,
6 2.06210828235846E-02, 1.94388240897861E-02, 1.8310633000863E-02,
7 1.74293213231963E-02, 1.65685837786612E-02, 1.57865285987918E-02,
8 1.50729501494096E-02, 1.44193250839955E-02, 1.3818480575342E-02,

```

C

TEST INPUT ARGUMENTS

C

```

NZ=0
KT=1
IF(N-1) 92,108,109
108 KT=2
109 NN=N
IFIX) 93,110,120
110 IF(ALPHA) 91,114,116
114 Y(1)=1.
IF(N.EQ.1) RETURN
11=2
GO TO 116
116 11=1
118 DO 119 I=1,N
119 Y(1)=0.

```

```

      RET=RN
120 C      NUE
      IF(ALPHA.LT.0.) GO TO 91
C
      DFN=DBLE(FLOAT(N))+DBLE(ALPHA)-1.0+0
      FNU=DFN
      X02=X+.5
      SX02=X02*X02
C
      DECISION TREE FOR REGION WHERE SERIES, ASYMPTOTIC EXPANSION FOR X
      TO INFINITY AND ASYMPTOTIC EXPANSION FOR NU TO INFINITY ARE
      APPLIED.
C
      IF(SX02.LE.(FNU+.1)) GO TO 850
      TA=AMAX1(20.,FNU)
      IF(X.GT.TA) GO TO 880
      IF(X.GT.12.) GO TO 860
      X02L=ALOG(X02)
      NS=SX02-FNU
      GO TO 852
850 FNU=FNU
      FNP1=FNU+.1
      X02L=ALOG(X02)
      IS=KT
      IF(X.LE.0.5) GO TO 134
      NS=0
852 DFN=DFN+DBLE(FLOAT(NS))
      FNU=DFN
      FNP1=FNU+.1
      IS=KT
      IF(N-1+NS.GT.0) IS=3
      GO TO 134
860 NS=AMAX1(36.-FNU,0.)
      DFN=DFN+DBLE(FLOAT(NS))
      FNU=DFN
      IS=KT
      IF(N-1+NS.GT.0) IS=3
      GO TO 130
880 CONTINUE
      RTA=RTA+RTX
      TAU=RTA+FNULIM(KT)
      IF(FNU.LE.TA) GO TO 500
129 FNU=FNU
      IS=KT
C
      UNIFORM ASYMPTOTIC EXPANSION FOR NU TO INFINITY
C
130 CONTINUE
      XX=X/FNU
      W2=1.-XX*XX
      ABW2=ABS(W2)
      RA=RTA+FNULIM(KT)
      IF(ABW2.GT.0.2775) GO TO 200
C
      CASES NEAR X=FN, ABS(1.-(X/FN)**2).LE.0.2775
      COEFFICIENTS OF ASYMPTOTIC EXPANSION BY SERIES
C
      ZETA AND TRUNCATION FOR A(ZETA) AND B(ZETA) SERIES
C
      KMAX IS TRUNCATION INDEX FOR A(ZETA) AND B(ZETA) SERIES=MAX(2,SA)
C
      SA=0.
      IF(ABW2.EQ.0.) GO TO 21
      SA=TOLS/ALOG(ABW2)

```

```

21 SB=SA
DO 21 I=1,5
  KM I)=AMAX1(SA,2.)
  SA=SA+SB
22 CONTINUE
  KB=KMAX(S)
  KLAST=KB-1
  SA=GAMA(KB)
  DO 24 K=1, KLAST
    KB=KB-1
    SA=SA+W2+GAMA(KB)
24 CONTINUE
  Z=W2+SA
  AZ=ABS(Z)
  RTZ=SORT(AZ)
  FN13=FN+CON2
  RTARY=RTZ+FN13
  ARY=RTARY+RTARY
  AZ2=AZ+RTZ+CON1
  ACZ=FN+AZ2
  IF(Z.LE.0.) GO TO 27
  TEST FOR UNDERFLOW, 1.E-280=EXP(-644.), ONE WORD LENGTH
  UP FROM UNDERFLOW LIMIT OF CDC 6600
  IF(ACZ.GT.ELIM2) GO TO 180
  ARY=ARY
27 PHI=SQRT(SORT(SA+SA+SA+SA))
  C
  C
  C
  B(ZETA) FOR S=0
  C
  C
  KB=KMAX(S)
  KLAST=KB-1
  SB=BETA(KB,1)
  DO 23 K=1, KLAST
    KB=KB-1
    SB=SB+W2+BETA(KB,1)
23 CONTINUE
  KSP1=1
  FN2=FN+FN
  RFN2=1./FN2
  RDN=1.
  ASUM=1.
  RELB=TOL*ABS(SB)
  BSUM=SB
  DO 25 KS=1,4
    KSP1=KSP1+1
    RDN=RDN+RFN2
  C
  C
  A(ZETA) AND B(ZETA) FOR S=1,2,3,4
  C
  C
  KB=KMAX(S-KS)
  KLAST=KB-1
  SA=ALFA(KB,KS)
  SB=BETA(KB,KSP1)
  DO 26 K=1, KLAST
    KB=KB-1
    SA=SA+W2+ALFA(KB,KS)
    SB=SB+W2+BETA(KB,KSP1)
26 CONTINUE
  TA=SA+RDN
  TB=SB+RDN
  ASUM=ASUM+TA
  BSUM=BSUM+TB
  IF(ABS(TA).LE.TOL.AND.ABS(TB).LE.RELB) GO TO 152
25 CONTINUE

```

```

152 CONTINUE
BS=-BSUM/(FN*FN13)
GD 400
C
C 200 CONTINUE
TAU=1./RA
T2=1./W2
IF(W2.GE.0.) GO TO 30
C
C CASES FOR (X/FN).GT.SORT(1.2775)
C
AZ32=ABS(RA-ATAN(RA))
ACZ=AZ32*FN
CZ=-ACZ
Z32=1.5-AZ32
RTZ=Z32*CON2
FN13=FN*CON2
RTARY=RTZ*FN13
ARY=-RTARY*RTARY
GO TO 150
30 CONTINUE
C
C CASES FOR (X/FN).LT.SORT(0.7225)
C
C AZ32=ABS(ALOG((1.+RA)/XX) -RA)
C
C TEST FOR UNDERFLOW, 1.E-280 = EXP(-644.), ONE WORD LENGTH
C UP FROM UNDERFLOW LIMIT OF CDC 6600
C
ACZ=AZ32*FN
CZ=ACZ
IF(ACZ.GT.ELIM2) GO TO 180
Z32=1.5-AZ32
RTZ=Z32*CON2
FN13=FN*CON2
RTARY=RTZ*FN13
ARY=-RTARY*RTARY
150 CONTINUE
PHI=SQRT((RTZ+RTZ)*TAU)
TB=1.
ASUM=1.
TFN=TAU/FN
UPOL(2)=(C(1,1)*T2+C(2,1))*TFN
RCZ=CON1/CZ
CRZ2=CON548*RCZ
BSUM=UPOL(2)*CRZ2
REL8=TOL*ABS(BSUM)
AP=TFN
KS=0
KP1=2
RZDEN=RCZ
DO 155 LR=2,8,2
C
C COMPUTE TWO U POLYNOMIALS FOR NEXT A(ZETA) AND B(ZETA)
C
LRP1=LR+1
DO 101 K=LR,LRP1
KS=KS+1
KP1=KP1+1
S1=C(1,K)
DO 102 J=2,KP1
S1=S1*T2+C(J,K)
102 CONTINUE
AP=AP*TFN
UPOL(KP1)=AP*S1
CR(KS)=BR(KS)*RZDEN

```

```

RZD=H-RZDEN=RCZ
DR( )=ARIKS)*RZDEN
101 CON.,INUE
SUMA=UPOL(LRP1)
SUMB=UPOL(LR+2)*UPOL(LRP1)*CRZ32
JU=LRP1
DO 151 JR=1,LR
JU=JU-1
SUMA=SUMA+CR(JR)*UPOL(JU)
SUMB=SUMB+DR(JR)*UPOL(JU)
151 CONTINUE
TB=-TB
IF(W2.GT.O.) TB=ABS(TB)
ASUM=ASUM+SUMA*TB
BSUM=BSUM+SUMB*TB
IF(ABS(SUMA).LE.TOL.AND.ABS(SUMB).LE.RELB) GO TO 165
155 CONTINUE
165 TB=RTARY
IF(W2.GT.O.) TB=-TB
BSUM=BSUM/TB
C
400 CONTINUE
CALL JAIRY(ARY,RTARY,ACZ,AT,DA1)
TEMP(15)=PHI*(AI+ASUM+DA1*BSUM)/FN13
GO TO (401,202,850), IS
402 TEMP(1)=TEMP(3)
KI=1
401 IS=2
DFN=DFN-1.D+0
FN=DFN
GO TO 130
C
C
C SERIES FOR (X/2)**2.LE.NU+1
C
134 CONTINUE
GLN=GMLN(FNP1)
ARG=FN*XO2-GLN
IF(ARG.LT.-ELIM1) GO TO 123
EARG=EXP(ARG)
300 CONTINUE
S=1.
AK=3.
T2=1.
T=1.
S1=FN
DO 125 K=1,17
S2=T2*S1
T=-T*SXO2/S2
S=S+T
IF(ABS(T).LT.TOL) GO TO 127
T2=T2+AK
AK=AK+2.
S1=S1+FN
125 CONTINUE
127 CONTINUE
TEMP(15)=S+EARG
GO TO (301,202,600), IS
301 EARG=EARG*FN/XO2
DFN=DFN-1.D+0
IS=2
GO TO 300
C
C
C SET UNDERFLOW VALUE AND UPDATE PARAMETERS
C
160 Y(NN)=0.

```

```

NW I-1
DA JFN-1.D+0
FN=DFN
IF (NN-1) 170,171,130
171 KT=2
IS=2
GO TO 130
123 Y(NN)=0.
NN=NN-1
FNP1=FN
DFN=DFN-1.D+0
FN=DFN
IF (NN-1) 170,172,173
172 KT=2
IS=2
173 IF(SX02.LE.FNP1) GO TO 133
GO TO 130
133 ARG=ARG-X02L+ALOG(FNP1)
IF(ARG.LT.-ELIM1) GO TO 123
GO TO 134
170 NZ=N-NN
RETURN
C
C BACKWARD RECURSION SECTION
C
202 CONTINUE
NZ=N-NN
IF(KT.EQ.2) GO TO 250
203 CONTINUE
BACKWARD RECUR FROM INDEX ALPHA+NN-1 TO ALPHA
Y(NN)=TEMP(1)
Y(NN-1)=TEMP(2)
IF(NN.EQ.2) RETURN
DX=X
TRX=2.D+0/DX
DTM=DFN+TRX
TM=DTM
K=NN+1
DO 230 I=3,NN
K=K-1
Y(K-2)=TM+Y(K-1)-Y(K)
DTM=DTM-TRX
TM=DTM
230 CONTINUE
RETURN
250 Y(1)=TEMP(2)
RETURN
C
C ASYMPTOTIC EXPANSION FOR X TO INFINITY WITH FORWARD RECURSION IN
C OSCILLATORY REGION X.GT.MAX(20, NU), PROVIDED THE LAST MEMBER
C OF THE SEQUENCE IS ALSO IN THE REGION.
C
500 CONTINUE
IN=ALPHA-TAU+2.
IF(JN.LE.0) GO TO 502
INP1=IN+1
DALPHA=ALPHA-FLOAT(INP1)
KT=1
GO TO 511
502 DALPHA=ALPHA
IN=0
511 IS=KT
512 ARG=X-PIDT-DALPHA-PDF
SA=SIN(ARG)
SB=COS(ARG)
RA=RTTP/RTX

```

```

503 ETX=B.*X
D( ALPHA
DX=X+DX
DTM=DX+DX
T2=DTM-1.D+0
T2=T2/ETX
S2=T2
REL8=TOL*ABS(T2)
T1=ETX
S1=1.
FN=1.
AK=B.
DO 504 K=1,13
T1=T1+ETX
FN=FN+AK
DX=FN
TRX=DTM-DX
AP=TRX
T2=-T2*AP/T1
S1=S1+T2
T1=T1+ETX
AK=AK+B.
FN=FN+AK
DX=FN
TRX=DTM-DX
AP=TRX
T2=T2*AP/T1
S2=S2+T2
IF(ABS(T2).LE.REL8) GO TO 505
AK=AK+B.
504 CONTINUE
505 TEMP(15)=RA*(S1+SB-S2*SA)
GO TO (506,507),15
506 DALPHA=DALPHA+1.
IS=2
TB=SA
SA=SB
SB=TB
GO TO 503
C
C FORWARD RECURSION SECTION
C
507 IF(KT.EQ.2) GO TO 250
S1=TEMP(1)
S2=TEMP(2)
TX=2./X
TM=DALPHA*TX
IF(IN.EQ.0) GO TO 520
C
C FORWARD RECUR TO INDEX ALPHA
C
DO 510 I=1,IN
S=S2
S2=TM+S2-S1
TM=TM+TX
S1=S
510 CONTINUE
IF(INN.EQ.1) GO TO 535
S=S2
S2=TM+S2-S1
TM=TM+TX
S1=S
520 CONTINUE
C
C FORWARD RECUR FROM INDEX ALPHA TO ALPHA+N-1
C

```

```

Y(1)=S1
Y(2)=S2
IF (.EQ.2) RETURN
DO -J0 I=3,NN
Y(I)=TM*Y(I-1)-Y(I-2)
TM=TM+TX
530 CONTINUE
RETURN
535 Y(1)=S2
RETURN

C
C BACKWARD RECURSION WITH NORMALIZATION BY
C ASYMPTOTIC EXPANSION FOR NU TO INFINITY OR POWER SERIES.
C
600 CONTINUE
C COMPUTATION OF LAST ORDER FOR SERIES NORMALIZATION
KM=AMAX1(3,-FN,0.)
TFN=FN+FLOAT(KM)
TA=(GLN+TFN-0.9189385332-0.0833333333/TFN)/(TFN+0.5)
TA=XO2L-TA
TB=-(1.-1.5/TFN)/TFN
IN=CE/(-TA+SQR1(TA+TA-CE+TB))+1.5
IN=IN+KM
GO TO 603

650 CONTINUE
C COMPUTATION OF LAST ORDER FOR ASYMPTOTIC EXPANSION NORMALIZATION
GLN=AZ32+RA
IF(ARY.GT.30.) GO TO 675
RDEN=(PP(4)*ARY+PP(3))*ARY+1.
RZDEN=PP(1)*PP(2)*ARY
TA=RZDEN/RDEN
IF(W2.LT.0.10) GO TO 651
TB=GLN/RTARY
GO TO 677

651 TB=(1.-259921049+0.1679894730*W2)/FN13
GO TO 677

675 CONTINUE
TA=CON1+TCE/ACZ
TA=((0.0493827160*TA-0.1111111111)*TA+0.6666666667)*TA*ARY
IF(W2.LT.0.10) GO TO 651
TB=GLN/RTARY
IN=TA/TB+1.5
IF(IN.GT.1N1M) GO TO 602
DX=FLOAT(IN)
DTM=DFN+DX
DX=X
TRA=2.D+0/DX
DTM=DTM+TRA
TM=DTM
TA=0.
TB=TOL
KK=1
605 CONTINUE
C BACKWARD RECUR UNINDEXED
C
DO 601 I=1,IN
S=TB
TB=TM+TB-TA
TA=S
DTM=DTM-TRA
TM=DTM
601 CONTINUE
C NORMALIZATION
IF(KK.NE.1) GO TO 604
TA=(TA/TB)*TEMP(3)

```



```

D{*****NSION DA:P(19),DAJN(19),DA(15),DB(18)
D{*****NSION DAK1(14),DAK2(24),DAK3(14)

DATA N1,N2,N3,N4/14,23,19,15/
DATA M1,M2,M3,M4/12,21,17,13/
DATA FPI12,CON1,CON2,CON3,CON4,COM5/
1 1.3089959899575E+00, 8.666666666666667E-01, 5.03154716196777E+00,
2 3.80004598967293E-01, 8.333333333333333E-01, 8.66025403784439E-01/
DATA AK1 / 2.20423090987793E-01, -1.25290242787700E-01,
1 1.03881163359194E-02, 8.22844152006343E-04, -2.34614345891226E-04,
2 1.63824280172116E-05, 3.06902589573189E-07, -1.2962199359332E-07,
3 8.22908158823688E-09, 1.53953368823298E-11, -3.39165465615682E-11,
4 2.03253257423626E-12, -1.10679546097884E-14, -5.16169497785080E-15/
DATA AK2 / 2.74366150869598E-01, 5.39790969736903E-03,
1 1.57339220621190E-03, 4.27427528348750E-04, -1.1212491739925E-04,
2 2.88763171318904E-05, -7.36804232370554E-06, 1.87290209741024E-06,
3 4.75892793962291E-07, 1.21130416955909E-07, -3.09245374270614E-08,
4 7.92454705282654E-09, -2.03902447167914E-09, 5.26863056595742E-10,
5 1.36704767633569E-10, 3.56141039013708E-11, -9.31388296548430E-12,
6 2.44464450473635E-12, -6.43840261990955E-13, 1.70106030559349E-13,
7 4.50760104503281E-14, 1.19774799164811E-14, -3.19077040865066E-15/
DATA AK3 / 2.80271447340791E-01, -1.78127042844379E-03,
1 4.03422579628999E-05, -1.63249965269003E-06, 9.21181482476768E-08,
2 6.52294330229155E-09, 5.47138404576546E-10, -5.24408251800260E-11,
3 5.60477904117209E-12, -6.5637524639313E-13, 8.31285761966247E-14,
4 1.12705134691063E-14, 1.62267976598129E-15, -2.46480324312426E-16/
DATA AJP / 7.78952966437581E-02, -1.84356363456801E-01,
1 3.01412605216174E-02, 3.053422324377608E-02, -4.95424702513079E-03,
2 1.72749552563952E-03, 2.43137637839190E-04, 5.04564777517082E-05,
3 6.16316582695208E-06, -9.03986745510768E-07, 9.70243778355884E-08,
4 1.09639453305205E-08, -1.04716330588766E-09, -9.60359441344646E-11,
5 8.25358789454134E-12, 6.36123439018768E-13, -4.96629614116015E-14,
6 3.29810288929615E-15, 2.35798252031104E-16/
DATA AJN / 3.80497887617242E-02, -2.45319541845546E-01,
1 1.65820623702696E-01, 7.49330045818789E-02, -2.63476288106641E-02,
2 5.92535597304981E-03, 1.44744409589804E-03, 2.18311831322215E-04,
3 4.10662077680304E-05, -4.66874894171766E-06, 7.15218807277160E-07,
4 6.52964770854633E-08, -8.44284027565946E-09, -6.44186158976978E-10,
5 7.20802286505285E-11, 4.72465431717846E-12, -4.66022632547045E-13,
6 2.67762710389189E-14, 2.36161316570019E-15/
DATA A / 4.90275424742791E-01, 1.57647277946204E-03,
1 9.66195963140306E-05, 1.35916080268815E-07, 2.98157342654859E-07,
2 1.86824767559979E-08, -1.03685737667141E-09, 3.28660818434328E-10,
3 2.57091410632780E-11, -2.32357655300677E-12, 9.57523279048255E-13,
4 1.20340828045719E-13, -2.90907716770715E-15, 4.55656454580149E-15,
5 9.99003874810259E-16/
DATA B / 2.78593552803079E-01, -3.52915691882584E-03,
1 2.31149677384994E-05, 4.71371842263560E-06, -1.12415907931333E-07,
2 2.00100301184339E-08, 2.60948075302193E-09, -3.55098136101216E-11,
3 3.50849978423875E-11, 5.83007187954202E-12, -2.04644828753326E-13,
4 1.10529179476742E-13, 2.87724778038775E-14, -2.88205111009939E-15,
5 3.32656311696166E-16/
DATA N1D,N2D,N3D,N4D/14,24,19,15/
DATA M1D,M2D,N3D,M4D/12,22,17,13/

DATA DAK1 / 2.04567842307887E-01, -6.61322739905664E-02,
1 8.49845800989287E-03, 3.12183491556289E-03, -2.70016489829432E-04,
2 6.35636298679387E-06, 3.02397712409509E-06, -2.18311195330089E-07,
3 5.36194289332826E-10, 1.13098035622310E-09, -7.43023834629073E-11,
4 4.28804170826891E-13, 2.23810925754539E-13, -1.39140135641182E-14/

```

DATA DAK2 / 2.93332343883230E-01, -8.06196784743112E-03,  
1 2.42540172333140E-03, -6.82297548850235E-04, 1.8578642775191E-04,  
2 7457447684059E-05, 1.32090681239497E-05, -3.4952824044 E-06,  
3 9.4362451078835E-07, -2.44732671521867E-07, 6.49307837648 E-08,  
4 1.72717621501538E-08, 4.60725763604656E-09, -1.23249055291550E-09,  
5 3.30620409488102E-10, -8.89252099772401E-11, 2.39773319878298E-11,  
6 -6.48013921153450E-12, 1.75510132023731E-12, -4.76303829833637E-13,  
7 1.29498241100810E-13, -3.52679622210430E-14, 9.62005151585923E-15,  
8 -2.62786314342292E-15/

DATA DAK3 / 2.84675828811349E-01, 2.53073072619080E-03,  
1 -4.83481130337976E-05, 1.84907283946343E-06, -1.01418491178576E-07,  
2 7.0592563457153E-09, -5.85325291400382E-10, 5.56357688831339E-11,  
3 -5.90889094779500E-12, 6.88574353784436E-13, -8.68588256452194E-14,  
4 1.17374762617213E-14, -1.68523146510923E-15, 2.55374773097056E-16/

DATA DAKP / 6.53219131311457E-02, -1.20262933688823E-01,  
1 9.78010236263823E-03, 1.67948429230505E-02, -1.97146140182132E-03,  
2 -8.45560295098867E-04, 9.42889620701976E-05, 2.25827860945475E-05,  
3 -2.29067870915987E-06, -3.76343991136919E-07, 3.45663933559565E-08,  
4 4.29611332003007E-09, -3.58673691214989E-10, 3.57245881361895E-11,  
5 2.72696091066336E-12, 2.26120653095771E-13, -1.58763205238303E-14,  
6 -1.12604374485125E-15, 7.31327529515367E-17/

DATA DAJN / 1.08594539632967E-02, 8.53313194857091E-02,  
1 -3.15277068113058E-01, -8.78420725294257E-02, 5.53251906976048E-02,  
2 9.41674060503241E-03, -3.32187026018996E-03, -4.11157343156826E-04,  
3 1.01297326891346E-04, 9.87833682208396E-06, -1.87312969812393E-06,  
4 -1.50798500131468E-07, 2.32687669525394E-08, 1.59599917419225E-09,  
5 -2.07665922668385E-10, -1.24103350500302E-11, 1.39631765331043E-12,  
6 7.39400971155740E-14, -7.32887475627500E-15/

DATA DA / 4.91627321104601E-01, 3.11164930427489E-03,  
1 6.23140762854081E-05, -4.61789776172142E-06, -6.13158880534626E-08,  
2 2.87295804656520E-08, -1.81959715372117E-09, -1.44752826642035E-10,  
3 4.53724043420422E-11, -3.99655065847223E-12, -3.24089119830323E-13,  
4 1.62098952568741E-13, -2.40765247974057E-14, 1.69384811284491E-16,  
5 8.17900786477396E-16/

DATA DB / -2.77571356944231E-01, 4.44212833419920E-03,  
1 -8.42328522190089E-05, -2.58040318418710E-06, 3.42389720217621E-07,  
2 -6.24286894709776E-09, -2.36377836844577E-09, 3.16991042656873E-10,  
3 -4.40995691658191E-12, -5.18674221093575E-12, 9.64874015137022E-13,  
4 -4.90190576608710E-14, -1.7725430678112E-14, 5.55950610442662E-15,  
5 -7.11793337579530E-16/

IF(X.LT.0.) GO TO 300  
IF(C.GT.5.) GO TO 200  
IF(X.GT.1.2) GO TO 150  
T=(X+X-1.2)\*CDN4  
TT = T + T

J=N1  
F1=AK1(J)  
F2=0.  
DO 105 I=1,M1  
J=J-1  
TEMP1=F1  
F1=TT\*F1-F2\*AK1(J)  
F2=TEMP1  
105 CONTINUE  
AT=FF1-F2\*AK1(1)

J=N1D  
F1=DAK1(J)  
F2=0.  
DO 106 I=1,M1D  
J=J-1

```

      TT = F1
      F1 = TT * F1 - F2 + DAK1(J)
      F2 = TEMP1
106      CONTINUE
      DAI = -(T * F1 - F2 + DAK1(1))
      RETURN
C
150      CONTINUE
      T = (X + X - CON2) * CON3
      TT = T + T
      J = N2
      F1 = DAK2(J)
      F2 = 0.
      DO 155 I=1,M2
        J = J-1
        TEMP1 = F1
        F1 = TT * F1 - F2 + DAK2(J)
        F2 = TEMP1
155      CONTINUE
      RTRX = SORT(RX)
      EC = EXP(-C)
      AI = EC * (T * F1 - F2 + DAK2(1)) / RTRX
      J = N2D
      F1 = DAK2(J)
      F2 = 0.
      DO 156 I=1,M2D
        J = J-1
        TEMP1 = F1
        F1 = TT * F1 - F2 + DAK2(J)
        F2 = TEMP1
156      CONTINUE
      DAI = EC * (T * F1 - F2 + DAK2(1)) * RTRX
      RETURN
C
200      CONTINUE
      T = 10. / C - 1.
      TT = T + T
      J = N1
      F1 = DAK3(J)
      F2 = 0.
      DO 205 I=1,M1
        J = J-1
        TEMP1 = F1
        F1 = TT * F1 - F2 + DAK3(J)
        F2 = TEMP1
205      CONTINUE
      RTRX = SORT(RX)
      EC = EXP(-C)
      AI = EC * (T * F1 - F2 + DAK3(1)) / RTRX
      J = N1D
      F1 = DAK3(J)
      F2 = 0.
      DO 206 I=1,M1D
        J = J-1
        TEMP1 = F1
        F1 = TT * F1 - F2 + DAK3(J)
        F2 = TEMP1
206      CONTINUE
      DAI = -RTRX * EC * (T * F1 - F2 + DAK3(1))
      RETURN
C
300      CONTINUE
      IF (C.GT.5.) GO TO 350
      T = 4 * C - 1.
      TT = T + T
      J = N3

```

```

F1=IP(J)
E1=V(J)
F2=...
E2=0.
DO 305 I=1,M3
J=J-1
TEMP1=F1
TEMP2=E1
F1=TT*F1-F2+AJ(P(J))
E1=TT*E1-E2+AJN(J)
F2=TEMP1
E2=TEMP2
305 CONTINUE
A1=(T*E1-E2+AJN(1))-X*(T*F1-F2+AJ(P(1)))
J=N3D
F1=DAJP(J)
E1=DAJN(J)
F2=0.
E2=0.
DO 306 I=1,M3D
J=J-1
TEMP1=F1
TEMP2=E1
F1=TT*F1-F2+DAJP(J)
E1=TT*E1-E2+DAJN(J)
F2=TEMP1
E2=TEMP2
306 CONTINUE
DA1=X*X*(T*F1-F2+DAJP(1))+T*(E1-E2+DAJN(1))
RETURN

C
350 CONTINUE
T=10./C-1.
TT=T*T
J=N4
F1=A(J)
E1=B(J)
F2=0.
E2=0.
DO 310 I=1,M4
J=J-1
TEMP1=F1
TEMP2=E1
F1=TT*F1-F2+A(J)
E1=TT*E1-E2+B(J)
F2=TEMP1
E2=TEMP2
310 CONTINUE
TEMP1=TT*F1-F2+A(1)
TEMP2=TT*E1-E2+B(1)
RTRX=SQRT(RX)
CV=C-FR12
CCV=CCS(CV)
SCV=5*IN(CV)
A1=(TEMP1*CCV-TEMP2*SCV)/RTRX
J=N4D
F1=DA(J)
E1=DB(J)
F2=0.
E2=0.
DO 311 I=1,M4D
J=J-1
TEMP1=F1
TEMP2=E1
F1=TT*F1-F2+DA(J)
E1=TT*E1-E2+DB(J)

```

```

F2=TEMP1
E7 MP2
CO, .NUE
TEMP1=T*F1-F2*DA(1)
TEMP2=T*E1-E2*DB(1)
E1=CCV*CONS+.5*SCV
E2=SCV*CONS-.5*CCV
DA1=(TEMP1*E1-TEMP2*E2)*RTRX
RETURN
END
SUBROUTINE BES1(X,ALPHA,KODE,N,Y,NZ)

```

WRITTEN BY D. E. AMOS AND S. L. DANIEL, JANUARY, 1975.

REFERENCE SAND-75-0152

# ABSTRACT

BES1 COMPUTES AN N MEMBER SEQUENCE OF J BESSEL FUNCTIONS  $I_{\text{SUB}}(\text{ALPHA}+K-1)/(X)$ ,  $K=1, \dots, N$  OR SCALED BESSEL FUNCTIONS  $\text{EXP}(-X) \cdot I_{\text{SUB}}(\text{ALPHA}+K-1)/(X)$ ,  $K=1, \dots, N$  FOR NON-NEGATIVE ALPHA AND X. A COMBINATION OF THE POWER SERIES, THE ASYMPTOTIC EXPANSION FOR X TO INFINITY, AND THE UNIFORM ASYMPTOTIC EXPANSION FOR NU TO INFINITY ARE APPLIED OVER SUBDIVISIONS OF THE (NU, X) PLANE. FOR VALUES NOT COVERED BY ONE OF THESE FORMULAE, THE ORDER IS INCREMENTED BY AN INTEGER SO THAT ONE OF THESE FORMULAE APPLY. BACKWARD RECURSION IS USED TO REDUCE ORDERS BY INTEGER VALUES. THE ASYMPTOTIC EXPANSION FOR X TO INFINITY IS USED ONLY WHEN THE ENTIRE SEQUENCE (SPECIFICALLY THE LAST MEMBER) LIES WITHIN THE REGION COVERED BY THE EXPANSION. LEADING TERMS OF THESE EXPANSIONS ARE USED TO TEST FOR OVER OR UNDERFLOW WHERE APPROPRIATE. IF A SEQUENCE IS REQUESTED AND THE LAST MEMBER WOULD UNDERFLOW, THE RESULT IS SET TO ZERO AND THE NEXT LOWER ORDER TRIED, ETC., UNTIL A MEMBER COMES ON SCALE OR ALL ARE SET TO ZERO. AN OVERFLOW CANNOT OCCUR WITH SCALING. BES1 CALLS FUNCTION GAMLN.

# DESCRIPTION OF ARGUMENTS

```

INPUT
X      - X, GE. 0
ALPHA  - ORDER OF FIRST MEMBER OF THE SEQUENCE, ALPHA, GE. 0
KODE   - A PARAMETER TO INDICATE THE SCALING OPTION
        KODE=1 RETURNS
        Y(K)= I/SUB(ALPHA+K-1)/(X),
              K=1, ..., N
        KODE=2 RETURNS
        Y(K)= EXP(-X) * I/SUB(ALPHA+K-1)/(X),
              K=1, ..., N
N      - NUMBER OF MEMBERS IN THE SEQUENCE, N, GE. 1

OUTPUT
Y      - A VECTOR WHOSE FIRST N COMPONENTS CONTAIN
        VALUES FOR I/SUB(ALPHA+K-1)/(X) OR SCALED
        VALUES FOR EXP(-X) * I/SUB(ALPHA+K-1)/(X),
        K=1, ..., N, DEPENDING ON KODE
NZ     - NUMBER OF COMPONENTS OF Y SET TO ZERO DUE TO
        UNDERFLOW.
        NZ=0, NORMAL RETURN, COMPUTATION COMPLETED
        NZ=NE.0, LAST NZ COMPONENTS OF Y SET TO ZERO,
        Y(K)=0., K=N-NZ+1, ..., N.

ERROR CONDITIONS
IMPROPER INPUT ARGUMENTS - A FATAL ERROR
OVERFLOW WITH KODE=1 - A FATAL ERROR
UNDERFLOW - A NON-FATAL ERROR(NZ,NE.0)

```

```

C
C      !      USES SUBROUTINES GAMLN, ERRCHK, ERRGET, ERRPT, ERR...
C      BE...
C      COMPIL DECKS BESI, GAMLN, ERRCHK
C
C      DOUBLE PRECISION DX, TRA, DTM, DFN
C      DIMENSION Y(1), TEMP(3)
C      DIMENSION C(11,10)
C      DIMENSION C1(11,8), C2(11,2)
C      EQUIVALENCE (C(1,1), C1(1,1))
C      EQUIVALENCE (C(1,9), C2(1,1))
C
C      DATA ELIM,TOL / 667. , 1.E-15 /
C
C      DATA RTPI,RTTPI / 1.59154943091895E-01, 3.98942280401433E-01/
C
C      DATA CE / 3.45387763900000E+01/
C
C      DATA INLM / 80 /
C
C      DATA C1 9(0.) /-2.0833333333333E-01, 1.2500000000000E-01,
1 3.3420138888888E-01, -4.0104166666666E-01,
2 7.0312500000000E-02, 8(0.) -1.02581259645062E+00,
3 1.8464626736111E+00, 8.9121093750000E-01, 7.3242187500000E-02,
4 7(0.) 4.6898442342525E+00, -1.12070026162230E-01,
5 8.78912353515625E+00, -2.36408691406250E+00, 1.1215205969375E-01,
6 6(0.) -2.8212072582002E+01, 8.46362176746007E+01,
7-9.18182415432400E+01, 4.2534987453885E+01, -7.36879435947963E+00,
8 2.27108001708984E-01, 5(0.) 2.12570130039217E+02,
9-7.65252468141182E+02, 1.0599045252800E+03, -6.99579627376133E+02,
A 2.18190511744212E+02, -2.64914304869516E+01, 5.72501420974731E-01,
B 4(0.) -1.91945766231841E+03, 8.0617218173731E+03,
C-1.35865500064341E+04, 1.16553333368645E+04, -5.30564697861340E+03,
D 1.20090291321635E+03, -1.08090919788395E+02, 1.7277250258446E+04,
E 3(0.) 2.02042813309661E+04, -9.69805983886375E+04,
F 1.92547001233532E+05, -2.03400177280416E+05, 1.2200464983017E+05,
G-4.11926549686976E+04, 7.10951430248936E+03, -4.93915304773086E+02,
H 6.07404200127348E+00, 2(0.) /
C
C      DATA C2 /-2.42919187900551E+05, 1.31176361466298E+06,
1-2.99801591853811E+06, 3.76327129765640E+06, -2.81356322659653E+06,
2 1.26836527332162E+06, -3.31645172484564E+05, 4.52187689813627E+04,
3-2.49983048181121E+03, 2.43805296995561E+01, 1(0.) 1(0.)
4 3.28446985307204E+06, -1.97088191184322E+07, 5.09526024926646E+07,
5-7.41051482115327E+07, 8.63445122747290E+07, -3.75671766607834E+07,
6 1.32887671664218E+07, -2.7856181280845E+06, 3.08186404612662E+05,
7-1.38866397537170E+04, 1.10017140269247E+02/
C
C      TEST INPUT ARGUMENTS
C
C      NZ=0
C      KT=1
C      IF(N=1) 92,108,109
108 KT=2
109 NN=N
C      IF(KODE.LT.1.OR.KODE.GT.2) GO TO 90
C      IF(X) 93,110,120
110 IF(ALPHA) 91,114,116
114 Y(1)=1.
C      IF(N.EQ.1) RETURN
C      I1=2
C      GO TO 116
116 I1=1
118 DO 119 I=1,N
119 Y(I)=0.

```

```

      RETURN
120 C  IF (ALPHA.LT.0.) GO TO 91
C
      DFN=DBLE(FLOAT(N))+DBLE(ALPHA)-1.D+0
      FNU=DFN
      IN=0
      X02=X+.5
      SX02=X02*X02
      ETA=KODE-1
      SX=ETX*X
C
      DECISION TREE FOR REGION WHERE SERIES, ASYMPTOTIC EXPANSION FOR X
      TO INFINITY AND ASYMPTOTIC EXPANSION FOR NU TO INFINITY ARE
      APPLIED.
C
      IF(SX02.LE.(FNU+1.)) GO TO 850
      IF(X.LE.12.) GO TO 870
      FN=0.55*FNU*FNU
      FN=AMAX1(17.,FN)
      IF(X.GE.FN) GO TO 500
      NS=AMAX1(36.,-FNU,0.)
      DFN=DFN+DBLE(FLOAT(NS))
      FN=DFN
      IS=KT
      KM=N-1+NS
      IF(KM.GT.0) IS=3
      GO TO 129
850 FN=FNU
      FNP1=FN+1.
      X02L=ALOG(X02)
      IS=KT
      IF(X.LE.0.5) GO TO 134
      NS=0
      DFN=DFN+DBLE(FLOAT(NS))
      FN=DFN
      FNP1=FN+1.
      IS=KT
      IF(N-1+NS.GT.0) IS=3
      GO TO 134
870 X02L=ALOG(X02)
      NS=SK02-FNU
      GO TO 852
129 CONTINUE
C
      OVERFLOW TEST ON UNIFORM ASYMPTOTIC EXPANSION
C
      IF(KODE.EQ.2) GO TO 130
155 IF(ALPHA.LT.1.) GO TO 156
      Z=X/ALPHA
      RA=SQRT(1.+Z*Z)
      GLN=ALOG((1.+RA)/Z)
      T=RA*(1.-ETX)+ETX/(Z+RA)
      ARG=ALPHA*(T-GLN)
      IF(ARG.GT.ELIM) GO TO 94
      IF(KM.EQ.0) GO TO 157
130 CONTINUE
C
      UNDERFLOW TEST ON UNIFORM ASYMPTOTIC EXPANSION
C
      Z=X/FN
      RA=SQRT(1.+Z*Z)
      GLN=ALOG((1.+RA)/Z)
      T=RA*(1.-ETX)+ETX/(Z+RA)
      ARG=FN*(T-GLN)
157 IF(ARG.LT.-ELIM) GO TO 180

```

```

GO TO 148
186 IF GT.ELIM) GO TO 94
GC J 130
C
C UNIFORM ASYMPTOTIC EXPANSION FOR NU TO INFINITY
C
199 IF(KM.NE.0) GO TO 198
Y(1)=TEMP(3)
RETURN
198 TEMP(1)=TEMP(3)
IN=NS
KT=1
200 CONTINUE
IS=2
DFN=DFN-1.D+0
FN=DFN
Z=X/FN
RA=SQRT(1.+Z*Z)
GLN=ALOG((1.+RA)/Z)
T=RA*(1.-ETX)*ETX/(Z+RA)
ARG=FN*(T-GLN)
148 COEF=EXP(ARG)
T1=T/T
T2=T/T
T1/FN
S2=1.
AP=1.
DO 140 K=1,10
KPI=K+1
S1=C(1,K)
DO 135 J=2,KPI
S1=S1*T2+C(J,K)
135 CONTINUE
AP=AP*T
AK=AP*S1
S2=S2+AK
IF(ABS(AK).LT.TOL) GO TO 145
140 CONTINUE
145 CONTINUE
TEMP(15)=SORT(T*RTPI)*COEF*S2
GO TO (200,202,650), 15
C
C SERIES FOR (X/2)**2.LE.NU+1
C
134 CONTINUE
GLN=GMLN(FNPI)
ARG=FN*LOG(-GLN-SX
IF(ARG.LT.-ELIM) GO TO 123
EARG=EXP(ARG)
300 CONTINUE
S=1.
AK=3.
T2=1.
T=1.
S1=FN
DO 125 K=1,17
S2=T2+S1
T=T*SX02/S2
S=S+T
IF(ABS(T).LT.TOL) GO TO 127
T2=T2+AK
AK=AK+2.
S1=S1+FN
125 CONTINUE
127 CONTINUE
TEMP(15)=S+EARG

```

```

GO TO (301,202,600),IS
301 EA' SARG=FN/X02
   DF1 .FN-1.D+0
   FN=DFN
   IS=2
   GO TO 300
C
C SET UNDERFLOW VALUE AND UPDATE PARAMETERS
C
180 Y(NN)=0.
   NN=NN-1
   DFN=DFN-1.D+0
   FN=DFN
   IF (NN-1) 170,171,130
171 KT=2
   IS=2
   GO TO 130
123 Y(NN)=0.
   NN=NN-1
   FNP1=FN
   DFN=DFN-1.D+0
   FN=DFN
   IF (NN-1) 170,172,173
172 KT=2
   IS=2
173 IF(SX02.LE.FNP1) GO TO 133
   GO TO 130
133 ARG=ARG-X02L+ALOG(FNP1)
   IF(ARG.LT.-ELIM) GO TO 123
   GO TO 134
170 NZ=N-NN
   RETURN
C
C BACKWARD RECURSION SECTION
C
202 CONTINUE
   NZ=N-NN
204 GO TO (203,250), KT
203 CONTINUE
   S1=TEMP(1)
   S2=TEMP(2)
   DX=X
   TRA=2.D+0/DX
   DTM=DFN+TRX
   TM=DTM
   IF(IN.EQ.0) GO TO 220
   BACKWARD RECUR TO INDEX ALPHA+NN-1
   DO 210 I=1,IN
   S=S2
   S2=TM+S2+S1
   S1=S
   DTM=DTM-TRX
   TM=DTM
210 CONTINUE
   Y(NN)=S1
   IF(NN.EQ.1) RETURN
   Y(NN-1)=S2
   IF(NN.EQ.2) RETURN
   GO TO 221
220 CONTINUE
   BACKWARD RECUR FROM INDEX ALPHA+NN-1 TO ALPHA
   Y(NN)=S1
   Y(NN-1)=S2
221 IF(NN.EQ.2) RET'
   K=NN+1
   DO 230 I=3,NN

```

```

K=K-1
Y(L)=TM*Y(K-1)+Y(K)
DFA JTM-TRX
TM=DTM
230 CONTINUE
250 Y(1)=TEMP(2)
RETURN
C
C ASYMPTOTIC EXPANSION FOR X TO INFINITY
C
500 CONTINUE
EARG=RTPL/SQRT(X)
IF(KODE.EQ.2) GO TO 502
501 IF(X.GT.ELIM) GO TO 94
EARG=EARG*EXP(X)
502 ETX=B.*X
IS=KT
IN=O
FN=FNU
503 DX=DFN+DFN
DTM=DX*DX
S1=ETX
TRX=S1
DX=-(DTM-1.D+O)/TRX
T=DX
TRX=1.D+O + DX
S=TRX
S2=1.
AK=B.
DO 504 K=1,25
S1=S1+ETX
S2=S2+AK
DX=S2
TRX=DTM-DX
AP=TRX
T=T*AP/S1
S=S+T
IF(ABS(T).LE.TOL) GO TO 505
AK=AK+B.
504 CONTINUE
505 TEMP(1S)=S+EARG
GO TO (508,204),IS
508 IS=2
DFN=DFN-1.D+O
FN=DFN
GO TO 503
C
C BACKWARD RECURSION WITH NORMALIZATION BY
C ASYMPTOTIC EXPANSION FOR NU TO INFINITY OR POWER SERIES.
C
600 CONTINUE
COMPUTATION OF LAST ORDER FOR SERIES NORMALIZATION
NM=AMAX1(3.-FN,0.)
TFN=FN+FLOAT(NM)
TA=(GLN+TFN-0.9189385332-0.0833333333/TFN)/(TFN+0.5)
TA=XO2L-TA
TB=-(1.-1./TFN)/TFN
IN=CE/(-TA+SQRT(TA*TA-CE*TB))+1.5
IN=IN+NM
GO TO 603
650 CONTINUE
COMPUTATION OF LAST ORDER FOR ASYMPTOTIC EXPANSION NORMALIZATION
IN=CE/(GLN+SQRT(GLN+GLN+1.*CE))+1.5
IF(IN.GT.INLIM) GO TO 199
603 DX=FLOAT(IN)

```

```

D1 1FN+DX
DX
TRX=2.D+0/DX
DTM=DTM+TRX
TM=DTM
TA=0.
TB=TOL
KK=1
605 CONTINUE
C
C BACKWARD RECUR UNINDEXED
C
DO 601 I=1.IN
S=TB
TB=TM+TB+TA
TA=S
DTM=DTM-TRX
TM=DTM
601 CONTINUE
C NORMALIZATION
IF(KK.NE.1) GO TO 604
TA=(TA/TB)*TEMP(3)
TB=TEMP(3)
KK=2
IN=NS
IF(NS.NE.0) GO TO 605
604 Y(NN)=TB
615 NZ=N-NN
IF(NN.EQ.1) RETURN
TB=TM+TB+TA
K=NN-1
Y(K)=TB
IF(NN.EQ.2) RETURN
DTM=DTM-TRX
TM=DTM
KM=K-1
C BACKWARD RECUR INDEXED
C
C
DO 602 I=1.KM
Y(K-1)=TM+Y(K)+Y(K+1)
DTM=DTM-TRX
TM=DTM
K=K-1
602 CONTINUE
RETURN
C
C
90 CONTINUE
CALL ERRCHK(42,42HIN BESI, SCALING OPTION, MODE, NOT 1 OR 2.)
RETURN
91 CONTINUE
CALL ERRCHK(38,38HIN BESI, ORDER, ALPHA, LESS THAN ZERO.)
RETURN
92 CONTINUE
CALL ERRCHK(25,25HIN BESI, N LESS THAN ONE.)
RETURN
93 CONTINUE
CALL ERRCHK(26,26HIN BESI, X LESS THAN ZERO.)
RETURN
94 CONTINUE
CALL ERRCHK(42,42HIN BESI, OVERFLOW, X TOO LARGE FOR MODE=1.)
RETURN
END
SUBROUTINE BESYN(X,NU,N,Y)

```

W/ TEN BY D.E. AMOS AND S.L. DANIEL, FEBRUARY, 1974.

REFERENCE SAND-75-0150

# ABSTRACT

BESYN IMPLEMENTS FORWARD RECURSION ON THE THREE TERM RECURSION RELATION FOR A SEQUENCE OF INTEGER ORDER BESSEL FUNCTIONS  $Y/\text{SUB}(NU+K-1)/(X)$ ,  $K=1, \dots, N$  FOR REAL  $X \geq 0$  AND A NON-NEGATIVE INTEGER  $NU$ . IF  $NU \leq LT$ ,  $NU$ IM, ORDERS 0 AND 1 ARE OBTAINED FROM FUNCTION BESY01 TO START THE RECURSION. IF  $NU \geq NULIM$ , THE UNIFORM ASYMPTOTIC EXPANSION IS USED FOR ORDERS  $NU$  AND  $NU+1$  TO START RECURSION.  $NULIM=100$  RESTRICTS FORWARD RECURSION TO RETAIN ACCURACY. AN OVERFLOW TEST IS MADE ON THE LEADING TERM OF THE ASYMPTOTIC EXPANSION BEFORE ANY EXTENSIVE COMPUTATION IS DONE. BESYN CALLS FUNCTION BESY01 AND SUBROUTINE ASYBES. BESY01 CALLS FUNCTION BESJ01. ASYBES CALLS SUBROUTINE YBAIRY.

# DESCRIPTION OF ARGUMENTS

INPUT  
 X -  $X \geq 0$   
 NU - ORDER OF THE INITIAL Y FUNCTION,  $NU=0, 1, 2, \dots$   
 N - NUMBER OF MEMBERS IN THE SEQUENCE,  $N \geq 1$

OUTPUT  
 Y - A VECTOR WHOSE FIRST N COMPONENTS CONTAIN VALUES FOR  $Y(K)=Y/\text{SUB}(NU+K-1)/(X)$ ,  $K=1, \dots, N$

ERROR CONDITIONS  
 IMPROPER INPUT ARGUMENTS - A FATAL ERROR  
 OVERFLOW - A FATAL ERROR

BESYN USES SUBROUTINES BESY01, BESJ01, ASYBES, YBAIRY, ERRCHK, ERRGET, ERRPRT, ERRSET, ERNSTCT

COMPILE DECKS BESYN, BESY01, BESJ01, ERRCHK

DIMENSION Y(1)

DATA NULIM / 100 /

DATA ELIM/667./

TEST INPUT ARGUMENTS

IF(MU.LT.0) GO TO 91

IF(X.LE.0.) GO TO 92

IF(N.LT.1) GO TO 93

NN=MINO(2,N)

FN=NU

FN=NU+N-1

IF(FN.LT.2.) GO TO 300

OVERFLOW TEST (LEADING EXPONENTIAL OF ASYMPTOTIC EXPANSION)

FOR THE LAST ORDER,  $NU+N-1 \geq 2$

XXN=X/FN

W2N=1.-XXN\*XXN

IF(W2N.LE.0.) GO TO 20

RAN=SQRT(W2N)

AZN=ALOG((1.+RAN)/XXN)-RAN

CN=FN+AZN

```

IF I.GT.ELIM) GO TO 94
20 IF J.LT.NULIM) GO TO 200
CALL ASYBES(FNU,NN,X,Y)
GO TO (25,27).NN
25 RETURN
27 TRA=2./X
TM=TRX*(FNU+1.)
GO TO 250

C
200 CONTINUE
S1=BESY01(X,0,DJ)
S2=BESY01(X,1,DJ)
TRA=2./X
TM=TRX
IN=NU-1
IF(IN) 220,211,203
FORWARD RECUR FROM 0 TO NU TO GET Y(1)
203 DO 210 I=1,IN
S=S2
S2=TM+S2-S1
S1=S
TM=TM+TRX
210 CONTINUE
FORWARD RECUR FROM NU TO NU+1 TO GET Y(2)
211 IF(N.EQ.1) GO TO 251
S=S2
S2=TM+S2-S1
S1=S
TM=TM+TRX
220 CONTINUE
FORWARD RECUR FROM NU+2 TO NU+N-1
Y(1)=S1
Y(2)=S2
250 CONTINUE
IF(N.EQ.2) RETURN
NM2=N-2
DO 230 I=1,NM2
Y(I+2)=TM+Y(I+1)-Y(I)
TM=TM+TRX
230 CONTINUE
RETURN
251 Y(1)=S2
RETURN

C
300 CONTINUE
NUM1=NU-1
DO 315 I=1,N
J=NUM1+I
ANS=BESY01(X,J,DJ)
Y(I)=ANS
315 CONTINUE
RETURN

C
C
C
91 CONTINUE
CALL ERRCHK(36,36HIN BESYN, ORDER, NU, LESS THAN ZERO.)
RETURN
92 CONTINUE
CALL ERRCHK(39,39HIN BESYN, X LESS THAN OR EQUAL TO ZERO.)
RETURN
93 CONTINUE
CALL ERRCHK(26,26HIN BESYN, N LESS THAN ONE.)
RETURN
94 CONTINUE

```

AD-A139 604

MATHEMATICAL MODELLING OF WAVEGUIDING TECHNIQUES AND  
ELECTRON TRANSPORT VOLUME 2(U) ARCON CORP WALTHAM MA  
S WOOLF ET AL JAN 84 RADC-TR-83-313-VOL-2

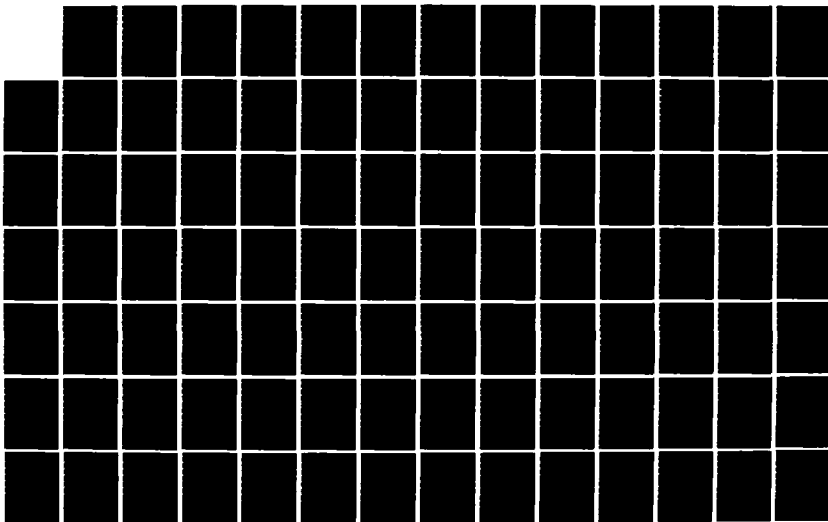
3/5

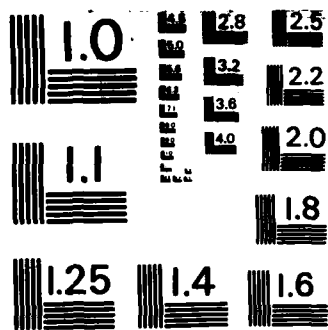
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F/G 9/2

NL





MICROCOPY RESOLUTION TEST CHART  
NATIONAL BUREAU OF STANDARDS-1963-A

```

CALL ERRCHK(53.53HIN BESYN, OVERFLOW, NU OR N TOO LARGE OR X TOO S
1MAY' )
RE: 'A
END
SUBROUTINE ASYBES(FNU,IN,X,Y)

```

```

      ASYBES COMPUTES Y BESSEL FUNCTIONS
      Y/SUB(NU+K-1)/(X), K=1,IN
      FOR X.GT.0., FNU.GE.32. AND IN=1,2

```

#### INPUT

```

FNU - ORDER OF FIRST BESSEL FUNCTION
IN - NUMBER OF Y FUNCTIONS DESIRED, IN=1 OR 2
X - ARGUMENT, X.GT.0

```

#### OUTPUT

```

Y - A VECTOR WHOSE FIRST IN COMPONENTS CONTAIN THE SEQUENCE
      Y(K)=Y/SUB(NU+K-1)/(X), K=1,IN

```

WRITTEN BY  
D. E. AMOS  
S. L. DANIEL

#### ABSTRACT

ASYBES IMPLEMENTS THE UNIFORM ASYMPTOTIC EXPANSION OF  
Y/SUB(FNU)/(4) FOR FNU.GE.32 AND REAL X.GT.0. THE FORM, EXCEPT  
FOR SIGN, IS IDENTICAL TO THAT IMPLEMENTED IN JBESS WHERE  
THE AI(X) AIRY FUNCTION IS REPLACED BY THE BI(X) FUNCTION.  
CORRESPONDINGLY, YBAIRY REPLACES JAIRY FOR THE AIRY FUNCTION  
EVALUATIONS. ASYBES CALLS YBAIRY.

```

DIMENSION Y(1)
DIMENSION C(11,10),ALFA(26,4),BETA(26,5)
DIMENSION C1(11,8),C2(11,2),ALFA1(26,2),ALFA2(26,2)
DIMENSION BETA1(26,2),BETA2(26,2),BETA3(26,1)
DIMENSION GAMMA(26),KMAX(5),AR(8),BR(10),UPOL(10)
DIMENSION CR(10),DR(10)

EQUIVALENCE(C(1,1),C1(1,1))
EQUIVALENCE(C(1,9),C2(1,1))
EQUIVALENCE(ALFA(1,1),ALFA1(1,1))
EQUIVALENCE(ALFA(1,3),ALFA2(1,1))
EQUIVALENCE(BETA(1,1),BETA1(1,1))
EQUIVALENCE(BETA(1,3),BETA2(1,1))
EQUIVALENCE(BETA(1,5),BETA3(1,1))

DATA ELIM1,ELIM2,FNULIM,TOL/ 667. , 644. , 32. , 1.E-15 /
DATA TOLS=LN(1.E-3)
DATA TOLS /-6.9077527898214E+00/

DATA UPOL(1),CON1,CON2,CON3,CON548 / 1.000000000000000E+00,
1 6.666666666666667E-01, 3.333333333333333E-01, 1.41421356237310E+00,
2 1.041666666666667E-01/

DATA AR / 8.35503472222222E-02, 1.28226574556327E-01,
1 2.91849026464140E-01, 8.81627267443758E-01, 3.32140828186277E+00,
2 1.49957629868626E+01, 7.89230130115865E+01, 4.74451538868264E+02/

DATA BR /-1.45833333333333E-01,-9.87413194444444E-02,
1-1.43312053915895E-01,-3.17227202678414E-01,-9.42428147957120E-01,
2-3.51120304082638E+00,-1.57272636203680E+01,-8.22814390971859E+01,
3-4.92355370523671E+02,-3.31621856854797E+03/

DATA C1 /-2.08333333333333E-01, 1.25000000000000E-01,

```



1 2.88501402231133E-03, 1.8008606761054E-03, 1.24753110589199E-03,  
 2 9.878876572938E-04, 7.1443042172787E-04, 5.717872817891E-04,  
 3 4.431007606482E-04, 3.93232385462917E-04, 3.348188893181E-04,  
 4 2.88952148495752E-04, 2.52211615549573E-04, 2.2280580798883E-04,  
 5 1.97541838033063E-04, 1.75836855019718E-04, 1.59316939661821E-04,  
 6 4.4347930197334E-04, 1.31448068119965E-04, 1.20245444949303E-04,  
 7 1.10449144504599E-04, 1.01828770740567E-04, 9.4199822404238E-05,  
 8 7.413054573834E-05, 8.13466262162801E-05, 7.59002269646219E-05,  
 9 1.49282953213429E-03, 8.78204709546389E-04, 5.02916549572035E-04,  
 A 2.9822138512746E-04, 1.7583996970783E-04, 1.040855046081E-04,  
 B 5.56141953048458E-05, 3.12038929076098E-05, 1.26089759880233E-05,  
 C 2.4892608575730E-07, 8.05996165414274E-06, 1.3650700926790823E-05,  
 D 1.73964125472928E-05, 1.98672978842134E-05, 2.14632637590823E-05,  
 E 2.23954659232457E-05, 2.28967783814713E-05, 2.30785389811078E-05,  
 F 2.30321976808099E-05, 2.28236073720349E-05, 2.25005881105292E-05,  
 G 2.20981015361991E-05, 2.16418427448104E-05, 2.11507649256221E-05,  
 H 2.06388749782171E-05, 2.01165241997082E-05/

C

DATA BETA2  
 1 5.52213076721293E-04, 4.47932581553385E-04,  
 2 7.952653992021E-04, 1.52468156198447E-04, 6.93271105657044E-05,  
 3 1.76258683069991E-05, 1.3574498634269E-05, 3.17972413350427E-05,  
 4 1.8861861696693E-05, 4.69004889379141E-05, 4.87665447413787E-05,  
 5 4.87010031186735E-05, 4.74755620890087E-05, 4.55813058138628E-05,  
 6 3.3309644511266E-05, 4.09230193157750E-05, 3.84822638603221E-05,  
 7 3.60857167535411E-05, 3.37793306123367E-05, 3.15888560772110E-05,  
 8 2.95269561750807E-05, 2.7597891482836E-05, 2.58008174666884E-05,  
 9 4.1308356761280E-05, 2.25823509518346E-05, 2.11479656768913E-05,  
 A 7.461779855960E-04, 4.77864567147321E-04, 3.20390228067038E-04,  
 B 1.61105018119962E-04, 4.25778101285435E-05, 3.44571294294868E-05,  
 C 7.97092684075675E-05, 1.03138236708272E-04, 1.12466775262202E-04,  
 D 1.3103642108481E-04, 1.08651634848774E-04, 1.01437951597662E-04,  
 E 9.292963959364E-05, 8.40293133016090E-05, 7.52727991349138E-05,  
 F 6.69632521975731E-05, 5.92564547323195E-05, 5.22169308826978E-05,  
 G 4.58539485165361E-05, 4.01445513891487E-05, 3.50481730031328E-05,  
 H 3.05157995034347E-05, 2.64956119950516E-05, 2.29363633690998E-05,  
 I 1.9789305664022E-05, 1.70091984636413E-05/

C

DATA BETA3  
 1 7.36465810572578E-04, 8.72790805146194E-04,  
 2 6.22614862573135E-04, 2.85998154194304E-04, 3.84737672879366E-06,  
 3 1.8790603636972E-04, 2.9760346584555E-04, 3.45998126832656E-04,  
 4 3.3382470916038E-04, 3.35715635775049E-04, 3.0432112789040E-04,  
 5 2.667223047613E-04, 2.27654214122820E-04, 1.89922611854562E-04,  
 6 1.5505891859094E-04, 1.23778240761874E-04, 9.62926147717644E-05,  
 7 2.5178327714425E-05, 5.22070028895634E-05, 3.50347750511901E-05,  
 8 7.0648976103552E-05, 8.70106096849767E-06, 1.13698686675100E-06,  
 9 1.6426474122779E-06, 1.56477785428873E-05, 2.08223629482467E-05/

C

DATA GAMA  
 1 2.9960524947437E-01, 2.51984209978975E-01,  
 2 1.54790300415656E-01, 1.10713062416159E-01, 8.57308395527395E-02,  
 3 6.9716131695868E-02, 5.86085671893714E-02, 5.04698873536311E-02,  
 4 4.2600580699155E-02, 3.93720661543510E-02, 3.54283195924455E-02,  
 5 2.1818857502098E-02, 2.94646240791158E-02, 2.71581677112934E-02,  
 6 2.5176827973862E-02, 2.34570755306079E-02, 2.19508390134907E-02,  
 7 2.06210828235646E-02, 1.94388240897881E-02, 1.83810633800683E-02,  
 8 1.74293213231963E-02, 1.65885837766612E-02, 1.57865285987919E-02,  
 9 1.50729501494086E-02, 1.44193250839955E-02, 1.38184805735342E-02/

C

130 CONTINUE

FN=FNU  
 DO 100 JN=1,IN  
 XX=X/FN  
 W2=1.-XX\*XX  
 ABW2=ABS(W2)  
 IF(ABW2.GT.0.2775) GO TO 200

C



```

KB=KB-1
S7  \W2+ALPA(KB,KS)
S8  J*W2+BETA(KB,NSP1)
26 CONTINUE
TA=SA+RDEN
TB=SB+RDEN
ASUM=ASUM+TA
BSUM=BSUM+TB
IF(ABS(TA).LE.TOL.AND.ABS(TB).LE.RELB) GO TO 152
25 CONTINUE
152 CONTINUE
BSUM=BSUM/(FN*FN13)
GO TO 400

C 200 CONTINUE
RA=SQRT(ABW2)
TAU=1./RA
T2=1./W2
IF(W2.GE.0.) GO TO 30
C CASES FOR (X/FN).GT.SQRT(1.2775)
AZ32=ABS(RA-ATAN(RA))
ACZ-AZ32*FN
CZ=-ACZ
Z32=1.5*AZ32
RTZ-Z32*CON2
FN13=FN*CON2
RTARY=RTZ*FN13
ARY=-RTARY*RTARY
GO TO 150
30 CONTINUE

C CASES FOR (X/FN).LT.SQRT(0.7225)
AZ32=ABS(ALOG((1.+RA)/XX) -RA)
TEST FOR OVERFLOW
ACZ-AZ32*FN
CZ=ACZ
IF(ACZ.GT.ELIM1) GO TO 94
Z32=1.5*AZ32
RTZ-Z32*CON2
FN13=FN*CON2
RTARY=RTZ*FN13
ARY=RTARY*RTARY
150 CONTINUE
PHI=SQRT((RTZ+RTZ)*TAU)
TB=1.
ASUM=1.
TFN=TAU/FN
UPOL(2)=(C(1,1)*T2+C(2,1))*TFN
RCZ=CON1/CZ
CRZ32=CON548*RCZ
BSUM=UPOL(2)*CRZ32
RELB=TOL*ABS(BSUM)
AP=TFN
NS=0
NP1=2
RZDEN=RCZ
DO 155 LR=2,6,2
C COMPUTE TWO U POLYNOMIALS FOR NEXT A(ZE/A) AND B(ZETA)
C
LRP1=LR+1

```

```

DO 101 K=LR,LRP1
  KS  +1
  KP1=AP1+1
  S1=C(1,K)
  DO 102 J=2,KP1
    S1=S1+2*C(J,K)
  102 CONTINUE
  AP=AP+TFN
  UPOL(KP1)=AP+S1
  CR(KS)=BR(KS)*RZDEN
  RZDEN=RZDEN*RCZ
  DR(KS)=AR(KS)*RZDEN
  101 CONTINUE
  SUMA=UPOL(LRP1)
  SUMB=UPOL(LR+2)*UPOL(LRP1)*CR232
  JU=LRP1
  DO 151 JR=1,LR
    JU=JU-1
    SUMA=SUMA+CR(JR)*UPOL(JU)
    SUMB=SUMB+DR(JR)*UPOL(JU)
  151 CONTINUE
  TB=-TB
  IF(W2.GT.0.) TB=ABS(TB)
  ASUM=ASUM+SUMA*TB
  BSUM=BSUM+SUMB*TB
  IF(ABS(SUMA).LE.TOL.AND.ABS(SUMB).LE.REL8) GO TO 165
  155 CONTINUE
  165 TB=RTARY
  IF(W2.GT.0.) TB=-TB
  BSUM=BSUM/TB
C
  400 CONTINUE
  CALL YBAIRY(ARY,RTARY,ACZ,BI,DBI)
  Y(JN)=PHI*(BI*ASUM+DBI*BSUM)/FN13
  FN=FN+1
  100 CONTINUE
  94 RETURN
  CALL ERRCHK(20,20HIN ASYBES, OVERFLOW.)
  RETURN
  END
  SUBROUTINE YBAIRY(X,RX,C,BI,DBI)
    YBAIRY COMPUTES THE AIRY FUNCTION BI(X)
    AND ITS DERIVATIVE DBI(X) FOR ASYBES
    INPUT
    X - ARGUMENT, COMPUTED BY ASYBES, X UNRESTRICTED
    RX - RX=SQRT(ABS(X)), COMPUTED BY ASYBES
    C - C=2.*(ABS(X)+1.5)/3., COMPUTED BY ASYBES
    OUTPUT
    BI - VALUE OF FUNCTION BI(X)
    DBI - VALUE OF THE DERIVATIVE DBI(X)
    WRITTEN BY
    D. E. AMOS
    S. L. DANIEL
    DIMENSION BK1(20),BK2(20),BK3(20),BK4(14)
    DIMENSION BJP(19),BJN(19),AA(14),BB(14)
    DIMENSION DBK1(21),DBK2(20),DBK3(20),DBK4(14)
    DIMENSION DBJP(19),DBJN(19),DAA(14),DBB(14)
C

```

DATA M1,M2,N3/20,19,14/  
DA M1,M2,N3/18,17,12/  
DA MID,N2D,N3D,M4D/21,20,19,14/  
DATA MID,N2D,N3D,M4D/19,18,17,12/  
DATA FPI12,SPI12,CON1,CON2,CON3/  
1 1.309959389575E+00, 1.83259571459405E+00, 6.6666666666667E-01,  
2 7.74148278841775E+00, 3.64786105490356E-01/  
DATA BK1 / 2.43202846447449E+00, 2.57132009754685E+00,  
1 1.02802341258618E+00, 3.4195817825872E-01, 8.4197862888284E-02,  
2 1.93077282587962E-02, 3.92687837130335E-03, 8.43026889948045E-04,  
3 1.14611403991141E-04, 1.741951788337086E-05, 2.41223629958355E-06,  
4 3.2452591983273E-07, 4.03508788540183E-08, 4.70875059642298E-09,  
5 5.3536743258589E-10, 5.70606721846334E-11, 5.80526363709933E-12,  
6 5.76338988616388E-13, 5.42103834518071E-14, 4.91857330301677E-15/  
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2 1.22269906524508E-08, -1.01381314386052E-09, 9.63639784237475E-11,
3-1.02344993379648E-11, 1.19284576554355E-12, -1.50443899103287E-13,
4 2.03299052379349E-14, -2.91890652008292E-15, 4.42322081975475E-16/

C
DATA DBJP / 1.1314087330745E-01, -2.08301511416328E-01,
1 1.69396341953138E-02, 2.90895212478621E-02, -3.41467131311549E-03,
2-1.46455339197417E-03, 1.63313272898517E-04, 3.9114532822162E-05,
3-3.96757190808115E-06, -8.51846913772395E-07, 5.98707495269280E-08,
4 7.44108654536549E-09, -6.21241056522832E-10, -6.1876801731326E-11,
5 4.7232348475324E-12, 3.91652459802532E-13, -2.74985937845226E-14,
6-1.95036497762750E-15, 1.26669643809444E-16/

C
DATA DBJN /-1.880912600688950E-02, -1.47798180826140E-01,
1 5.46075900433171E-01, 1.52146932663116E-01, -9.58260412266886E-02,
2-1.63102731696130E-02, 5.75364806680105E-03, 7.12145408252655E-04,
3-1.754521116846724E-04, -1.71063171685128E-05, 3.24435580531680E-06,
4 2.61190663932884E-07, -4.03026865912779E-08, -2.76435165853895E-09,
5 3.59687929062312E-10, 2.14953306456051E-11, -2.41849311903901E-12,
6-1.28068004920751E-13, 1.26939834401773E-14/

C
DATA DAA / 2.77571356944231E-01, -4.44212833419920E-03,
1 8.42328522190089E-05, 2.58040318418710E-06, -3.42389720217621E-07,
2 6.24286894709776E-09, 2.36377836844577E-09, -3.16991042656673E-10,
3 4.40995691658191E-12, 5.18674221093575E-12, -9.64874015137022E-13,
4 4.90190576608710E-14, 1.77253430678112E-14, -5.55950610442662E-15/

C
DATA DBB / 4.91627321104601E-01, 3.11164930427489E-03,
1 8.23140762854081E-05, -4.61769776172142E-06, -6.13158880534626E-08,
2 2.82795804656520E-08, -1.81959715372117E-09, -1.4475282642035E-10,
3 4.53724043420422E-11, -3.99655065847223E-12, -3.24089119830323E-13,
4 1.62098952568741E-13, -2.40785247974057E-14, 1.69384811284491E-16/

C
AX=ABS(X)
RX=SQRT(AX)
C=CON1*AX*RX
IF(X.LT.0.) GO TO 300
IF(C.GT.8.) GO TO 200
IF(C.GT.8.) GO TO 200

```

```

IF(X.GT.2.5) GO TO 150
T = (X-2.5)*.4
J=N1
F1=BK1(J)
F2=0.
DO 105 I=1,M1
  J=J-1
  TEMP1=F1
  F1=TT*F1-F2+BK1(J)
  F2=TEMP1
105 CONTINUE
  B1=TT*F1-F2+BK1(1)
  J=N1D
  F1=DK1(J)
  F2=0.
  DO 106 I=1,M1D
    J=J-1
    TEMP1=F1
    F1=TT*F1-F2+DK1(J)
    F2=TEMP1
106 CONTINUE
    DB1=TT*F1-F2+DK1(1)
    RETURN
150 CONTINUE
  RTRX=SQRT(RX)
  T=(X+X-CON2)*CON3
  TT=TT+T
  J=N1
  F1=BK2(J)
  F2=0.
  DO 155 I=1,M1
    J=J-1
    TEMP1=F1
    F1=TT*F1-F2+BK2(J)
    F2=TEMP1
155 CONTINUE
    B1=(TT*F1-F2+BK2(1))/RTRX
    EX=EXP(C)
    B1=B1*EX
    J=N2D
    F1=DK2(J)
    F2=0.
    DO 156 I=1,M2D
      J=J-1
      TEMP1=F1
      F1=TT*F1-F2+DK2(J)
      F2=TEMP1
156 CONTINUE
      DB1=(TT*F1-F2+DK2(1))/RTRX
      DB1=DB1*EX
      RETURN
C
200 CONTINUE
  RTRX=SQRT(RX)
  T=16./C-1.
  TT=TT+T
  J=N1
  F1=BK3(J)
  F2=0.
  DO 205 I=1,M1
    J=J-1
    TEMP1=F1
    F1=TT*F1-F2+BK3(J)
    F2=TEMP1
205 CONTINUE

```

```

S1=7*F1-F2+BK3(1)
J=J-1
F1=BK3(J)
F2=0.
DO 206 I=1,M2D
  J=J-1
  TEMP1=F1
  F1=TT*F1-F2+BK3(J)
  F2=TEMP1
206 CONTINUE
D1=7*F1-F2+BK3(1)
TC=C+C
EX=EXP(C)
IF(TC.GT.35.) GO TO 214
T=10./C-1.
TT=TT+T
J=N3
F1=BK4(J)
F2=0.
DO 210 I=1,M3
  J=J-1
  TEMP1=F1
  F1=TT*F1-F2+BK4(J)
  F2=TEMP1
210 CONTINUE
S2=7*F1-F2+BK4(1)
B1=(S1+EXP(-TC)*S2)/RTRX
B1=BI*EX
J=N4D
F1=BK4(J)
F2=0.
DO 211 I=1,M4D
  J=J-1
  TEMP1=F1
  F1=TT*F1-F2+BK4(J)
  F2=TEMP1
211 CONTINUE
D2=7*F1-F2+BK4(1)
DB1=RTRX*(D1+EXP(-TC)*D2)
DB1=DBI*EX
RETURN
214 B1=EX*S1/RTRX
DB1=EX*RTRX*D1
RETURN
C
300 CONTINUE
IF(C.GT.5.) GO TO 350
T=4*C-1.
TT=TT+T
J=N2
F1=BJP(J)
E1=BJN(J)
F2=0.
E2=0.
DO 305 I=1,M2
  J=J-1
  TEMP1=F1
  TEMP2=E1
  F1=TT*F1-F2+BJP(J)
  E1=TT*E1-E2+BJN(J)
  F2=TEMP1
  E2=TEMP2
305 CONTINUE
B1=(T*E1-E2+BJN(1))-AX*(T*F1-F2+BJP(1))
J=N3D
F1=BJP(J)

```

```

E1=DBUN(J)
F2
E2
DO 306 I=1,M3D
J=J-1
TEMP1=F1
TEMP2=E1
F1=TT*F1-F2*DBUN(J)
E1=TT*E1-E2*DBUN(J)
F2=TEMP1
E2=TEMP2
306 CONTINUE
DBI=X*X*(T*F1-F2*DBUN(1))+T*E1-E2*DBUN(1))
RETURN

C
350 CONTINUE
RTRX=SQRT(RX)
T=10./C-1.
TT=T+T
J=N3
F1=AA(J)
E1=BB(J)
F2=0.
E2=0.
DO 310 I=1,M3
J=J-1
TEMP1=F1
TEMP2=E1
F1=TT*F1-F2*AA(J)
E1=TT*E1-E2*BB(J)
F2=TEMP1
E2=TEMP2
310 CONTINUE
TEMP1=T*F1-F2*AA(1)
TEMP2=T*E1-E2*BB(1)
CV=C-FP112
BI=(TEMP1*COS(CV)+TEMP2*SIN(CV))/RTRX
J=N4D
F1=DAA(J)
E1=DOB(J)
F2=0.
E2=0.
DO 311 I=1,M4D
J=J-1
TEMP1=F1
TEMP2=E1
F1=TT*F1-F2*DAA(J)
E1=TT*E1-E2*DOB(J)
F2=TEMP1
E2=TEMP2
311 CONTINUE
TEMP1=T*F1-F2*DAA(1)
TEMP2=T*E1-E2*DOB(1)
CV=C-SP112
DBI=(TEMP1*COS(CV)-TEMP2*SIN(CV))/RTRX
RETURN
END
SUBROUTINE BESKN (X,MU,KODE,N,Y,NZ)
C
C WRITTEN BY D.E. AMOS AND S.L. DANIEL, FEBRUARY, 1974.
C
C REFERENCE SAND-75-0151
C
C ABSTRACT
C BESKN IMPLEMENTS FORWARD RECURSION ON THE THREE TERM
C RECURSION RELATION FOR A SEQUENCE OF INTEGER ORDER BESSEL

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```

NN=MINO(2,ND)
FY IUD
FN JD=ND-1
FNN=FN
IF(FN.LT.2.) GO TO 300

C OVERFLOW TEST (LEADING EXPONENTIAL OF ASYMPTOTIC EXPANSION)
C FOR THE LAST ORDER, NU=N-1,GE,NULIM
C
ZN=X/FN
RTZ=RTZ/(1.+ZN*ZN)
GLN=ALOG((1.+RTZ)/ZN)
T=RTZ*(1.-ETX)+ETX/(ZN+RTZ)
CN=FN*(T-GLN)
IF(CN.GT.ELIM) GO TO 94
IF(NUD.LT.NULIM(NN)) GO TO 200
IF(NN.EQ.1) GO TO 10
S CONTINUE

C UNDERFLOW TEST (LEADING EXPONENTIAL OF ASYMPTOTIC EXPANSION)
C FOR THE FIRST ORDER, NU,GE,NULIM
C
FN=FNU
ZN=X/FN
RTZ=RTZ/(1.+ZN*ZN)
GLN=ALOG((1.+RTZ)/ZN)
T=RTZ*(1.-ETX)+ETX/(ZN+RTZ)
CN=FN*(T-GLN)
10 IF(CN.LT.-ELIM) GO TO 95
20 CONTINUE

C ASYMPTOTIC EXPANSION FOR ORDERS NU AND NU+1,GE,NULIM
C
CALL ASKBES(KODE,FNU,NM,X,RTZ,CN,Y)
GO TO (96,27). NM
27 TRX=2./X
TM=TRX*(FNU+1.)
GO TO 250

C 200 CONTINUE
IF(KODE.EQ.2) GO TO 201

C UNDERFLOW TEST (LEADING EXPONENTIAL OF ASYMPTOTIC EXPANSION IN X)
C FOR ORDER 0
C
204 IF(X.GT.ELIM) GO TO 95
201 S1=BESK01(X,0,KODE,MZ)
S2=BESK01(X,1,KODE,MZ)
TRX=2./X
TM=TRX
HERE NU=0 OR 1 IMPLIES N,GE,2. OR NU,GE,2 IMPLIES N,GE,1
IN=NUD-1
IF(IN) 220,211,203
FORWARD RECUR FROM 0 TO NU TO GET Y(1)
C 203 DO 210 I=1,IN
S=S2
S2=TM+S2+S1
S1=S
TM=TM+TRX
S1=S
TM=TM+TRX
210 CONTINUE
C FORWARD RECUR FROM NU TO NU+1 TO GET Y(2)
211 IF(ND.EQ.1) GO TO 251
S=S2
S2=TM+S2+S1
S1=S
TM=TM+TRX

```

```

C 220 CONTINUE
C 221 Y(1)=S1
C 222 Y(2)=S2
C 250 CONTINUE
C IF(ND.EQ.2) GO TO 96
C NM2=ND-2
C DO 230 I=1,NM2
C Y(I+2)=TM*Y(I+1)+Y(I)
C TM=TM*TRX
C 230 CONTINUE
C GO TO 96
C 251 Y(1)=S2
C GO TO 96
C 300 CONTINUE
C FN.LT.2 IMPLIES THE CASES NU=0, N=1 OR 2
C NU=1, N=1
C GO TO (309,310),KODE
C 309 IF(X.GT.ELIM) GO TO 95
C 310 CONTINUE
C NUM1=NUD-1
C DO 315 I=1,ND
C J=NUM1+I
C ANS=BESK01(X,J,KODE,M2)
C Y(I)=ANS
C 315 CONTINUE
C GO TO 96
C 95 NUD=NUD+1
C NO=NO-1
C IF(ND.EQ.0) GO TO 96
C NM=MIN0(2,ND)
C FNU=NUD
C IF(FNN.LT.2.) GO TO 95
C IF(MUD.LT.NULIM(NN)) GO TO 200
C GO TO 5
C 96 NZ=N-ND
C IF(NZ.EQ.0) RETURN
C IF(ND.EQ.0) GO TO 98
C DO 97 I=1,ND
C J=N-I+1
C K=ND-I+1
C Y(J)=Y(K)
C 97 CONTINUE
C 98 DO 99 I=1,NZ
C Y(I)=0.
C 99 CONTINUE
C RETURN
C 90 CONTINUE
C CALL ERRCHK(43,43HIN BESKN, SCALING OPTION, KODE, NOT 1 OR 2.)
C RETURN
C 91 CONTINUE
C CALL ERRCHK(36,36HIN BESKN, ORDER, NU, LESS THAN ZERO.)
C RETURN
C 92 CONTINUE

```

```

CA)  ERRCHK(39.39*IN BESKN, X LESS THAN OR EQUAL TO ZERO.)
      N
93  CONTINUE
      CALL ERRCHK(26.26*IN BESKN, N LESS THAN ONE.)
      RETURN
94  CONTINUE
      CALL ERRCHK(53.53*IN BESKN, OVERFLOW, NU OR N TOO LARGE OR X TOO S
      1MALL.)
      RETURN
      END
SUBROUTINE ASKBES(KODE,FNU,IN,X,RA,ARG,Y)
CDC 6600 ROUTINE
JANUARY 1974

```

ASKBES COMPUTES K BESSEL FUNCTIONS  
 $K/\text{SUB}(FNU+1-1)/(X)$ ,  $I=1, IN$   
 OR SCALED BESSEL FUNCTIONS  
 $\text{EXP}(X)*K/\text{SUB}(FNU+1-1)/(X)$ ,  $I=1, IN$   
 FOR  $X.GT.0.$ ,  $FNU.GE.32.$ , AND  $IN=1$  OR  $2$

#### INPUT

KODE - A PARAMETER TO INDICATE THE SCALING OPTION  
 KODE=1 RETURNS  $Y(I)=K/\text{SUB}(FNU+1-1)/(X)$ ,  $I=1, IN$   
 KODE=2 RETURNS  $Y(I)=\text{EXP}(X)*K/\text{SUB}(FNU+1-1)/(X)$ ,  $I=1, IN$   
 FNU - ORDER OF FIRST BESSEL FUNCTION  
 IN - NUMBER OF K FUNCTIONS DESIRED,  $IN=1$  OR  $2$   
 X - ARGUMENT,  $X.GT.0.$   
 RA -  $\text{SORT}(1+Z/2)$ ,  $Z=X/FNU$   
 ARG - ARGUMENT OF THE LEADING EXPONENTIAL

#### OUTPUT

Y - A VECTOR WHOSE FIRST IN COMPONENTS CONTAIN THE SEQUENCE  
 $Y(I)=K/\text{SUB}(FNU+1-1)/(X)$ ,  $I=1, IN$  OR  
 $Y(I)=\text{EXP}(X)*K/\text{SUB}(FNU+1-1)/(X)$ ,  $I=1, IN$   
 DEPENDING ON KODE

WRITTEN BY  
 D. E. AMOS  
 S. L. DANIEL

ABSTRACT  
 ASKBES IMPLEMENTS THE UNIFORM ASYMPTOTIC EXPANSION OF  
 $K/\text{SUB}(FNU)/(X)$  FOR  $FNU.GE.32$  AND REAL  $X.GT.0.$  THE FORM, EXCEPT  
 FOR AN ALTERNATION OF SIGN IN THE EXPANSION, IS IDENTICAL TO  
 THAT IMPLEMENTED IN BESI.

DIMENSION Y(1)  
 DIMENSION C(11,10), C1(11,8), C2(11,2)  
 EQUIVALENCE (C(1,1), C1(1,1))  
 EQUIVALENCE (C(1,9), C2(1,1))  
 DATA TOL/1.E-15/  
 DATA RTPIO2/1.25331413731550/

DATA C1 9(0.) /-2.08333333333333E-01, 1.25000000000000E-01,  
 1 7.03125000000000E-02, 3.34201388888889E-01, -4.01041666666667E-01,  
 2 1.84646267361111E+00, -8.91210937500000E-01, 7.32421875000000E-02,  
 3 7(0.) /4.66958442342825E+00, -1.12070026162230E+01,  
 4 8.7891235315625E+00, -2.36408691406250E+00, 1.12152099609375E-01,  
 5 6(0.) /-2.82120725582002E+01, 8.46362176746007E+01,  
 6 7-9.18182415432400E+01, 4.2534987453885E+01, -7.36879435947963E+00,  
 8 2.27108001708984E-01, 5(0.) /2.12570130039217E+02,

```

9-7. 45252489141182E+02, 1.05998045282800E+03, -6.995796277376133E+02,
A 2( 190511744312E+02, -2.6491430489516E+01, 5.725014209747 -01,
4(0.) -1.91945766231841E+03, 8.061722181737 +03,
C-1.35865500664341E+04, 1.16553933368645E+04, -5.30564697861340E+03,
D 1.20090291321635E+03, -1.08090919788395E+02, 1.7272750258446E+00,
3(0.) 2.02042913309661E+04, -9.69805983886375E+04,
F 1.92547001232532E+05, -2.03400177280416E+05, 1.22200464983017E+05,
G-4.11926549688976E+04, 7.10951430248936E+03, -4.93915304773088E+02,
H 6.07404200127348E+00, 2(0.)

```

C

```

DATA C2
-2.42919187900551E+05, 1.31176361466298E+06,
1-2.99801591853811E+06, 3.76327129765640E+06, -2.81356322658653E+06,
2 1.2683652732162E+06, -3.31645172484564E+05, 4.52187689813627E+04,
3-2.49983048181121E+03, 2.43805296995561E+01, 1(0.)
4 3.28446985307204E+06, -1.97068191184322E+07, 5.09526024926646E+07,
5-7.41051482115327E+07, 6.63445122747290E+07, -3.75671766607634E+07,
6 1.32887671664218E+07, -2.78561812808645E+06, 3.08186404612662E+05,
7-1.38860897537170E+04, 1.10017140269247E+02/

```

C

```

FN=FNH
ETX=KODE-1
DO 200 JN=1, IN
IF(JN.EQ.1) GC TO 148
FN=FN+1.
Z=X/FN
RA=SQRT(1.+Z*Z)
GLN=ALOG((1.+RA)/Z)
T=RA*(1.-ETX)+ETX/(Z+RA)
ARG=-FN*(T-GLN)
148 COEF=EXP(ARG)
T=1./RA
T2=T*T
T3=T/FN
S2=1.
AP=1.
DO 140 K=1,10
KPI=K+1
S1=C(1,K)
DO 135 J=2,KPI
S1=S1*T2+C(J,K)
135 CONTINUE
AP=AP*T
AK=AP*S1
S2=S2*AK
IF(ABS(AK).LT.TOL) GO TO 145
140 CONTINUE
145 CONTINUE
Y(JN)=SQRT(T)*RTPID2*COEF*S2
200 CONTINUE
RETURN
END
FUNCTION GAMLN(X)

```

GMLN 1  
GMLN 2  
GMLN 4

WRITTEN BY D. E. AMOS, SEPTEMBER, 1977.

REFERENCES

SAND-77-1518

COMPUTER APPROXIMATIONS BY J.F. MART, ET AL., SIAM SERIES IN  
APPLIED MATHEMATICS, WILEY, 1968, P.135-136.

NBS HANDBOOK OF MATHEMATICAL FUNCTIONS, AMS 55, BY  
M. ABRAMOWITZ AND I.A. STEGUN, DECEMBER, 1955, P.257.

GMLN 6  
GMLN 7  
GMLN 8

ABSTRACT

GAMLN COMPUTES THE NATURAL LOG OF THE GAMMA FUNCTION FOR



```

6 2 0564940971863E+02, 2.64921649798553E+02, 2.69291097651070E+02, GMLN 61
7 1 673124285694E+02, 2.78067573440366E+02, 2.82474292687E+02, GMLN 62
8 2 6893133295427E+02, 2.91323950094270E+02, 2.957666013507E+02, GMLN 63
9 3 00220948847014E+02, 3.0468685676569E+02, 3.09164193580147E+02, GMLN 64
A 3.12652829949879E+02, 3.18152639620209E+02, 3.22663499128726E+02, GMLN 65
B 3.27185287703775E+02, 3.31717887196928E+02, 3.36281181979198E+02, GMLN 66
C 3.40815058870799E+02, 3.45379407062267E+02, 3.49954118040770E+02, GMLN 67
D 3.5453988519441E+02, 3.59134205369575E+02, GMLN 68
C IF(X) 90.90.5 GMLN 69
5 NDX=X
T=X-FLOAT(NDX)
IF(T.EQ.0.0) GO TO 51
DX=X-LIM1-X
IF(DX.LT.0.0) GO TO 40
C RATIONAL CHEBYSHEV APPROXIMATION ON 2.LT.X.LT.3 FOR GAMMA(X)
C
C NXM=NDX-2
PX=PCOE(1)
DO 10 I=2,9
10 PX=T*PX+PCOE(I)
QX=QCOE(1)
DO 15 I=2,4
15 QX=T*QX+QCOE(I)
DGAM=PX/QX
IF(NXM.GT.0) GO TO 22
IF(NXM.EQ.0) GO TO 25
C BACKWARD RECURSION FOR 0.LT.X.LT.2
C
C DGAM=DGAM/(1.+T)
IF(NXM.EQ.-1) GO TO 25
DGAM=DGAM/T
GMLN=ALOG(DGAM)
RETURN
C FORWARD RECURSION FOR 3.LT.X.LT.8
C
C 22 XX=2.+T
DO 24 I=1,NXM
DGAM=DGAM*XX
24 XX=XX+1.
25 GMLN=ALOG(DGAM)
RETURN
C X.GT.XLIM1
C
C 40 RX=1./X
RX=RX*RX
IF((X-XLIM2).LT.0.) GO TO 41
PX=Q(1)*RX+Q(2)
GMLN=PX*RX+(X-.5)*ALOG(X)-X*RTWPIL
RETURN
C X.LT.XLIM2
C
C 41 PX=P(1)
SUM=(X-.5)*ALOG(X)-X
DO 42 I=2,5
PX=PX*RX+P(I)
42 CONTINUE
GMLN=PX*RX+SUM+RTWPIL
RETURN
C TABLE LOOK UP FOR INTEGER ARGUMENTS LESS THAN OR EQUAL 100.

```

```

C  51 IF (X.GT.100) GO 10,40
    GAM=N=GLN(NDX)
    RETURN
90  CALL ERRCHK(49,49HIN GAMLN, ARGUMENT IS LESS THAN OR EQUAL TO ZERO
1.)
    RETURN
    END
GMLN 119
GMLN 120
GMLN 121
GMLN 122
GMLN 123
GMLN 124
GMLN 125

```

**Program Listing**

**- GRATING -**

**Subroutines: PDECOL, PLOTN**

**(Reference: Vol. I, Section III)**



WHICH OUTPUT INFORMATION IS DESIRED (SOLUTION ARRAYS, FIELDS, ETC.) ARE  
 OR EACH CARD-IMAGE GIVES A TIME INCREMENT AND THE NUMBER (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12) OF  
 CONSECUTIVE INCREMENTS OF THIS SIZE. TOGETHER THESE INCREMENTS CON-  
 STITUTE A SET OF STEPS. AT THE SUCCESSFUL COMPLETION OF SUCH A SET OF  
 STEPS, CONTROL RETURNS TO STATEMENT 3 FOR DEFINITION OF A NEW STEP OR A  
 NEW SET OF STEPS. (INFORMATION ON PLOTTING AND BEGINNING THE ERASE MODE  
 IS ALSO READ HERE; SEE BELOW.) A BLANK CARD HERE IS THE NORMAL MEANS  
 OF TERMINATING PROGRAM EXECUTION.

A TYPICAL DATA-SET MIGHT BE (SEE COMMENTS AT STATEMENTS 1, 2, AND 3):

```

C----- 100 12 5 4 0 4 2
C----- 1.00 9 0.04 0.02 1.00 NOTE DEFAULT VALUES OF G1, G2
C----- .10E-08 10 PLOT IS READ AS ZERO
C----- .10E-07 9 4 PLOT RHO, E AT T = 1.0E-06
C----- .10E-06 9
C----- .10E-05 9
C----- .10E-04 9
C----- .10E-03 9
C----- .10E-02 9
C----- .10E-01 9
C----- .10E+00 9
C----- .10E+01 9 4ERASE
C----- .10E+02 10
C----- .10E+01 9
C----- .10E+00 9 4
C----- .10E+01 9 PLOT RHO, E AT T = 1
C----- PLOT RHO, E AT T = 1
C----- PLOT RHO, E AT T = 10
C----- ERASE MODE BEGINS: T MSD FROM ERASE
C----- PLOT RHO, E AT T = 1
C----- LAST SET OF STEPS ENDS AT T = 10
C----- BLANK CARD TERMINATES EXECUTION
  
```

SUBROUTINES CONTAINED ON THE MAIN SOURCE FILE:

```

C BLOCK DATA
C SUBR. STACK
C SUBR. POUT
C SUBR. CHECK
C SUBR. F
C FUNCTION LGT
C SUBR. BNDRY
C SUBR. UNIT
C SUBR. SCFLD
C SUBR. EFELD
C SUBR. ESTOR
C SUBR. RERR
C SUBR. INT
C SUBR. ALI
C SUBR. PEAK
  
```

DEFINES CERTAIN PROGRAM CONSTANTS  
 PREPARES DATA FOR PLOTTING  
 PRINTS SOLUTION ARRAYS, FIELDS, ETC.  
 GIVES ANALYTICAL SOLUTION FOR U WITH NO DRIFT, DIFFUSION  
 GIVES RHS OF PDES FOR GIVEN X, T (REQD BY PDECOL)  
 ILLUMINATION FUNCTION FOR A GIVEN X  
 SETS BOUNDARY CONDITIONS FOR A GIVEN T (REQD BY PDECOL)  
 SETS INITIAL CONDITIONS FOR A GIVEN X (REQD BY PDECOL)  
 CALCULATES E AT BREAK-POINTS, AT THE CURRENT T  
 CALCULATES E FOR GIVEN X, T, BY INTERPOLATION  
 EXTRAPOLATES E AT THE BREAK-POINTS TO A LARGER T  
 FINDS AVERAGE ABSOLUTE ERROR IN EXTRAPOLATED VALUES OF E  
 SELECTS AND ORDERS POINTS FOR INTERPOLATION SUBROUTINE  
 PERFORMS INTERPOLATION  
 FINDS APPROXIMATE MAX (OR MIN) OF TABULATED FUNCTION

SUBPROGRAMS CONTAINED IN THE PACKAGE PDECOL ARE DESCRIBED IN THAT LISTING  
 SUBROUTINE PLOTN IS DESCRIBED IN THE PLOTN LISTING

NOTES ON PARAMETERS USED FOR TIME AND TIME-INCREMENTS:

T-----CURRENT WORKING TIME, CHOSEN BY THE PACKAGE (TO ADVANCE THE  
 TIME-INTEGRATION PROCESS AS QUICKLY AS POSSIBLE CONSISTENT  
 WITH MAINTAINING THE REQUIRED ACCURACY). WORKING-TIME STEPS  
 ARE COUNTED BY VARIABLE NSTEP.

TOUT---OUTPUT TIME, SPECIFIED BY USER, AT WHICH SOLUTIONS ARE DE-  
 Sired. OUTPUT-TIME STEPS ARE COUNTED BY VARIABLE KNT.

DELT---SIZE OF OUTPUT-TIME STEPS (READ IN AT STATEMENT 3)  
 DTUSED---SIZE OF WORKING-TIME STEPS (DETERMINED BY THE PACKAGE AND  
 PRINTED AS "DT" UNDER "TIME-STEP-SEQUENCE" ON TAPES).  
 DT-----INITIAL WORKING-TIME STEP USED ONLY ON FIRST CALL TO PDECOL  
 DTZ-----SIZE OF FIRST THREE STEPS AFTER BEGINNING OF ERASE MODE.  
 WORKING TIME COINCIDES WITH OUTPUT TIME FOR THESE STEPS.

10 ---INITIAL TIME (ZERO)  
 11 ---ERASE-TIME: TIME MEASURED FROM TO WHEN ERASE MOD .EGINS  
 12 ---CURRENT OUTPUT TIME, MEASURED FROM TO OR TER  
 13 ---VALUE OF T AT PREVIOUS WORKING-TIME STEP

# NOTES ON THE DETERMINATION OF THE SPACE-CHARGE FIELD:

THE ELECTRIC FIELD CONTAINS AN APPLIED-FIELD COMPONENT (READ IN AS B) AND A SPACE-CHARGE COMPONENT (DETERMINED AT EACH STEP BY INTEGRATING RHO, THE SPACE-CHARGE, FROM MINUS INFINITY TO X). SINCE THE FIELD APPEARS AS COEFFICIENTS IN THE PARTIAL DIFFERENTIAL EQUATIONS (PDES), THE EQUATIONS ARE COUPLED NONLINEAR INTEGRO-DIFFERENTIAL EQUATIONS. TO MAKE THESE TRACTABLE THE INTEGRAL IS REPLACED BY AN "ANTICIPATED" VALUE OF THE SPACE-CHARGE FIELD. IT IS VERY IMPORTANT THAT THIS "ANTICIPATED" VALUE NOT BE GREATLY IN ERROR.

BOTH THE "ANTICIPATED" AND THE "ACTUAL" (SEE BELOW) VALUES OF THE SPACE-CHARGE FIELD ARRAYS ARE EVALUATED AT THE BREAK-POINTS, XBKPT. WHATEVER INTERMEDIATE SPATIAL POINTS ARE REQUIRED ARE OBTAINED BY INTERPOLATION.

AT THE END OF EACH WORKING-TIME STEP AND (IF T.NE.TOUT, AS IS USUALLY THE CASE) ALSO AT EACH OUTPUT-TIME STEP, THE "ACTUAL" FIELD ARRAY IS DETERMINED. THIS IS DONE IN SUBROUTINE SCFLD BY NUMERICAL INTEGRATION OF THE SOLUTION ARRAYS. (RHO IS PROPORTIONAL TO THE DIFFERENCE OF THE TWO SOLUTIONS, V-U.)

SUPPOSE THE MOST RECENT STEP TO BE COMPLETED IS THE NTH ONE, CORRESPONDING TO TIME T. THE "ACTUAL" FIELD FOR THIS STEP (AS WELL AS ALL PREVIOUS ONES) HAS THEREFORE BEEN DETERMINED FROM THE SOLUTION ARRAYS. AT THE BEGINNING OF STEP N+1, T IS AUGMENTED TO T+DT, A NEW "ANTICIPATED" VALUE OF THE FIELD ARRAY IS COMPUTED BY TAKING THE PREVIOUS THREE "ACTUAL" FIELD ARRAYS AND EXTRAPOLATING TO THE NEW VALUE OF T. (THESE ARRAYS, STORED AS ES, CORRESPOND TO STEPS N-2, N-1, AND N; THE THREE TIMES ARE STORED IN ARRAY TE.)

WHEN STEP N+1 IS COMPLETE A NEW "ACTUAL" FIELD ARRAY IS COMPUTED. AN ERROR PARAMETER, DIS, GIVES THE AVG ABSOLUTE ERROR IN THE "ANTICIPATED" FIELD FOR STEP N+1 BY COMPARING THE ARRAY E TO THIS NEW "ACTUAL" FIELD ARRAY. (DIS IS ACTUALLY COMPUTED AT THE BEGINNING OF STEP N+2.) SINCE THE "ACTUAL" FIELD IS DETERMINED AFTER A CALCULATION WHICH USED THE "ANTICIPATED" FIELD, IT IS IMPORTANT TO MINIMIZE THIS ERROR. THIS CAN BE DONE (AT A COST IN CP TIME) BY USING SMALL VALUES OF EPS, AND/OR BY FORCING THE PROGRAM TO TAKE SMALL WORK-TIME STEPS. (SEE PDECOL)

## VARIABLES WITH SINGLE ASSIGNED PURPOSES IN THE MAIN PROGRAM:

A DIFFUSION LENGTH (READ IN AT STATEMENT 2)  
 B DRIFT LENGTH (READ IN AT STATEMENT 2)  
 DIS DISCREPANCY BETWEEN "ANTICIPATED" FIELD AND "ACTUAL" FIELD  
 DT DTZ, ETC---SEE COMMENTS ABOVE  
 E ARRAY FOR STORING VALUES OF "ANTICIPATED" SPACE-CHARGE FIELD  
 EP ERROR PARAMETER FOR SUBROUTINE PEAK  
 EPS TIME-DISCRETIZATION ERROR PARAMETER (SEE COMMENTS IN PDECOL)  
 ER CHARACTER CONSTANT NEEDED TO INITIATE ERASE MODE  
 ERASE CHARACTER VARIABLE NEEDED TO INITIATE ERASE MODE  
 ES ARRAY TO STORE VALUES OF "ACTUAL" SPACE-CHARGE FIELD  
 G1 RECOMBINATION FREQUENCY (READ IN AT STATEMENT 2)  
 G2 EXCITATION FREQUENCY (READ IN AT STATEMENT 2)  
 ICNT # OF TIMES FUNCTION F IS CALLED  
 INDEX CONTROL CODE FOR PDECOL (SEE COMMENTS IN PDECOL)  
 INDX VALUE USED FOR INDEX FOR MOST OF CALCULATION (READ IN AT STATEMENT 1)



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117 FCAT(= M =.F6.2)
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119 FCAT(= B =.E10.4)
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NNT = 0
TQ = 0.0
TC = TO
NF = 22
NC = 10
ER = SHERASE
TER = 0.0
ICNT = C
INDEX = 1
LWOK(1) = 27500
LWOK(2) = 1210

C PRINT THE PROGRAM PARAMETERS' INPUT VALUES
C
WRITE(6,102)
WRITE(6,103)NINT
WRITE(6,107)NQRD
WRITE(6,108)NQRD
WRITE(6,109)NCC
WRITE(6,110)'F'
WRITE(6,113)NXX
WRITE(6,113)NXH
WRITE(6,103)INTR
WRITE(6,111)'I'
WRITE(6,112)'A'

C $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C READ IN THE PHYSICAL PARAMETERS
C
2 READ(5,100)XR,NCHC,A,B,M,R,G1,G2

C NCHC IS THE RIGHT BOUNDARY (IN UNITS OF 2*PI/KG)
C XR IS THE NUMBER OF HALF-CYCLES IN THE INTERFERENCE PATTERN
C A IS THE DIFFUSION LENGTH (UNITS OF INTERFERENCE-PATTERN SPACING)
C B IS THE DRIFT LENGTH (DUE TO THE APPLIED FIELD) (SAME UNITS)
C G1 IS THE RECOMBINATION FREQUENCY (UNITS OF DIELECTRIC RELAX FREQ)
C G2 IS THE EXCITATION FREQUENCY (UNITS OF DIELECTRIC RELAX FREQ)
C R IS THE RATIO OF OCCUPIED TO UNOCCUPIED DONOR-LEVELS AT TZERO
C M IS THE INTERFERENCE-PATTERN STRENGTH (0 < M < 1)

DEF: 5
DEF: 9
DEF: 9
DEF: 0
DEF: 0
DEF: 2.5E+7
DEF: 1.6E-2
DEF: 1
DEF: .5

IF(A.LT.0.0)A = 4.3E-02
IF(B.LT.0.0)B = 1.6E-02
IF(R.EQ.0.0)R = 1.0
IF(M.EQ.0.0)M = 0.5
IF(G1.EQ.0.0)G1 = 2.500E+07
IF(G2.EQ.0.0)G2 = 1.625E-02
IF(XR.LE.0.0)XR = 5.0
IF(NCHC.LE.0.0)NCHC = 9

C PRINT THE PHYSICAL PARAMETERS' INPUT VALUES
C
WRITE(6,115)NOHC
WRITE(6,116)XR
WRITE(6,117)M
WRITE(6,118)A
WRITE(6,119)B
WRITE(6,120)R
WRITE(6,121)G1
WRITE(6,122)G2

C $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C THE ASSUMED INTENSITY PATTERN, LGT(X), IS PERIODIC IN THE MIDDLE
C OF THE CRYSTAL (A COSINE FUNCTION CORRESPONDING TO THE INTERFERENCE
C PATTERN) AND A CONSTANT AT THE ENDS (TO VALIDATE THE NEUMANN BOUN-
```







```

CALL PEAK(SPACE(MBEG),XBKPT(MBEG),MMUM,YMIN,DY,EP,1)
YT = YMAX - YMIN
WR = EIG,(IOS)*T,NSTEP,TIME1,TCUR,U(1,IC,1).U(2,IC,1).ES(IC,1),
YT,XMAX,YMAX,YMIN
IF(ITER.NE.K)GO TO 365
DO 350 I = 1,NPTS
SPACE(I) = U(1,I,1)
CALL POUT(4,TCUR,SPACE,TER,1,NPTS,NO,PRT(1))
IF(B.GT.0.0)GO TO 359
YT = 10.0/G1
IF(A.GT.0.0.OR.TOUT.GT.YT)GO TO 359
DO 355 J = 1,NPTS
VT = XBKPT(J)
CALL CHECK(YT,TOUT,YMAX)
355 SPACE(J) = SPACE(I) - YMAX
CALLS,POUT(4,TOUT,SPACE,TER,1,NPTS,NO,PRT(13))
359 DO 360 I = 1,NPTS
CALL POUT(4,TCUR,SPACE,TER,1,NPTS,NO,PRT(3))
360 SPACE(I) = U(2,I,1)
CALL POUT(4,TCUR,SPACE,TER,1,NPTS,NO,PRT(11))
365 DO 370 I = 1,NPTS
370 SPACE(I) = U(2,I,1) - U(1,I,1)
CALL POUT(4,TCUR,SPACE,TER,1,NPTS,NO,PRT(11))
IF(PLC.GT.0.AND.K.EQ.ITER)CALL STACK(SPACE,WORK,M,TCUR,PRT(1)),
1,PLC,7)
DO 380 I = 1,NPTS
380 SPACE(I) = ES(1,ID)
CALL POUT(4,TCUR,SPACE,TER,1,NPTS,NO,PRT(5))
IF(PLC.GT.0.AND.K.EQ.ITER)CALL STACK(SPACE,WORK,M,TCUR,PRT(5)),
12,PLC,7)
C      C
C      C      RESET INDEX
C      C      INDEX = INDY
C      C      IF(K.EQ.(ITER-1).AND.ERASE.EQ.ER)INDEX = 2
C      C
C      C      400 CONTINUE
C      C
C      C      $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C      C      RESET FOR ERASE MODE IF DESIRED. STRATEGY IS TO SET INDEX = 3 AND
C      C      ADVANCE THREE SMALL STEPS FROM THE FINAL OUTPUT TIME OF THE WRITE
C      C      MODE (WHICH WILL HAVE BEEN REACHED EXACTLY). THEN THE PROGRAM IS
C      C      ALLOWED TO TAKE OVER AND CHOOSE ITS OWN TIME STEPS. (UNLESS INDX
C      C      HAD BEEN SET EQUAL TO 2 AT THE BEGINNING).
C      C
C      C      IF(ERASE.NE.ER)GO TO 3
C      C      DELT = DTZ
C      C      DT = DTZ
C      C      INDEX = 3
C      C      ITER = 1
C      C      IF(KNT-3)399,480,500
C      C
C      C      480 ERASE = 5H
C      C      INDEX = INDX
C      C      TOUT = TER
C      C      ER = 5H BOMB
C      C      GO TO 3
C      C
C      C      INITIALIZE CERTAIN ARRAYS AND PARAMETERS FOR THE ERASE MODE
C      C
C      C      500 M = 0.0
C      C      PLC = 0
C      C      TER = TOUT
C      C      KNT = 0
C      C      DIS = 0.0

```



```

C      SUBROUTINE TO PREPARE FOR PLOTTING THE SPACE-CHARGE OR THE
C      SPACE-CHARGE FIELD. STACK DETERMINES XPLOT, THE ARRAY OF VALUES
C      OF X, AND THE INTERFERENCE PATTERN AT THESE VALUES OF X. THEN IT
C      CALLS ROUTINES TO DETERMINE RHO AND E (UNLESS PLC = 1, IN WHICH
C      CASE THEY ARE DETERMINED IN MAIN) AND STORES THEM IN SPACE. LAST
C      OF ALL, IT SCALES THE VARIABLES SO THAT THE INTERFERENCE PATTERN
C      FILLS THE TOP 20 PERCENT OF THE PLOT REGION, AND CALLS PLOTN.
C
C      INTEGER PLC,PLL
C      REAL M,LGT
C      DIMENSION XB-PT(201),E(201),ES(201,3),TE(3)
C      DIMENSION PLVAR(201,4),STORE(201),XPLOT(201),SPACE(201),WRT(2)
C      DIMENSION UL(2,201),SCICH(4),WORK(1)
C      COMMON/EFIELD/XBKPT,E,ES,TE,TPRV,TER,DIS,NPTS,INTR,ICNT
C      EQUIVALENCE (PLVAR(1),U(1))
C
C      INITIALIZATION OF PARAMETERS. THE FIRST CALL SHOULD OCCUR WITH
C      PLC = 0. ON THIS CALL (ONLY) T, WRT(1), AND WRT(2) PROVIDE IN-
C      FORMATION ON WHERE TO BEGIN ANY EXPANDED PLOTS---THAT IS, WHAT
C      VALUE XMIN SHOULD HAVE.
C
C      IF(PLC.GT.0)GO TO 200
C      PLL = 0
C      ER = M
C      XMIN = WRT(2) + 0.8 + T + WRT(1)
C      XMIN = -XMIN
C      DO 180 I = 2,NPTS
C      IF(XMIN.LT.XBKPT(I))GO TO 190
C      180 CONTINUE
C      190 XMIN = XBKPT(I-1)
C      RETURN
C
C      SECTION TO SET UP THE ARRAY, XPLOT, AND CALCULATE LGT(X)
C
C      200 IF(PLC.EQ.PLL)GO TO 300
C      PLL = PLC
C      IF(PLC.GT.1)GO TO 220
C      DO 210 I = 1,NPTS
C      X = XBKPT(I)
C      STORE(I) = LGT(X)
C      210 XPLOT(I) = X
C      MAX = NPTS
C      GO TO 350
C
C      PLC > 1: SET MAX AND XMAX SO THAT THE RANGE IS XMIN-ABS(XMIN) OR
C      XMIN-N (WHERE N IS A NONNEGATIVE INTEGER). CHOOSING THE GREATEST
C      SUCH RANGE THAT FITS INTO 161 PRINT POSITIONS. OTHERWISE PLOT 161
C      POINTS STARTING AT XMIN
C
C      220 IF(PLC.GT.10)PLC = 10
C      DX = (XBKPT(2) - XBKPT(1))/PLC
C      XMAX = ABS(XMIN)
C      XMAX = (XMAX - XMIN)/DX + 1.0 + 1.0E-10
C      IF(MAX.LE.161)GO TO 230
C      I = XMAX + 1.0 + 1.0E-10
C      XMAX = I
C      DO 225 J = 1,I
C      MAX = (XMAX - XMIN)/DX + 1.0 + 1.0E-10
C      IF(MAX.LE.161)GO TO 230
C      225 XMAX = XMAX - 1.0
C      MAX = 161
C      XMAX = XMIN + (MAX - 1)*DX
C      230 CONTINUE
C      DO 240 I = 1,MAX
C      X = XMIN + (I-1)*DX

```

```

      XPLOT(I) = X
240 S (I) = LGT(X)
C
C   STACK THE PROPER VARIABLE IN THE ARRAY SPACE.
C
300 IF(PLC.EQ.1)GO TO 350
   IF(LVL.EQ.2)GO TO 320
C
C   CALL VALUES(XPLOT,U,SCATCH,2,MAX,MAX,0,WORK)
   DO 310 I = 1,MAX
310 SPACE(I) = U(2,I,1) - U(1,I,1)
   GO TO 350
320 DO 330 I = 1,MAX
   X = XPLOT(I)
   DX = TPRV
   CALL EFIELD(X,DX,Q)
330 SPACE(I) = Q
   ICNT = ICNT - MAX
C
C   SCALE THE INTERFERENCE-PATTERN DATA
C
350 AMAX = -1.0E+321
   AMIN = 1.0E+321
   DO 360 I = 1,MAX
   Q = SPACE(I)
   IF(Q.GT.AMAX)AMAX = Q
   IF(Q.LT.AMIN)AMIN = Q
360 X = 11.*(AMAX - AMIN)/(94.*EM)
   DX = (107.*AMAX - 13.*AMIN)/94. - X
   Q = (95.*AMAX - AMIN)/94.
C
C   DO 370 I = 1,MAX
   PLVAR(1,4) = 0.0
   PLVAR(1,3) = 0
   PLVAR(1,2) = X*STORE(1) + DX
370 PLVAR(1,1) = SPACE(I)
   Q = 8H (WRITE)
   IF(TER.GT.0.0)Q = 8H (ERASE)
C
C   PRINT THE INFORMATION DESIRED, AND CALL PLOTN
C
   DX = (XBKPT(2) - XBKPT(1))/PLC
   IF(PLC.GT.1)WRITE(KFILE,100)XMIN,PLC,MAX,XMAX,DX
   WRITE(KFILE,101)T,Q,WRT(1),WRT(2)
C
C   CALL PLOTN(PLVAR,1,MAX,1,201,4,1,4,KFILE)
   WRITE(KFILE,101)T,Q,WRT(1),WRT(2)
   WRITE(KFILE,102)
C
C   100 FORMAT(= RANGE IS=,8X,=X(1) = XMIN=,F6.2,= WITH PLC=,I2,/,
      116X,=X(=,I3,=) = XMAX=,F6.2,= DX=,F7.4)
   101 FORMAT(= T=,E10.4,AB,32X,2A10,27X,=INTERFERENCE PATTERN=)
   102 FORMAT(1H1)
C
C   RETURN
   END
C
C   SUBROUTINE POUT(FOUT,T,X,TER,MIN,MAX,NO,WRT)
C
C   SUBROUTINE TO PRINT AN ARRAY, X, OR A PORTION OF IT
C
C   INTEGER FOUT
C   DIMENSION X(1),WRT(2)
C
C   100 FORMAT(1X,13,10E11,4,15)
   101 FORMAT(2X,=T=,E10.4,AB,/)
   102 FORMAT(/,2X,2A10)

```

```

C      BH (WRITE)
C      IF (EQ.0.0)MM = BH
C      IF (TER.GT.0.0)MM = BH (ERASE)
C      IF (NO.GT.10)NO = 10
C      J = 1
C      NL = (MAX - MIN + 1)/NO
C      IF ((NO-NL).EQ.(MAX-MIN+1))J = 0
C      NL = NL + J
C      WRITE(FOOT,102)WRT(1),WRT(2)
C      WRITE(FOOT,101)T,MM
C      DO 200 I = 1,NL
C      K = MIN + NO - (I-1)
C      L = K + NO - 1
C      IF (L.GT.MAX)L = MAX
C      IF (NO.LT.10)GO TO 199
C      IF ((L-K).LT.9)GO TO 199
C      WRITE(FOOT,100)K,(X(J),J=K,L),L
C      GO TO 200
C      199 WRITE(FOOT,100)K,(X(J),J=K,L)
C      200 CONTINUE
C      RETURN
C      END
C      SUBROUTINE CHECK(X,T,U)
C      SUBROUTINE TO CALCULATE THE VALUE OF U, AT A SINGLE POINT X AND
C      A TIME T, IN THE ABSENCE OF DRIFT AND DIFFUSION
C      REAL LGT
C      COMMON/A/DUM(3),G1,G2,R,DVM(S)
C      F = LGT(X)
C      A = R*G2
C      B = G1 + G2*F
C      C = -F*G1
C      ARG = SQRT(B*B - 4.0*A*C)
C      AL = B/ARG
C      CP = 2.0*A*AL/B
C      CPP = CP/(1.0 + AL)
C      CP = CP/(1.0 - AL)
C      F = EXP(-ARG*T)
C      U = (1.0 - F)/(CP + CPP*F)
C      RETURN
C      END
C      SUBROUTINE F(T,X,U,UX,UX,FVAL,NPDE)
C      DIMENSION U(NPDE),UX(NPDE),UX(NPDE),FVAL(NPDE)
C      REAL LGT
C      COMMON/A/PI,A,B,G1,G2,R,DUM(S)
C      THESE (FVAL) ARE THE RIGHT-HAND SIDES OF THE PDES TO BE EVALUATED
C      CALL EFIED(X,T,XZ)
C      SUM = LGT(X)*(G1 - G2*U(2)) - G1*U(1) - R*G2*U(1)*U(2)
C      SVM = (G1*B + XZ)*UX(1) + U(2)*U(1) - U(1)*U(1)
C      FVAL(1) = A*A*G1*UX(1) + SVM + SUM
C      FVAL(2) = SUM
C      RETURN
C      END
C      REAL FUNCTION LGT(X)
C      FUNCTION TO ESTABLISH LIGHT-INTENSITY FROM INTERFERENCE PATTERN
C      REAL M
C      COMMON/A/PI,DUM(5),M,XB,S1,S2,PH

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```

IF(ABS(X).LT.X9)GO TO 200
R1 = 11--3
S1 = S1
Y = -XB
IF(X.LT.0.0)GO TO 100
S = S2
Y = XB
100 ARG = R*(X-Y)**2
Y = 0.0
IF(ARG.LT.800.)Y = EXP(-ARG)
LGT = 1.0 + S-M*Y
RETURN
C 200 LGT = 1.0 + M-COS(2.0*PI*X - PH)
RETURN
END
SUBROUTINE BNDRY(T,X,U,UX,DBOU,DBOXX,DZDT,NPDE)
C
C DIMENSION U(NPDE),UX(NPDE),DZDT(NPDE)
C DIMENSION DBOU(NPDE),DBOXX(NPDE),DBOXX(NPDE)
C COMMON/A/DUM(3),G1,DVM(7)
C
C FROM THE BOUNDARY CONDITIONS OF THE FORM B(U,UX) = Z(T), FIND THE
C PARTIAL DERIVATIVES OF BOTH SIDES. THE BOUNDARY CONDITIONS ARE
C THE SAME AT XL AND XR.
C
C DBOU(1,1) = 1.0
C DBOU(1,2) = -1.0
C DBOU(2,1) = 0.0
C DBOU(2,2) = 0.0
C DBOXX(1,1) = 0.0
C DBOXX(1,2) = 0.0
C DBOXX(2,1) = 1.0
C DBOXX(2,2) = 0.0
C DZDT(1) = 0.0
C DZDT(2) = 0.0
C
C RETURN
C END
SUBROUTINE UINIT(X,U,NPDE)
C
C SET INITIAL CONDITIONS, REPRESENTING ZERO CONDUCTION-ELECTRON DENSITY
C ZERO OCCUPIED DONOR-LEVEL DENSITY
C
C DIMENSION U(NPDE)
C U(1) = 0.0
C U(2) = 0.0
C RETURN
C END
SUBROUTINE SCFLD(T,WORK)
C
C SUBROUTINE SCFLD EVALUATES THE ELECTRIC FIELD (REALLY, A NORMALIZED
C FIELD; SEE NOTES) DUE TO THE NET SPACE-CHARGE INSIDE THE CRYSTAL, BY
C INTEGRATING THE NET CHARGE DENSITY FROM XL (THE LEFT BOUNDARY) TO X.
C THE FIELD, ES(1), IS OBTAINED AT THE (NINT+1) BREAKPOINTS XBKPT(1),
C THE COMPLETE INTEGRAL IS DONE IN PIECES, CONSISTING OF THE INTERVALS
C BETWEEN CONSECUTIVE BREAKPOINTS. THE SUM OF ALL INTEGRALS UP TO AND
C INCLUDING THE 1TH ONE IS ES(1). THE BREAKPOINTS ARE ASSUMED TO BE E-
C QUALLY-SPACED. WHETHER THE FIELD IS STORED IN ES(1,2) OR ES(1,3) DE-
C PENDS ON WHETHER T > TOUT (K = 2) OR T < TOUT (K = 1). IN THE FORMER
C CASE, SCFLD IS CALLED FROM PDECOL; IN THE LATTER, FROM MAIN.
C
C DIMENSION XBKPT(201),E(201),ES(201,3),XPT(16),U(2,16,1),SCFCH(4)
C DIMENSION WORK(1),TE(3)
C DIMENSION CS(4,8),NS(4,8)
C COMMON/EFIELD/XBKPT,E,ES,TE,TPRV,TER,DIS,NPTS,INTR,ICNT

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```

C      CCM/WM/MTS/CS.WS
C      (
      IRK = INTR/2
      INN = INTR/4
      SUM = 0.0
      K = 2
      IF (T.LT.TE(3)) K = 1
      DO 140 J = 1,K
        TE(J) = TE(J+1)
      DO 140 I = 1,NPTS
140   ES(I,J) = ES(I,J+1)
155   ES(I,K+1) = 0.0
      XU = XBKPT(1)
      B = XBKPT(2) - XBKPT(1)
C      LOOP TO COMPUTE THE INTEGRALS IN EACH OF THE INTERVALS
C
C      DO 200 I = 2,NPTS
C
C      SET INTEGRATION LIMITS FOR THE ITH INTERVAL
C
C      XL = XU
C      XU = XBKPT(I)
C
C      OBTAIN THE POINTS, XPT, AT WHICH THE ITH INTEGRAND WILL NEED TO
C      BE EVALUATED. INTR-POINT GAUSSIAN QUADRATURE WILL BE USED.
C
C      A = .5*(XU+XL)
C      DO 160 J = 1,IRR
C      C = CS(INN,J)*B
C      XPT(2*J-1) = A + C
160   XPT(2*J) = A - C
C
C      EVALUATE THE DEPENDENT VARIABLES, U (U AND V) AT THESE POINTS
C
C      CALL VALUES(XPT,U,SCCH,2,16,INTR,0,WORK)
C
C      EVALUATE THE INTEGRAND, V - U, AT EACH POINT REQUIRED BY THE
C      QUADRATURE FORMULA
C
C      DO 170 J = 1,INTR
170   U(2,J,1) = U(2,J,1) - U(1,J,1)
C
C      EVALUATE THE INTEGRAL BY SUMMING THE WEIGHTED FUNCTION VALUES
C
C      Y = 0.0
C      DO 180 J = 1,IRR
C      W = WS(INN,J)
180   Y = Y + W*(U(2,2*J-1,1) + U(2,2*J,1))
C      Y = Y*B
C
C      SUM, THE TOTAL INTEGRAL, INCLUDES CONTRIBUTIONS FROM INTERVALS 1,2...I
C
C      SUM = SUM + Y
200   ES(1,K+1) = SUM
      IF (K.EQ.2) CALL MERR
      RETURN
      END
      SUBROUTINE EFIELD(X,T,Y)
C
C      SUBROUTINE TO CALCULATE THE SPACE-CHARGE FIELD AT X, T. ARRAY E,
C      WHICH IS SET UP IN ESTOR, CORRESPONDS TO THE (ANTICIPATED) FIELD AT
C      ALL THE BREAK-POINTS, AT THE TIME OF INTEREST. THE VALUE OF THE FIELD
C      AT X IS FOUND FROM THIS ARRAY BY INTERPOLATION.
C

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```

C      DIMENSION E(201),XBKPT(201),ES(201,3),TE(3)
C      CC( N/EFIELD,XBKPT,E,ES,TE,TPRV,TER,DIS,NPTS,INTR,ICNT) (
C      NOT ESTOR TO UPDATE THE ARRAY. E, TO THE CURRENT TIME IF THIS HAS
C      NOT ALREADY BEEN DONE
C
C      ICNT = ICNT + 1
C      IF(T.EQ.TPRV)GO TO 90
C      CALL ESTOR(T)
C
C      SELECT THE RANGE (MIN, MAX) OF THE INDEX (I) TO BE SEARCHED FOR
C      THOSE TABLE VALUES (XBKPT(I)) NEAREST X.
C
C      90 NDIM = 3
C      EPS = 1.E-03
C      Y = (X - XBKPT(1))/(XBKPT(NPTS) - XBKPT(1))
C      Y = Y*NPTS - 2
C      MIN = Y
C      IF(MIN.LT.1)MIN = 1
C      MAX = MIN + 6
C      IF(MAX.LE.NPTS)GO TO 100
C      MAX = NPTS
C      MIN = MAX - 6
C      100 CONTINUE
C
C      CALL THE SUBROUTINE TO DO THE NDIM-POINT INTERPOLATION. THE RESULTANT
C      VALUE IS Y. THE ARRAY WORK IN INT HAS NOTHING TO DO WITH THE ARRAY
C      OF THE SAME NAME IN THE OTHER ROUTINES.
C
C      CALL INT(E,XBKPT,X,MIN,MAX,NDIM,EPS,Y)
C      RETURN
C      END
C      SUBROUTINE ESTOR(I)
C
C      THIS SUBROUTINE TAKES THE CONTENTS OF ES (CONTAINING THE SPACE-CHARGE
C      FIELD AT THE XBKPT AT TIMES TE(1), TE(2), AND TE(3)) AND DOES AN INTER-
C      POLATION OR EXTRAPOLATION TO TIME T. THE RESULTS ARE STORED IN ARRAY E
C
C      DIMENSION E(201),ES(201,3),TE(3),EE(3),XBKPT(201)
C      COMMON/EFIELD/XBKPT,E,ES,TE,TPRV,TER,DIS,NPTS,INTR,ICNT
C      COMMON/GEARQ/DTUSED,NQ,NSTEP,NF,NJ
C
C      EPS = 1.E-03
C      IF(TPRV.LT.0.0)JCNT = 0
C      I = ICNT - JCNT
C      DTU = T - TPRV
C      TPRV = T
C      DT = T - TER
C      DO 90 J = 1,3
C      EE(J) = TE(J) - TER
C      WRITE(3,700)NSTEP,DT,DTU,ICNT,I,EE(1),EE(2),EE(3),DIS
C      700 FORMAT(1X,NSTEP=,I3.0,T=,E10.4,DT=,E10.4,ICNT=,I6,
C      115.2X,3E10.4,DIS=,E10.4)
C      JCNT = ICNT
C      DT = TE(2) - TE(1)
C      IF(DT.EQ.0.0)RETURN
C      DO 100 I = 1,NPTS
C      DO 95 J = 1,3
C      EE(J) = ES(I,J)
C      CALL INT(EE,TE,T,1,3,EPS,Y)
C      95 EE(J) = ES(I,J)
C      100 E(I),Y
C      RETURN
C      END
C      SUBROUTINE RERR
C
C      DIMENSION XBKPT(201),E(201),ES(201,3),TE(3)

```

```

COMMON/EFIELD/XBKPT,E,ES,TE,TPRV,TER,DIS,NPTS,INTR,ICNT
C
C      CALCULATE THE AVERAGE ABSOLUTE ERROR IN THE SPACE-CHARGE FIELD USED IN
C      SUBROUTINE F AT THE CURRENT STEP BY COMPARING E (THE FIELD OBTAINED BY
C      EXTRAPOLATION FROM THE 'CORRECT' VALUES AT THE PREVIOUS TWO STEPS) WITH
C      ES (THE 'CORRECT' VALUE OBTAINED BY INTEGRATING RHO AFTER THE CURRENT
C      STEP IS COMPLETE.)
C
DIS = 0.0
DO 100 I = 1,NPTS
100 DIS = DIS + ABS(E(I) - ES(I,3))
DIS = DIS/NPTS
RETURN
END
SUBROUTINE INT(YS,XS,X,MIN,MAX,NDIM,EPS,Y)
C
C      SUBROUTINE TO SELECT AND ORDER THE NDIM POINTS TO BE USED FOR
C      AITKEN-LAGRANGE INTERPOLATION BY SUBROUTINE ALI
C
XS = NDIM ABSCISSAE NEAREST X
YS = NDIM ORDINATES CORRESPONDING TO XS
C
X = POINT AT WHICH FUNCTION IS TO BE EVALUATED BY INTERPOLATION
MIN,MAX = RANGE OF TABLE TO BE SEARCHED FOR NDIM POINTS NEAREST X
NDIM = NUMBER OF POINTS USED IN INTERPOLATION PROCEDURE
EPS = DESIRED ACCURACY OF INTERPOLATION
Y = RESULTANT FUNCTION VALUE
C
DIMENSION ARG(9),VAL(9)
DIMENSION YS(1),XS(1),WORK(201)
B = 0.0
DO 5 N = MIN,MAX
DEL = ABS(XS(N) - X)
IF(DEL.LT.B)GO TO 5
B = DEL
5 WORK(N) = DEL
B = B + 1.0
DO 10 J = 1,NDIM
DEL = B
DO 7 N = MIN,MAX
IF(WORK(N).GE.DEL)GO TO 7
I = N
DEL = WORK(N)
7 CONTINUE
VAL(J) = YS(I)
ARG(J) = XS(I)
10 WORK(I) = B
CALL ALI(X,ARG,VAL,Y,EPS,NDIM,IER)
RETURN
END
SUBROUTINE ALI(X,ARG,VAL,Y,EPS,NDIM,IER)
C
C      SUBROUTINE ALI PERFORMS NDIM-POINT INTERPOLATION ACCORDING TO THE
C      AITKEN-LAGRANGE METHOD. IT IS DESCRIBED IN THE IBM SCIENTIFIC SUB-
C      ROUTINE PACKAGE.
C
X = POINT AT WHICH FUNCTION IS TO BE EVALUATED BY INTERPOLATION
ARG = NDIM ABSCISSAE NEAREST X
VAL = NDIM ORDINATES CORRESPONDING TO ARG
Y = DESIRED FUNCTION VALUE
EPS = DESIRED ACCURACY OF INTERPOLATION
NDIM = NUMBER OF POINTS USED IN INTERPOLATION PROCEDURE
IER = 0 IF SUBROUTINE OPERATES PROPERLY
C
DIMENSION ARG(9),VAL(9)

```

```

C      I = 2
C      DELT2 = 0.0
C      IF (NDIM - 1) 9,7,1
C
C      START OF AITKEN-LOOP
C
C      1 DO 6 J = 2,NDIM
C        DELT1 = DELT2
C        IEND = J - 1
C        DO 2 I = 1,IEND
C          H = ARG(I) - ARG(J)
C          IF (H) 2,13,2
C          2 VAL(J) = (VAL(I) * (X - ARG(J)) - VAL(I) * (X - ARG(I))) / H
C          DELT2 = ABS(VAL(J) - VAL(IEND))
C          IF (J - 2) 6,6,3
C          3 IF (DELT2 - EPS) 10,10,4
C          4 IF (J - 6) 6,5,5
C          5 IF (DELT2 - DELT1) 6,11,11
C          6 CONTINUE
C        END OF AITKEN-LOOP
C      C
C      7 J = NDIM
C      8 Y = VAL(J)
C      9 RETURN
C
C      THERE IS SUFFICIENT ACCURACY WITHIN NDIM - 1 ITERATION STEPS
C
C      10 IER = 0
C      GO TO 8
C
C      TEST VALUE DELT2 STARTS OSCILLATING
C
C      11 IER = 1
C      12 J = IEND
C      GO TO 8
C
C      THERE ARE 2 IDENTICAL ARGUMENTS IN VECTOR ARG
C
C      13 IER = 3
C      GO TO 12
C      END
C      SUBROUTINE PEAK(Y,X,NO,YMAX,XMAX,EP,K)
C
C      DIMENSION Y(41),X(41)
C
C      FIND THE APPROXIMATE MAXIMUM OF A TABULATED FUNCTION, Y(X), BY INTER-
C      POLATION. THE POINTS, X, ARE ASSUMED EQUALLY SPACED. THE LOCATION OF
C      THE MAXIMUM, XMAX, IS FOUND TO WITHIN EP, AND YMAX IS THE APPROXIMATE
C      VALUE OF Y AT XMAX.
C
C      THE MINIMUM OF A TABULATED FUNCTION CAN BE FOUND BY SETTING K > 0. FOR
C      K = 0, THE MAXIMUM IS LOCATED.
C
C      IF (K.EQ.0) GO TO 145
C      DO 140 I = 1,NO
C      140 Y(I) = -Y(I)
C      145 NDIM = 3
C      EPS = 1.0E-03
C      NINT = 20
C      YMAX = -1.0E+321
C      DO 150 I = 1,NO
C      150 Y(I) = 1,NO
C      Q = Y(1)
C      IF (Q.LE.YMAX) GO TO 150
C      YMAX = Q

```

```

150  I      .NUE
      IF(IC.EQ.NO)IC = NO - 1
      IF(IC.EQ.1)IC = 2
      DX = (X(2)-X(1))/NINT
      DO 160 I = 1,5
      IF(DX.LE.EP)GO TO 170
      NINT = NINT*2
160  DX = DX/2.0
170  XMAX = X(IC)
      Q = (Y(IC-1) - Y(IC+1))/Y(IC)
      MIN = IC - 1
      MAX = IC + 1
      IF(ABS(Q).LT.EPS)GO TO 210
      IF(Q.GT.0.0)IC = IC - 1
      Q = Y(IC)
      DO 180 I = 1,NINT
      XMAX = X(IC) + I*DX
      CALL INT(Y,X,XMAX,MIN,MAX,NDIM,EPS,YMAX)
      IF(YMAX-Q)190,200,180
180  Q = YMAX
      DX = 0.0
190  XMAX = XMAX - DX/2.0
200  XMAX = XMAX - DX/2.0
210  CALL INT(Y,X,XMAX,MIN,MAX,NDIM,EPS,YMAX)

      IF(K.EQ.0)RETURN
      DO 220 I = 1,NO
220  Y(I) = -Y(I)
      YMAX = -YMAX
      RETURN
      END

```

C

```

SUBROUTINE PDECOL (TO, TOUT, DT, XBKPT, EPS, NINT, KORD, NCC, NPDE, MF,
INDEX, WORK, IWORK)

```

```

C-----
C
C THIS IS THE MARCH 24, 1978 VERSION OF PDECOL.
C
C THIS PACKAGE WAS CONSTRUCTED SO AS TO CONFORM TO AS MANY ANSI-FORTRAN
C RULES AS WAS CONVENIENTLY POSSIBLE. THE FORTRAN USED VIOLATES ANSI
C STANDARDS IN THE TWO WAYS LISTED BELOW....
C
C 1. SUBSCRIPTS OF THE GENERAL FORM C*V1 + V2 + V3 ARE USED
C (POSSIBLY IN A PERMUTED ORDER), WHERE C IS AN INTEGER CONSTANT
C AND V1, V2, AND V3 ARE INTEGER VARIABLES.
C
C 2. ARRAY NAMES APPEAR SINGLY IN DATA STATEMENTS IN THE ROUTINES
C BSPLVN AND COSET.
C
C MACHINE DEPENDENT FEATURES.....
C
C THIS VERSION OF PDECOL WAS DESIGNED FOR USE ON CDC MACHINES WITH
C A WORD LENGTH OF 60 BITS. WE DO NOT RECOMMEND THE USE OF PDECOL WITH
C WORD LENGTHS OF LESS THAN 48 BITS. THE MOST IMPORTANT MACHINE
C AND WORD LENGTH DEPENDENT CONSTANTS ARE DEFINED IN THE BLOCK DATA
C AND IN SUBROUTINES COLPNT AND COSET. THE USER SHOULD CHECK THESE
C CAREFULLY FOR APPROPRIATENESS FOR HIS LOCAL SITUATION. THE FORTRAN
C FUNCTIONS USED BY EACH ROUTINE ARE LISTED IN THE COMMENTS TO
C FACILITATE CONVERSION TO DOUBLE PRECISION.
C-----
C
C PDECOL IS THE DRIVER ROUTINE FOR A SOPHISTICATED PACKAGE OF
C SUBROUTINES WHICH IS DESIGNED TO SOLVE THE GENERAL SYSTEM OF
C NPDE NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS OF AT MOST SECOND
C ORDER ON THE INTERVAL (XLEFT,XRIGHT) FOR T .GT. TO WHICH IS OF THE
C FORM....
C
C DU/DT = F ( T, X, U, UX, UXX )
C
C WHERE
C
C U = ( U(1), U(2), ... , U(NPDE) )
C UX = ( UX(1), UX(2), ... , UX(NPDE) )
C UXX = ( UXX(1), UXX(2), ... , UXX(NPDE) )
C
C EACH U(K) IS A FUNCTION OF THE SCALAR QUANTITIES T AND X.
C U(K) REPRESENTS THE FIRST PARTIAL DERIVATIVE OF U(K) WITH RESPECT
C TO THE VARIABLE X. UXX(K) REPRESENTS THE SECOND PARTIAL DERIVATIVE
C OF U(K) WITH RESPECT TO THE VARIABLE X, AND DU/DT IS THE VECTOR OF
C PARTIAL DERIVATIVES OF U WITH RESPECT TO THE TIME VARIABLE T.
C F REPRESENTS AN ARBITRARY VECTOR VALUED FUNCTION WHOSE NEDS
C COMMENTS DEFINE THE RESPECTIVE PARTIAL DIFFERENTIAL EQUATIONS OF
C THE PDE SYSTEM. SEE SUBROUTINE F DESCRIPTION BELOW.
C
C BOUNDARY CONDITIONS
C
C DEPENDING ON THE TYPE OF PDE(S), 0, 1, OR 2 BOUNDARY CONDITIONS
C ARE REQUIRED FOR EACH PDE IN THE SYSTEM. THESE ARE IMPOSED AT XLEFT
C AND/OR XRIGHT AND EACH MUST BE OF THE FORM....
C

```



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C      CONDITION. THIS ROUTINE CAN BE STRUCTURED AS FOLLOWS...
C      ( IF COMMON BLOCK /ENDPT/ IS NOT A PART OF POECOL AND
C      ( NOT BE SUPPLIED AND DEFINED BY THE USER.
C      COMMON /ENDPT/ LEFT
C      IF( X.NE. XLEFT ) GO TO 10
C      HERE DEFINE AND SET PROPER VALUES FOR DBOUX(K,J), OBOUX(K,J),
C      AND ODOUX(K,J) FOR K,J = 1 TO NPDE FOR THE LEFT BOUNDARY POINT
C      X = XLEFT.
C      RETURN
C 10 CONTINUE
C      HERE DEFINE AND SET PROPER VALUES FOR DBOUX(K,J), OBOUX(K,J),
C      AND ODOUX(K,J) FOR K,J = 1 TO NPDE FOR THE RIGHT BOUNDARY POINT
C      X = XRIGHT.
C      RETURN
C      END
C 3) SUBROUTINE UNIT1( X, U, NPDE )
C      DIMENSION UNIT1( X, U, NPDE )
C      THIS ROUTINE IS USED TO PROVIDE THE PDE PACKAGE WITH THE
C      NEEDED INITIAL CONDITION FUNCTION VALUES. THE PACKAGE
C      PROVIDES A VALUE OF THE INPUT VARIABLE X, AND THE USER IS TO
C      DEFINE THE PROPER INITIAL VALUES (AT T = T0) FOR ALL OF THE
C      PDE COMPONENTS, I.E.
C      U(X) = DESIRED INITIAL VALUE OF PDE COMPONENT U(X) AT
C      X AND T = 0 FOR K = 1 TO NPDE.
C      NOTE... THE INCOMING VALUE OF X WILL BE A COLLOCATION POINT
C      VALUE. THE INITIAL CONDITIONS AND BOUNDARY CONDITIONS
C      MUST BE CONSISTENT (SEE ABOVE).
C      RETURN
C      END

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#### OPTIONAL USER SUPPLIED SUBROUTINE

```

C IF THE USER DESIRES TO USE THE MF = 11 OR 21 OPTION IN ORDER TO SAVE
C ABOUT 10-20 PERCENT IN EXECUTION TIME (SEE BELOW), THEN THE USER MUST
C PROVIDE THE FOLLOWING SUBROUTINE WHICH PROVIDES INFORMATION ABOUT THE
C DERIVATIVES OF THE FUNCTION F ABOVE. THIS PROVIDES FOR MORE EFFICIENT
C JACOBIAN MATRIX GENERATION. ON MOST COMPUTER SYSTEMS, THE USER WILL
C BE REQUIRED TO SUPPLY THIS SUBROUTINE AS A DUMMY SUBROUTINE IF THE
C OPTIONS MF = 12 OR 22 ARE USED (SEE BELOW).
C 1) SUBROUTINE DERIVE( T, X, U, UX, UXX, DFOU, DFOUX, DFOUXX, NPDE )
C      DIMENSION U( NPDE ), UX( NPDE ), UXX( NPDE )
C      THE PACKAGE PROVIDES VALUES OF THE INPUT VARIABLES T, X, U, UX,
C      AND UXX. AND THE USER SHOULD CONSTRUCT THIS ROUTINE TO PROVIDE
C      THE FOLLOWING CORRESPONDING VALUES OF THE OUTPUT ARRAYS
C      DFOU, DFOUX, AND DFOUXX FOR K,J = 1 TO NPDE...
C      DFOU(K,J) = PARTIAL DERIVATIVE OF THE K-TH COMPONENT OF THE
C      PDE DEFINING FUNCTION F WITH RESPECT TO THE
C      VARIABLE U(J).
C      DFOUX(K,J) = PARTIAL DERIVATIVE OF THE K-TH COMPONENT OF THE
C      PDE DEFINING FUNCTION F WITH RESPECT TO THE
C      VARIABLE UX(J).
C      DFOUXX(K,J) = PARTIAL DERIVATIVE OF THE K-TH COMPONENT OF THE
C      PDE DEFINING FUNCTION F WITH RESPECT TO THE
C      VARIABLE UXX(J).
C      NOTE... THE INCOMING VALUE OF X WILL BE A COLLOCATION POINT
C      VALUE.
C      RETURN
C      END

```

#### METHODS USED

THE PACKAGE PDECOL IS BASED ON THE METHOD OF LINES AND USES A  
 FINITE ELEMENT COLLOCATION PROCEDURE WITH PIECEWISE POLYNOMIAL  
 AS TRIAL SPACE FOR THE DISCRETIZATION OF THE SPATIAL VARIABLE  
 X. THE COLLOCATION PROCEDURE REDUCES THE PDE SYSTEM TO A SEMI-  
 DISCRETE SYSTEM WHICH THEN DEPENDS ONLY ON THE TIME VARIABLE T.  
 THE TIME INTEGRATION IS THEN ACCOMPLISHED BY USE OF SLIGHTLY  
 MODIFIED STANDARD TECHNIQUES (SEE REFS. 1,2).

#### PIECEWISE POLYNOMIALS

THE USER IS REQUIRED TO SELECT THE PIECEWISE POLYNOMIAL SPACE  
 WHICH IS TO BE USED TO COMPUTE HIS APPROXIMATE SOLUTION. FIRST, THE  
 ORDER, KORD, OF THE POLYNOMIALS TO BE USED MUST BE SPECIFIED  
 (KORD = POLYNOMIAL DEGREE + 1). NEXT, THE NUMBER OF PIECES  
 (INTERVALS) NINT, INTO WHICH THE SPATIAL DOMAIN (XLEFT,XRIGHT) IS  
 TO BE DIVIDED, IS CHOSEN. THE NINT + 1 DISTINCT BREAKPOINTS OF  
 THE DOMAIN MUST BE DEFINED AND SET INTO THE ARRAY XBKPT IN  
 STRICTLY INCREASING ORDER, I.E.  
 C XLEFT,XBKPT(1) .LT. XBKPT(2) .LT. ... .LT. XBKPT(NINT+1)=XRIGHT.  
 C THE APPROXIMATE SOLUTION AT ANY TIME T WILL BE A POLYNOMIAL OF  
 C ORDER KORD OVER EACH SUBINTERVAL (XBKPT(I),XBKPT(I+1)). THE  
 C NUMBER OF CONTINUITY CONDITIONS, NCC, TO BE IMPOSED ACROSS ALL OF  
 C THE BREAKPOINTS IS THE LAST PIECE OF USER SUPPLIED DATA WHICH IS  
 C REQUIRED TO UNIQUELY DETERMINE THE DESIRED PIECEWISE POLYNOMIAL  
 C SPACE. FOR EXAMPLE, NCC = 2 WOULD REQUIRE THAT THE APPROXIMATE  
 C SOLUTION (MADE UP OF THE SEPARATE POLYNOMIAL PIECES) AND ITS FIRST  
 C SPATIAL DERIVATIVE BE CONTINUOUS AT THE BREAKPOINTS AND HENCE ON  
 C THE ENTIRE DOMAIN (XLEFT,XRIGHT). NCC = 3 WOULD REQUIRE THAT THE  
 C APPROXIMATE SOLUTION AND ITS FIRST AND SECOND SPATIAL DERIVATIVES  
 C BE CONTINUOUS AT THE BREAKPOINTS, ETC. THE DIMENSION OF THIS LINEAR  
 C SPACE IS KNOWN AND FINITE AND IS NCPTS = KORD\*NINT - NCC\*(NINT-1).  
 C THE WELL-KNOWN B-SPLINE BASIS (SEE REF. 3) FOR THIS SPACE IS USED  
 C BY PDECOL AND IT CONSISTS OF NCPTS KNOWN PIECEWISE POLYNOMIAL  
 C FUNCTIONS BF(I,X) FOR I=1 TO NCPTS, WHICH DO NOT DEPEND ON THE  
 C TIME VARIABLE T. WE WISH TO EMPHASIZE THAT THE PIECEWISE POLYNOMIAL  
 C SPACE USED IN PDECOL (WHICH IS SELECTED BY THE USER) WILL DETERMINE  
 C THE MAGNITUDE OF THE SPATIAL DISCRETIZATION ERRORS IN THE COMPUTED  
 C APPROXIMATE SOLUTION. THE PACKAGE HAS NO CONTROL OVER ERRORS  
 C INTRODUCED BY THE USER'S CHOICE OF THIS SPACE. SEE INPUT PARAMETERS  
 C BELOW.

#### COLLOCATION OVER PIECEWISE POLYNOMIALS

THE BASIC ASSUMPTION MADE IS THAT THE APPROXIMATE SOLUTION  
 SATISFIES

$$U(T,X) = \sum_{i=1}^{NCPTS} C(i,T) * BF(i,X)$$

WHERE THE UNKNOWN COEFFICIENTS C DEPEND ONLY ON THE TIME T AND  
 THE KNOWN BASIS FUNCTIONS DEPEND ONLY ON X (WE HAVE ASSUMED THAT  
 NCPTS = 1 FOR CONVENIENCE). SO, AT ANY GIVEN TIME T THE APPROX-  
 IMATE SOLUTION IS A PIECEWISE POLYNOMIAL IN THE USER CHOSEN SPACE.  
 THE SEMI-DISCRETE EQUATIONS (ACTUALLY ORDINARY DIFFERENTIAL  
 EQUATIONS) WHICH DETERMINE THE COEFFICIENTS C ARE OBTAINED BY  
 REQUIRING THAT THE ABOVE APPROXIMATE U(T,X) SATISFY THE PDE AND  
 BOUNDARY CONDITIONS EXACTLY AT A SET OF NCPTS COLLOCATION POINTS  
 (SEE COLPNT). THUS, PDECOL ACTUALLY COMPUTES THE BASIS FUNCTION  
 COEFFICIENTS RATHER THAN SPECIFIC APPROXIMATE SOLUTION VALUES.

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# USE OF POECOL

POECOL IS CALLED ONCE FOR EACH DESIRED OUTPUT VALUE (TOUT) OF THE  
 TIME T, AND IT IN TURN MAKES REPEATED CALLS TO THE CORE INTEGRATOR,  
 STIFB, WHICH ADVANCES THE TIME BY TAKING SINGLE STEPS UNTIL  
 T.GE. TOUT. INTERPOLATION TO THE EXACT TIME TOUT IS THEN DONE.  
 SEE TOUT BELOW.

## SUMMARY OF SUGGESTED INPUT VALUES

IT IS OF COURSE IMPOSSIBLE TO SUGGEST INPUT PARAMETER VALUES WHICH  
 ARE APPROPRIATE FOR ALL PROBLEMS. THE FOLLOWING SUGGESTIONS ARE TO  
 BE USED ONLY IF YOU HAVE NO IDEA OF BETTER VALUES FOR YOUR PROBLEM.

DT = 1.E-10  
 XBKPT = CHOOSE NINT+1 EQUALLY SPACED VALUES SUCH THAT XBKPT(1) =  
 XLEFT AND XBKPT(NINT+1) = XRIGHT.  
 EPS = 1.E-4  
 NINT = ENOUGH SO THAT ANY FINE STRUCTURE OF THE PROBLEM MAY BE  
 RESOLVED.  
 KORO = 4  
 NCC = 2  
 MP = 22  
 INDEX = 1 (ON FIRST CALL ONLY, THEN 0 THEREAFTER).

## THE INPUT PARAMETERS ARE..

TO = THE INITIAL VALUE OF T, THE INDEPENDENT VARIABLE  
 (USED ONLY ON FIRST CALL).  
 TOUT = THE VALUE OF T AT WHICH OUTPUT IS DESIRED NEXT. SINCE  
 THE PACKAGE CHOOSES ITS OWN TIME STEP SIZES, THE  
 INTEGRATION WILL NORMALLY GO SLIGHTLY BEYOND TOUT  
 AND THE PACKAGE WILL INTERPOLATE TO T = TOUT.  
 DT = THE INITIAL STEP SIZE IN T, IF INDEX = 1, OR, THE  
 MAXIMUM STEP SIZE ALLOWED (MUST BE .GT. 0). IF INDEX = 3.  
 USED FOR INPUT ONLY WHEN INDEX = 1 OR 3. SEE BELOW.  
 XBKPT = THE ARRAY OF PIECEWISE POLYNOMIAL BREAKPOINTS.  
 THE NINT+1 VALUES MUST BE STRICTLY INCREASING WITH  
 XBKPT(1) = XLEFT AND XBKPT(NINT+1) = XRIGHT (USED ONLY  
 ON FIRST CALL).  
 EPS = THE RELATIVE TIME ERROR BOUND (USED ONLY ON THE  
 FIRST CALL, UNLESS INDEX = 4). SINGLE STEP ERROR  
 ESTIMATES DIVIDED BY CMAX(I) WILL BE KEPT LESS THAN  
 EPS IN ROOT-MEAN-SQUARE NORM. THE VECTOR CMAX OF WEIGHTS  
 IS COMPUTED IN POECOL. INITIALLY CMAX(I) IS SET TO  
 ABS(C(I)), WITH A DEFAULT VALUE OF 1 IF ABS(C(I)) .LT. 1.  
 THEREAFTER, CMAX(I) IS THE LARGEST VALUE  
 OF ABS(C(I)) SEEN SO FAR, OR THE INITIAL CMAX(I) IF  
 THAT IS LARGER. TO ALTER EITHER OF THESE, CHANGE THE  
 APPROPRIATE STATEMENTS IN THE DO-LOOPS ENDING AT  
 STATEMENTS 50 AND 130 BELOW. THE USER SHOULD EXERCISE  
 SOME DISCRETION IN CHOOSING EPS. IN GENERAL, THE  
 OVERALL RUNNING TIME FOR A PROBLEM WILL BE GREATER IF  
 EPS IS CHOSEN SMALLER. THERE IS USUALLY LITTLE REASON TO  
 CHOOSE EPS MUCH SMALLER THAN THE ERRORS WHICH ARE BEING

INTRODUCED BY THE USER'S CHOICE OF THE POLYNOMIAL SPACE.  
 SEE RELATED COMMENTS CONCERNING CMAX BELOW STATEMENT 40.  
 NINT = THE NUMBER OF SUBINTERVALS INTO WHICH THE SPATIAL DOMAIN  
 (XLEFT, XRIGHT) IS TO BE DIVIDED (MUST BE .GE. 1)  
 (USED ONLY ON FIRST CALL).  
 KORD = THE ORDER OF THE PIECEWISE POLYNOMIAL SPACE TO BE USED.  
 ITS VALUE MUST BE GREATER THAN 2 AND LESS THAN 21. FOR  
 FIRST ATTEMPTS WE RECOMMEND KORD = 4. IF THE SOLUTION  
 IS SMOOTH AND MUCH ACCURACY IS DESIRED, HIGHER VALUES  
 MAY PROVE TO BE MORE EFFICIENT. WE HAVE SELDOM USED  
 VALUES OF KORD IN EXCESS OF 8 OR 9, THOUGH THEY ARE  
 AVAILABLE FOR USE IN PROCOL (USED ONLY ON FIRST CALL).  
 NCC = THE NUMBER OF CONTINUITY CONDITIONS TO BE IMPOSED ON THE  
 APPROXIMATE SOLUTION AT THE BREAKPOINTS IN AKRPT.  
 NCC MUST BE GREATER THAN 1 AND LESS THAN KORD. WE  
 RECOMMEND THE USE OF NCC = 2.  
 SINCE THEORY PREDICTS THAT DRAMATICALLY MORE  
 ACCURATE RESULTS CAN OFTEN BE OBTAINED USING THIS CHOICE  
 (USED ONLY ON FIRST CALL).  
 NPDE = THE NUMBER OF PARTIAL DIFFERENTIAL EQUATIONS IN THE SYSTEM  
 TO BE SOLVED (USED ONLY ON FIRST CALL).  
 MF = THE METHOD FLAG (USED ONLY ON FIRST CALL, UNLESS  
 INDEX = 1). ALLOWED VALUES ARE 11, 12, 21, 22.  
 FOR FIRST ATTEMPTS WE RECOMMEND THE USE OF MF = 22.  
 MF HAS TWO DECIMAL DIGITS. METH AND MITER  
 (MF = 10\*METH + MITER).  
 METH IS THE BASIC METHOD INDICATOR..  
 METH = 1 MEANS THE ADAMS METHODS (GENERALIZATIONS OF  
 CRANK-NICOLSON).  
 METH = 2 MEANS THE BACKWARD DIFFERENTIATION  
 FORMULAS (BDF), OR STIFF METHODS OF GEAR.  
 MITER IS THE ITERATION METHOD INDICATOR  
 AND DETERMINES HOW THE JACOBIAN MATRIX IS  
 TO BE COMPUTED..  
 MITER = 1 MEANS CHORD METHOD WITH ANALYTIC JACOBIAN.  
 FOR THIS USER SUPPLIES SUBROUTINE DERIVF.  
 SEE DESCRIPTION ABOVE.  
 MITER = 2 MEANS CHORD METHOD WITH JACOBIAN CALCULATED  
 INTERNALLY BY FINITE DIFFERENCES. SEE  
 SUBROUTINES PSETIB AND DIFF.  
 INDEX = INTEGER USED ON INPUT TO INDICATE TYPE OF CALL,  
 WITH THE FOLLOWING VALUES AND MEANINGS..  
 0 THIS IS THE FIRST CALL FOR THIS PROBLEM.  
 1 THIS IS NOT THE FIRST CALL FOR THIS PROBLEM,  
 AND INTEGRATION IS TO CONTINUE.  
 2 SAME AS 0 EXCEPT THAT TOUT IS TO BE HIT  
 EXACTLY (NO INTERPOLATION IS DONE). SEE NOTE  
 BELOW. ASSUMES TOUT .GE. THE CURRENT T.  
 IF TOUT IS .LT. THE CURRENT TIME, THEN TOUT IS  
 RESET TO THE CURRENT TIME AND CONTROL IS  
 RETURNED TO THE USER. A CALL TO VALUES WILL  
 PRODUCE ANSWERS FOR THE NEW VALUE OF TOUT.  
 3 SAME AS 0 EXCEPT CONTROL RETURNS TO CALLING  
 PROGRAM AFTER ONE STEP. TOUT IS IGNORED AND  
 DT MUST BE SET .GT. 0 TO A MAXIMUM ALLOWED  
 DT VALUE.. SEE ABOVE.  
 4 THIS IS NOT THE FIRST CALL FOR THE PROBLEM,  
 AND THE USER HAS RESET EPS AND/OR MF.  
 SINCE THE NORMAL OUTPUT VALUE OF INDEX IS 0,  
 IT NEED NOT BE RESET FOR NORMAL CONTINUATION.  
 NOTE.. THE PACKAGE MUST HAVE TAKEN AT LEAST ONE SUCCESSFUL TIME  
 STEP BEFORE A CALL WITH INDEX = 2 OR 4 IS ALLOWED.  
 AFTER THE INITIAL CALL, IF A NORMAL RETURN OCCURRED AND A NORMAL  
 CONTINUATION IS DESIRED, SIMPLY RESET TOUT AND CALL AGAIN.  
 ALL OTHER PARAMETERS WILL BE READY FOR THE NEXT CALL.

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C A CHANGE OF PARAMETERS WITH INDEX = 4 CAN BE MADE AFTER
C EITHER SUCCESSFUL OR AN UNSUCCESSFUL RETURN PROVIDED AT LEAST
C ONE SUCCESSFUL TIME STEP HAS BEEN MADE.
C
C WORK = FLOATING POINT WORKING ARRAY FOR PDECOL. WE RECOMMEND
C THAT IT BE INITIALIZED TO ZERO BEFORE THE FIRST CALL
C TO PDECOL. ITS TOTAL LENGTH MUST BE AT LEAST
C
C KORD = 4*NPDE + 9*NPDE**2 + NCPTS*(3*KORD + 2) +
C NPDE*NCPTS*(3*ML + MAXDER + 7)
C
C WHERE ML AND MAXDER ARE DEFINED BELOW (SEE STORAGE
C ALLOCATION).
C
C IWORK = INTEGER WORKING ARRAY FOR PDECOL. THE FIRST TWO
C LOCATIONS MUST BE DEFINED AS FOLLOWS...
C IWORK(1) = LENGTH OF USERS ARRAY IWORK
C IWORK(2) = LENGTH OF USERS ARRAY IWORK
C THE TOTAL LENGTH OF IWORK MUST BE AT LEAST
C NCPTS*(NPDE + 1).
C
C OUTPUT
C
C THE SOLUTION VALUES ARE NOT RETURNED DIRECTLY TO THE USER BY PDECOL.
C THE METHODS USED IN PDECOL COMPUTE BASIS FUNCTION COEFFICIENTS. SO
C THE USER (AFTER A RETURN FROM PDECOL) MUST CALL THE PACKAGE ROUTINE
C VALUES TO OBTAIN HIS APPROXIMATE SOLUTION VALUES AT ANY DESIRED SPACE
C POINTS X AT THE TIME T = TOUT. SEE THE COMMENTS IN SUBROUTINE VALUES
C FOR DETAILS ON HOW TO PROPERLY MAKE THE CALL.
C
C EXECUTION ERROR MESSAGES WILL BE PRINTED BY PDECOL ON LOGICAL UNIT
C LOUT WHICH IS THE ONLY VARIABLE IN THE COMMON BLOCK /LOUNIT/. A
C DEFAULT OF LOUT = 3 IS SET IN THE BLOCK DATA.
C
C THE COMMON BLOCK /GEAR/ CONTAINS THE VARIABLES DTUSED, NUSED,
C NSTEP, NFE, AND NLE AND CAN BE ACCESSED EXTERNALLY BY THE USER IF
C DESIRED. RESPECTIVELY, IT CONTAINS THE STEP SIZE LAST USED (SUCCESS-
C FULLY), THE ORDER LAST USED (SUCCESSFULLY), THE NUMBER OF STEPS TAKEN
C SO FAR, THE NUMBER OF RESIDUAL EVALUATIONS (RES CALLS) SO FAR,
C AND THE NUMBER OF MATRIX EVALUATIONS (PSETIB CALLS) SO FAR.
C DIFFUN CALLS ARE COUNTED IN WITH RESIDUAL EVALUATIONS.
C
C THE OUTPUT PARAMETERS ARE..
C DT = THE STEP SIZE USED LAST, WHETHER SUCCESSFULLY OR NOT.
C TOUT = THE OUTPUT VALUE OF T. IF INTEGRATION WAS SUCCESSFUL,
C AND THE INPUT VALUE OF INDEX WAS NOT 3, TOUT IS
C UNCHANGED FROM ITS INPUT VALUE. OTHERWISE, TOUT
C IS THE CURRENT VALUE OF T TO WHICH THE INTEGRATION
C HAS BEEN COMPLETED.
C INDEX = INTEGER USED ON OUTPUT TO INDICATE RESULTS.
C WITH THE FOLLOWING VALUES AND MEANINGS...
C 0 INTEGRATION WAS COMPLETED TO TOUT OR BEYOND.
C -1 THE INTEGRATION WAS HALTED AFTER FAILING TO PASS THE
C ERROR TEST EVEN AFTER REDUCING DT BY A FACTOR OF
C 1.E10 FROM ITS INITIAL VALUE.
C -2 AFTER SOME INITIAL SUCCESS, THE INTEGRATION WAS
C HALTED EITHER BY REPEATED ERROR TEST FAILURES OR BY
C A TEST ON EPS. TOO MUCH ACCURACY HAS BEEN REQUESTED.
C -3 THE INTEGRATION WAS HALTED AFTER FAILING TO ACHIEVE
C CORRECTOR CONVERGENCE EVEN AFTER REDUCING DT BY A
C FACTOR OF 1.E10 FROM ITS INITIAL VALUE.
C -4 SINGULAR MATRIX ENCOUNTERED. PROBABLY DUE TO STORAGE
C OVERWRITES.
C -5 INDEX WAS 4 ON INPUT, BUT THE DESIRED CHANGES OF
C PARAMETERS WERE NOT IMPLEMENTED BECAUSE TOUT
C WAS NOT BEYOND T. INTERPOLATION TO T = TOUT WAS
C PERFORMED AS ON A NORMAL RETURN. TO TRY AGAIN,
C SIMPLY CALL AGAIN WITH INDEX = 4 AND A NEW TOUT.

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C      ILLEGAL INDEX VALUE.
C      ILLEGAL EPS VALUE.
C      -8  AN ATTEMPT TO INTEGRATE IN THE WRONG DIRECTION. THE
C          SIGN OF DT IS WRONG RELATIVE TO TO AND TOUT.
C      -9  DT .EQ. 0.0.
C      -10 ILLEGAL NINT VALUE.
C      -11 ILLEGAL KORD VALUE.
C      -12 ILLEGAL NCC VALUE.
C      -13 ILLEGAL NPDE VALUE.
C      -14 ILLEGAL MF VALUE.
C      -15 ILLEGAL BREAKPOINTS - NOT STRICTLY INCREASING.
C      -16 INSUFFICIENT STORAGE FOR WORK OR IWORK.
C-----
C
C SUMMARY OF ALL PACKAGE ROUTINES
C
C PDECL - STORAGE ALLOCATION, ERROR CHECKING, INITIALIZATION, REPEATED
C        CALLS TO STIFB TO ADVANCE THE TIME.
C
C INTERP - INTERPOLATES COMPUTED BASIS FUNCTION COEFFICIENTS TO THE
C          DESIRED OUTPUT TIMES, TOUT, FOR USE BY VALUES.
C
C INITAL - INITIALIZATION, GENERATION AND STORAGE OF PIECEWISE POLY-
C          NOMIAL SPACE BASIS FUNCTION VALUES AND DERIVATIVES. DETER-
C          MINES THE BASIS FUNCTION COEFFICIENTS OF THE PIECEWISE
C          POLYNOMIALS WHICH INTERPOLATE THE USERS INITIAL CONDITIONS.
C
C COLPNT - GENERATION OF REQUIRED COLLOCATION POINTS.
C
C BSPLVD - B-SPLINE PACKAGE ROUTINES WHICH ALLOW FOR EVALUATION OF
C          ANY B-SPLINE BASIS FUNCTION OR DERIVATIVE VALUE.
C
C BSPLVN -
C          INTERV
C
C VALUES - GENERATION AT ANY POINT(S) OF VALUES OF THE COMPUTED
C           APPROXIMATE SOLUTION AND ITS DERIVATIVES WHICH ARE
C           PIECEWISE POLYNOMIALS. THE SUBROUTINE IS CALLED ONLY BY
C           THE USER.
C
C STIFB - CORE INTEGRATOR. TAKES SINGLE TIME STEPS TO ADVANCE THE
C         TIME. ASSEMBLES AND SOLVES THE PROPER NONLINEAR EQUATIONS
C         WHICH ARE RELATED TO USE OF ADAMS OR GEAR TYPE INTEGRATION
C         FORMULAS. CHOOSES PROPER STEP SIZE AND INTEGRATION FORMULA
C         ORDER TO MAINTAIN A DESIRED ACCURACY. DESIGNED FOR ODE
C         PROBLEMS OF THE FORM  $A * (DY/DT) = G(T,Y)$ .
C
C COSET - GENERATES INTEGRATION FORMULA AND ERROR CONTROL COEFFICIENTS.
C
C RES - COMPUTES RESIDUAL VECTORS USED IN SOLVING THE NONLINEAR
C        EQUATIONS BY A MODIFIED NEWTON METHOD.
C
C DIFFUN - COMPUTES  $A_{i+1} * G(T,Y)$  WHERE A AND G ARE AS ABOVE (STIFB).
C
C ADDA - ADDS THE A MATRIX TO A GIVEN MATRIX IN BAND FORM.
C
C EVAL - EVALUATES THE COMPUTED PIECEWISE POLYNOMIAL SOLUTION AND
C        DERIVATIVES AT COLLOCATION POINTS.
C
C GFUN - EVALUATES THE FUNCTION  $G(T,Y)$  BY CALLING EVAL AND THE USER
C        SUBROUTINES F AND BNDRY.
C
C PSETIB - GENERATES PROPER JACOBIAN MATRICES REQUIRED BY THE MODIFIED
C          NEWTON METHOD.
C
C DIFFF - PERFORMS SAME ROLE AS THE USER ROUTINE DERIVF. COMPUTES
C          DERIVATIVE APPROXIMATIONS BY USE OF FINITE DIFFERENCES.

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```

C DECB ( PERFORM AN LU DECOMPOSITION AND FORWARD AND BACKWARD (
C SOLB SUBSTITUTION FOR SOLVING BANDED SYSTEMS OF LINEAR EQUATIONS.
C -----
C
C STORAGE ALLOCATION
C
C SINCE PDECOL IS A DYNAMICALLY DIMENSIONED PROGRAM, MOST OF ITS
C WORKING STORAGE IS PROVIDED BY THE USER IN THE ARRAYS WORK AND IWORK.
C THE FOLLOWING GIVES A LIST OF THE ARRAYS WHICH MAKE UP THE CONTENTS
C OF WORK AND IWORK, THEIR LENGTHS, AND THEIR USES. WHEN MORE THAN ONE
C NAME IS GIVEN, IT INDICATES THAT DIFFERENT NAMES ARE USED FOR THE
C SAME ARRAY IN DIFFERENT PARTS OF THE PROGRAM. THE DIFFERENT NAMES
C OCCUR BECAUSE PDECOL IS AN AMALGAMATION OF SEVERAL OTHER CODES
C WRITTEN BY DIFFERENT PEOPLE AND WE HAVE TRIED TO LEAVE THE SEPARATE
C PARTS AS UNCHANGED FROM THEIR ORIGINAL VERSIONS AS POSSIBLE.
C
C NAMES LENGTH USE
C -----
C
C BC 4*NPDE**2 BOUNDARY CONDITION INFORMATION.
C WORK
C
C A 3*NORD*NCPTS BASIS FUNCTION VALUES AT COLLOCATION POINT
C WORK(IW1)
C
C XT NCPTS + NORD BREAKPOINT SEQUENCE FOR GENERATION OF BASIS
C WORK(IW2) FUNCTION VALUES.
C
C XC NCPTS COLLOCATION POINTS.
C WORK(IW3)
C
C CHAX NPDE*NCPTS VALUES USED IN ESTIMATING TIME
C CHMAX NPDE*NCPTS INTEGRATION ERRORS.
C WORK(IW4)
C
C ERROR NPDE*NCPTS TIME INTEGRATION ERRORS.
C WORK(IW5)
C
C SAVE1 NPDE*NCPTS WORKING STORAGE FOR THE TIME INTEGRATION
C WORK(IW6) METHOD.
C
C SAVE2 NPDE*NCPTS WORKING STORAGE FOR THE TIME INTEGRATION
C WORK(IW7) METHOD.
C
C SAVE3 NPDE*NCPTS WORKING STORAGE FOR THE TIME INTEGRATION
C WORK(IW8) METHOD.
C
C UVAL 3*NPDE WORKING STORAGE FOR VALUES OF U, UX, AND
C WORK(IW9) UX AT ONE POINT.
C
C Y NPDE*NCPTS* CURRENT BASIS FUNCTION COEFFICIENT VALUES
C WORK(IW10) (MAXDER+1) AND THEIR SCALED TIME DERIVATIVES.
C
C DFDU NPDE**2 WORKING STORAGE USED TO COMPUTE THE
C WORK(IW11) JACOBIAN MATRIX.
C
C DFDUX NPDE**2 WORKING STORAGE USED TO COMPUTE THE
C WORK(IW12) JACOBIAN MATRIX.
C
C DFDUXX NPDE**2 WORKING STORAGE USED TO COMPUTE THE
C WORK(IW13) JACOBIAN MATRIX.

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C DBDU ( )
C WORK ( )
C NPDE**2 BOUNDARY CONDITION INFORMATION. (
C DBDUX
C WORK(1W15) BOUNDARY CONDITION INFORMATION.
C DZDT
C WORK(1W16) BOUNDARY CONDITION INFORMATION.
C PW
C WORK(1W17) STORAGE AND PROCESSING OF THE JACOBIAN
C NCPTS (3*ML+1) MATRIX.
C ILEFT NCPTS POINTERS TO BREAKPOINT SEQUENCE FOR
C IWORK GENERATION OF BASIS FUNCTION VALUES.
C IPIV NPDE*NCPTS PIVOT INFORMATION FOR THE LU DECOMPOSED
C IWORK(1W18) JACOBIAN MATRIX PW.
C WHERE...
C NCPTS = KCRD-NINT - NCC-(NINT-1)
C ML = NPDE*(KORD+IQAD-1) - 1
C IQAD = 1 IF KORD = 3 AND A NULL BOUNDARY CONDITION EXISTS
C IQAD = 0 OTHERWISE
C MAXDER = 5 UNLESS OTHERWISE SET BY THE USER INTO /OPTION/.
C THE COMMON BLOCK /OPTION/ CONTAINS THE TWO VARIABLES NOGAUS AND
C MAXDER. NOGAUS IS SET .EQ. 0 IN THE BLOCK DATA. IT CAN BE CHANGED
C TO BE SET .EQ. 1 IF THE GAUSS-LEGENDRE COLLOCATION POINTS ARE NOT
C DESIRED WHEN NCC = 2 (SEE ABOVE AND COLNT). MAXDER IS SET
C .EQ. 5 IN THE BLOCK DATA AND ITS VALUE REPRESENTS THE
C MAXIMUM ORDER OF TIME INTEGRATION FORMULA ALLOWED. ITS VALUE
C AFFECTS THE STORAGE REQUIRED IN WORK AND MAY BE CHANGED IF
C DESIRED. SEE COSET FOR RESTRICTIONS. THESE CHANGES MAY BE MADE BY
C THE USER BY ACCESSING /OPTION/ IN HIS CALLING PROGRAM (BEFORE THE
C FIRST CALL TO PDECOL) OR BY CHANGING THE DATA STATEMENT IN
C THE BLOCK DATA.
C-----
C COMMUNICATION
C EACH SUBROUTINE IN THE PACKAGE CONTAINS A COMMUNICATION SUMMARY
C AS INDICATED BELOW.
C PACKAGE ROUTINES CALLED.. EVAL,INITAL,INTERP,STIFB
C USER ROUTINES CALLED.. BNDY
C CALLED BY.. USERS MAIN PROGRAM
C FORTRAN FUNCTIONS USED.. ABS,AMAX1,FLOAT, SORT
C-----
C DIMENSION WORK(1),IWORK(1),XBKPT(1)
C COMMON /GEAR0/ DTUSED,NQUSED,NSTEP,NFE,NJE
C COMMON /GEAR1/ T,DTC,DTMX,DTMX,EPSC,URUND,N,MFC,KFLAG,JSTART
C COMMON /GEAR9/ EPSJ,RO,ML,RU,WM,NM1,NOML,NOW
C COMMON /OPTION/ NOGAUS,MAXDER
C COMMON /SIZES/ NIN,KOR,NC,NPD,NCPTS,NEON,IQAD
C COMMON /ISTART/ IW1,IW2,IW3,IW4,IW5,IW6,IW7,IW8,IW9,IW10,
C * IW11,IW12,IW13,IW14,IW15,IW16,IW17,IW18
C COMMON /OUNTIT/ LOUT
C IF (INDEX .EQ. 0) GO TO 60
C IF (INDEX .EQ. 2) GO TO 70
C IF (INDEX .EQ. 4) GO TO 80
C IF (INDEX .EQ. 3) GO TO 90
C-----

```

C SEVERAL CHECKS ARE MADE HERE TO DETERMINE IF THE INPUT PARAMETERS  
 C HAVE "AL" VALUES. ERROR CHECKS ARE MADE ON INDEX, EPS, (TO-TO-TO)\*DT,  
 C DT, N, KORD, NCC, NPDE, MF, WHETHER THE BREAKPOINT SEQUENCE  
 C STRICTLY INCREASING, AND WHETHER THERE IS SUFFICIENT STORAGE  
 C PROVIDED FOR WORK AND IWORK. PROBLEM DEPENDENT PARAMETERS ARE  
 C CALCULATED AND PLACED IN COMMON.

```

IERID = -6
IF (INDEX.NE.1) GO TO 320
IERID = IERID - 1
IF (EPS.LE.0.) GO TO 320
IERID = IERID - 1
IF ((TO-TO-TO)*DT.GT.0.) GO TO 320
IERID = IERID - 1
IF (DT.EQ.0.0) GO TO 320
IERID = IERID - 1
NIN = NINT
IF (NIN.LT.1) GO TO 320
IERID = IERID - 1
KOR = KORD
IF (KOR.LT.3.OR.KOR.GT.20) GO TO 320
IERID = IERID - 1
NCC = NCC
IF (NCC.LT.2.OR.NCC.GE.KOR) GO TO 320
IERID = IERID - 1
NPDE = NPDE
NPDE2 = NFO*NPDE
IF (NPDE.LT.1) GO TO 320
IERID = IERID - 1
IF (MF.NE.22.AND.MF.NE.21.AND.MF.NE.12.AND.MF.NE.11) GO TO 320
IERID = IERID - 1
DO 10 K=1,NIN
  IF (XBKPT(K).GE.XBKPT(K+1)) GO TO 320
10 CONTINUE
NCPTS = KOR + (NIN - 1) * (KOR - NCC)
NEON = NPDE * NCPTS
ML = (KOR-1)*NPDE - 1
MW = ML
MW = ML + ML + 1
NOW = NEON*MW
IWSAVE = IWORK(1)
IISAVE = IWORK(2)
IW1 = 4*NPDE2 + 1
IW2 = IW1 + 3*KORD*NCPTS
IW3 = IW2 + NCPTS + KORD
IW4 = IW3 + NCPTS
IW5 = IW4 + NEON
IW6 = IW5 + NEON
IW7 = IW6 + NEON
IW8 = IW7 + NEON
IW9 = IW8 + NEON
IW10 = IW9 + 3*NPDE
IW11 = IW10 + NEON*(MAXDER+1)
IW12 = IW11 + NPDE2
IW13 = IW12 + NPDE2
IW14 = IW13 + NPDE2
IW15 = IW14 + NPDE2
IW16 = IW15 + NPDE2
IW17 = IW16 + NPDE
IW18 = NCPTS + 1
IERID = IERID - 1
IWMSTOR = IW17 + NEON*(3*ML+1) - 1
IISAVE = IW18 + NEON - 1
IF (IWSAVE.LT.IWMSTOR.OR.IISAVE.LT.IIISTOR) GO TO 335
C PERFORM INITIALIZATION TASKS. IF KORD.EQ.3 THEN CALCULATE THE BAND-

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C WIDTH OF THE ASSOCIATED MATRIX PROBLEM BY DETERMINING THE TYPE OF
C BOUNDARY CONDITIONS, THEN CHECK FOR SUFFICIENT STORAGE AGAIN. (
-----
      CALL INITAL(KOR,WORK(IW1),WORK(IW6),XBKPT,WORK(IW2),WORK(IW3),
      *      WORK(IW7),WORK(IW8),IWORK)
      IF(IQUAD.NE.0) GO TO 280
      IF(KOR.NE.3) GO TO 40
      CALL EVAL(I,NPDE,WORK(IW6),WORK(IW9),WORK(IW1),IWORK)
      CALL BNDRY(TO,WORK(IW3),WORK(IW9),WORK(IW9+NPDE),WORK(IW14),
      *      WORK(IW15),WORK(IW16),NPDE)
      DO 20 K=1,NPDE
      *      I = K + NPDE*(K-1) - 1
      *      IF(WORK(IW14+I).EQ.0.0.AND.WORK(IW15+I).EQ.0.0)
      *      *      IQUAD = 1
      *      20 CONTINUE
      *      CALL EVAL(NCPTS,NPDE,WORK(IW6),WORK(IW9),WORK(IW1),IWORK)
      *      CALL BNDRY(TO,WORK(IW3+NCPTS-1),WORK(IW9),WORK(IW9+NPDE),
      *      *      WORK(IW14),WORK(IW15),WORK(IW16),NPDE)
      *      DO 30 K=1,NPDE
      *      *      I = K + NPDE*(K-1) - 1
      *      *      IF(WORK(IW14+I).EQ.0.0.AND.WORK(IW15+I).EQ.0.0)
      *      *      *      IQUAD = 1
      *      *      30 CONTINUE
      *      *      ML = ML + IQUAD*NPDE
      *      *      MU = ML
      *      *      MW = ML + ML + 1
      *      *      NM = NEON*MW
      *      40 CONTINUE
      *      INKSTOR = IW17 + NEON*(3*ML+1) - 1
      *      IF ( INKSAVE .LT. INKSTOR ) GO TO 335
      *      -----
      *      C IF INITIAL VALUES OF CMAX OTHER THAN THOSE SET BELOW ARE DESIRED,
      *      C THEY SHOULD BE SET HERE. ALL CMAX(I) MUST BE POSITIVE.
      *      C HAVING PROPER VALUES OF CMAX FOR THE PROBLEM BEING SOLVED IS AS
      *      C IMPORTANT AS CHOOSING EPS (SEE ABOVE), SINCE ERRORS ARE
      *      C MEASURED RELATIVE TO CMAX. IF VALUES FOR DTMIN OR DTMAX, THE
      *      C BOUNDS ON ABS(DT), OTHER THAN THOSE BELOW ARE DESIRED, THEY
      *      C SHOULD BE SET BELOW.
      *      -----
      *      DO 50 I = 1,NEON
      *      *      I1 = I - 1
      *      *      WORK(IW4+I1) = ABS(WORK(IW6+I1))
      *      *      IF (WORK(IW4+I1).LT.1.) WORK(IW4+I1) = 1.
      *      50 WORK(IW10+I1) = WORK(IW6+I1)
      *      *      N = NEON
      *      *      T = TO
      *      *      DTC = DT
      *      *      DTMIN = ABS(DT)
      *      *      DTUSED = 0.
      *      *      EPSC = EPS
      *      *      MFC = MF
      *      *      JSTART = 0
      *      *      EPSJ = SORT(UROUND)
      *      *      NM1 = NEON - 1
      *      *      NOML = NEON*ML
      *      *      NHCUT = 0
      *      *      KFLAG = 0
      *      *      TOUTP = TO
      *      *      IF ( TO.EQ.TOUT ) GO TO 360
      *      60 DTMX = ABS(TOUT-TOUTP)*10.
      *      *      GO TO 140
      *      *      C
      *      *      70 DTMX = ABS(TOUT-TOUTP)*10.
      *      *      IF ((T-TOUT)*DTC.GE.0.) GO TO 340
      *      *      GO TO 150
      *      *      C

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80 IF (T-TOUT)*DTC .GE. 0.) GO TO 300
   IF T = -1
     EPSC = EPS
     KFC = MF
     GO TO 100
C
90 DTMX = DT
100 IF ((T+DTC) .EQ. T) WRITE(LOUT,110)
110 FORMAT(36H WARNING.. T + DT = T ON NEXT STEP.)
C
C TAKE A TIME STEP BY CALLING THE INTEGRATOR.
C
   CALL STIFIB (NEON,WORK(I410),WORK(I44),WORK(I45),WORK(I46),
   *            WORK(I47),WORK(I48),WORK(I417),WORK(I418),WORK(I419))
C
   KGO = 1 - KFLAG
   GO TO (120, 160, 220, 260, 280), KGO
C KFLAG = 0, -1, -2, -3, -4
C
120 CONTINUE
C
C NORMAL RETURN FROM INTEGRATOR.
C
C THE WEIGHTS CHAI(I) ARE UPDATED. IF DIFFERENT VALUES ARE DESIRED,
C THEY SHOULD BE SET HERE. A TEST IS MADE FOR EPS BEING TOO SMALL
C FOR THE MACHINE PRECISION.
C
C ANY OTHER TESTS OR CALCULATIONS THAT ARE REQUIRED AFTER EVERY
C STEP SHOULD BE INSERTED HERE.
C
C IF INDEX = 3, SAVE1 IS SET TO THE CURRENT C VALUES ON RETURN.
C IF INDEX = 2, DT IS CONTROLLED TO HIT TOUT (WITHIN ROUNDOFF
C ERROR), AND THEN THE CURRENT C VALUES ARE PUT IN SAVE1 ON RETURN.
C FOR ANY OTHER VALUE OF INDEX, CONTROL RETURNS TO THE INTEGRATOR
C UNLESS TOUT HAS BEEN REACHED. THEN INTERPOLATED VALUES OF C ARE
C COMPUTED AND STORED IN SAVE1 ON RETURN.
C IF INTERPOLATION IS NOT DESIRED, THE CALL TO INTERP SHOULD BE
C REMOVED AND CONTROL TRANSFERRED TO STATEMENT 340 INSTEAD OF 360.
D = 0.
DO 130 I = 1, NEON
  II = I - 1
  AVI = ABS(WORK(I410+II))
  WORK(I44+II) = AMAX1(WORK(I44+II), AVI)
130 D = D + (AVI/WORK(I44+II))*.2
  D = D - (UROUND/EPS)*.2
  IF (D .GT. FLOAT(NEON)) GO TO 240
  IF (INDEX .EQ. 3) GO TO 340
  IF (INDEX .EQ. 2) GO TO 150
140 IF ((T-TOUT)*DTC .LT. 0.) GO TO 100
  CALL INTERP(TOUT,WORK(I410),NEON,WORK(I46))
  GO TO 360
C
150 IF ((T+DTC)-TOUT)*DTC .LE. 0.) GO TO 100
  IF (ABS(T-TOUT) .LE. 100.*UROUND*DTMX) GO TO 340
  IF ((T-TOUT)*DTC .GE. 0.) GO TO 340
  DTC = (TOUT - T)*(1. - 4.*UROUND)
  JSTART = -1
  GO TO 100
C
C ON AN ERROR RETURN FROM INTEGRATOR, AN IMMEDIATE RETURN OCCURS IF
C KFLAG = -2 OR -4, AND RECOVERY ATTEMPTS ARE MADE OTHERWISE.
C TO RECOVER, DT AND DTMX ARE REDUCED BY A FACTOR OF .1 UP TO 10
C TIMES BEFORE GIVING UP.
C
160 WRITE (LOUT,170) T

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170 FORMAT(//35H KFLAG = -1 FROM INTEGRATOR AT T = .E16.8/
* / M ERROR TEST FAILED WITH ABS(DT) = DTMIN/)
180 IF (NHOUT .EQ. 10) GO TO 200
NHOUT = NHOUT + 1
DTMIN = .1*DTMIN
DTC = .1*DTC
*WRITE (LOUT,190) DTC
190 FORMAT(25H DT HAS BEEN REDUCED TO .E16.8.
* 26H AND STEP WILL BE RETRIED//)
JSTART = -1
GO TO 100
C
200 *RITE (LOUT,210)
210 FORMAT(//44H PROBLEM APPEARS UNSOLVABLE WITH GIVEN INPUT//)
GO TO 340
C
220 WRITE (LOUT,230) T,DTC
230 FORMAT(//35H KFLAG = -2 FROM INTEGRATOR AT T = .E16.8, 6H DT =
* .E16.8/52H THE REQUESTED ERROR IS SMALLER THAN CAN BE HANDLED//)
GO TO 340
C
240 *RITE (LOUT,250) T
250 FORMAT(//37H INTEGRATION HALTED BY DRIVER AT T = .E16.8/
* 56H EPS TOO SMALL TO BE ATTAINED FOR THE MACHINE PRECISION//)
KFLAG = -2
GO TO 340
C
260 WRITE (LOUT,270) T
270 FORMAT(//35H KFLAG = -3 FROM INTEGRATOR AT T = .E16.8/
* 45H CORRECTOR CONVERGENCE COULD NOT BE ACHIEVED//)
GO TO 180
C
280 WRITE (LOUT,290)
290 FORMAT(//28H SINGULAR MATRIX ENCOUNTERED.
* 35H PROBABLY DUE TO STORAGE OVERWRITES//)
KFLAG = -4
GO TO 340
C
300 WRITE (LOUT,310) T,TOUT,DTC
310 FORMAT(//45H INDEX = -1 ON INPUT WITH (T-TOUT)*DT .GE. 0./
* 4H T = .E16.8,9H TOUT = .E16.8,8H DTC = .E16.8/
* 44H INTERPOLATION WAS DONE AS ON NORMAL RETURN./
* 41H DESIRED PARAMETER CHANGES WERE NOT MADE.)
CALL INTERP(TOUT,WORK(IW10),NEQN,WORK(IW6))
INDEX = -5
RETURN
C
320 *RITE (LOUT,330) IERID
330 FORMAT(//24H ILLEGAL INPUT...INDEX = ,13//)
INDEX = IERID
RETURN
C
335 WRITE (LOUT,336) IWSIOR,IWSAVE,IISTOR,IISAVE
336 FORMAT(//21H INSUFFICIENT STORAGE/24H WORK MUST BE OF LENGTH,
* 110,5X,12HYOU PROVIDED,110/24H IWORK MUST BE OF LENGTH,110,5X,
* 12HYOU PROVIDED,110//)
INDEX = IERID
RETURN
C
340 TOUT = T
DO 350 I = 1,NEQN
I1 = I - 1
350 WORK(IW6+I1) = WORK(IW10+I1)
360 INDEX = KFLAG
TOUTP = TOUT
DT = DTUSED

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IF (KFLAG .NE. 0) DT = DTC
RE" RN
EX
SUBROUTINE VALUES(X,USOL,SCATCH,NDIM1,NDIM2,NPTS,NDERV,WORK)
C-----
C SUBROUTINE VALUES COMPUTES THE SOLUTION U AND THE FIRST NDERV
C DERIVATIVES OF U AT THE NPTS POINTS X AND AT TIME TOUT AND RETURNS
C THEM IN THE ARRAY USOL. THIS ROUTINE MUST BE USED TO OBTAIN
C SOLUTION VALUES SINCE PDECOL DOES NOT RETURN ANY SOLUTION VALUES
C TO THE USER. SEE PDECOL.
C
C THE CALLING PARAMETERS ARE...
C X = AN ARBITRARY VECTOR OF SPATIAL POINTS OF LENGTH NPTS AT
C WHICH THE SOLUTION AND THE FIRST NDERV DERIVATIVE VALUES
C ARE TO BE CALCULATED. IF X .LT. XLEFT ( X .GT. XRIGHT )
C THEN THE PIECEWISE POLYNOMIAL OVER THE LEFTMOST ( RIGHT-
C MOST ) INTERVAL IS EVALUATED TO CALCULATE THE SOLUTION
C VALUES AT THIS UNUSUAL VALUE OF X. SEE PDECOL.
C
C USOL = AN ARRAY WHICH CONTAINS THE SOLUTION AND THE FIRST
C NDERV DERIVATIVES OF THE SOLUTION AT ALL THE POINTS IN
C THE INPUT VECTOR X. IN PARTICULAR, USOL(J,I,K) CONTAINS
C THE VALUE OF THE (K-1)-ST DERIVATIVE OF THE J-TH PDE
C COMPONENT AT THE I-TH POINT OF THE X VECTOR FOR
C J = 1 TO NPDE, I = 1 TO NPTS, AND K = 1 TO NDERV+1.
C
C SCATCH = A USER SUPPLIED WORKING STORAGE ARRAY OF LENGTH AT LEAST
C KORD*(NDERV+1). SEE BELOW AND PDECOL FOR DEFINITIONS OF
C THESE PARAMETERS.
C
C NDIM1 = THE FIRST DIMENSION OF THE OUTPUT ARRAY USOL IN THE CALLING
C PROGRAM. NDIM1 MUST BE .GE. NPDE.
C
C NDIM2 = THE SECOND DIMENSION OF THE OUTPUT ARRAY USOL IN THE
C CALLING PROGRAM. NDIM2 MUST BE .GE. NPTS.
C
C NPTS = THE NUMBER OF POINTS IN THE X VECTOR.
C
C NDERV = THE NUMBER OF DERIVATIVE VALUES OF THE SOLUTION THAT ARE
C TO BE CALCULATED. NDERV SHOULD BE LESS THAN KORD SINCE
C THE KORD-TH DERIVATIVE OF A POLYNOMIAL OF DEGREE KORD-1
C IS EQUAL TO ZERO. SEE PDECOL.
C
C WORK = THE USERS WORKING STORAGE ARRAY WHICH IS USED IN THIS CASE
C TO PROVIDE THE CURRENT BASIS FUNCTION COEFFICIENTS AND THE
C PIECEWISE POLYNOMIAL BREAKPOINT SEQUENCE.
C
C PACKAGE ROUTINES CALLED... BSPLVD,INTERV
C USER ROUTINES CALLED... NONE
C CALLED BY... USERS MAIN PROGRAM
C FORTRAN FUNCTIONS USED... NONE
C
C-----
C DIMENSION USOL(NDIM1,NDIM2,NDERV),X(NPTS),SCATCH(1),WORK(1)
C COMMON /SIZES/ NINT,KORD,NCC,NPDE,NCPDE,NCPDS,NEON,IQUAD
C COMMON /ISTART/ IM1,IM2,IM3,IM4,IM5,IM6,IDUM(12)
C DATA ILEFT/O/, NFLAG/O/
C NDERV1 = NDERV + 1
C DO 20 IPTS=1,NPTS
C CALL INTERV(WORK(IM2),NCPDS,X(IPTS),ILEFT,NFLAG)
C CALL BSPLVD(WORK(IM2),KORD,X(IPTS),ILEFT,SCATCH,NDERV1)
C IK = ILEFT - KORD
C DO 10 M=1,NDERV1
C I1 = (M-1)*KORD
C DO 10 K=1,NPDE
C USOL(K,IPTS,M) = 0.

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DO 10 I=1,KORD
  I2 = (I+IK-1)*NPDE + I46 - 1
  ( USOL(K,IPTS,M) = USOL(K,IPTS,M) + WORK(12+K) = SCF( 1+I1) )
10 CONTINUE
20 CONTINUE
RETURN
END
BLOCK DATA
C-----
C IN THE FOLLOWING DATA STATEMENT, SET..
C LOUT = THE LOGICAL UNIT NUMBER FOR THE OUTPUT OF MESSAGES DURING
C THE INTEGRATION.
C NCGAUS = SET .EQ. 1 IF THE GAUSS-LEGENDRE COLLOCATION POINTS ARE
C NOT DESIRED WHEN NCC = 2 (SEE PDECOL AND COLPNT).
C MAXDER = SET .EQ. 5. ITS VALUE REPRESENTS THE MAXIMUM ORDER OF
C THE TIME INTEGRATION ALLOWED. ITS VALUE AFFECTS THE STORAGE
C REQUIRED IN WORK AND MAY BE CHANGED IF DESIRED
C (SEE CCOSET FOR RESTRICTIONS).
C UROUND = THE UNIT ROUNDOFF OF THE MACHINE, I.E. THE SMALLEST
C POSITIVE U SUCH THAT 1. + U .NE. 1. ON THE MACHINE.
C-----
COMMON /GEAR1 DUM(S),UROUND,LDUM(4)
COMMON /OPTIC1 NCGAUS,MAXDER
COMMON /IOUNT1// LOUT
DATA LOUT,NCGAUS,MAXDER,UROUND/3,0,5,7,1E-15/
END
SUBROUTINE INITIAL(K,A,RHS,X,XT,XC,PW,IPIV,ILEFT)
C-----
C INITIAL IS CALLED ONLY ONCE BY PDECOL TO PERFORM INITIALIZATION TASKS.
C THESE TASKS INCLUDE - 1) DEFINING THE PIECEWISE POLYNOMIAL SPACE
C BREAKPOINT SEQUENCE. 2) CALLING THE SUBROUTINE COLPNT TO DEFINE THE
C REQUIRED COLLOCATION POINTS. 3) DEFINING THE PIECEWISE POLYNOMIAL SPACE
C BASIS FUNCTION VALUES (PLUS FIRST AND SECOND DERIVATIVE VALUES) AT
C THE COLLOCATION POINTS, AND 4) DEFINING THE INITIAL BASIS FUNCTION
C COEFFICIENTS WHICH DETERMINE THE PIECEWISE POLYNOMIAL WHICH
C INTERPOLATES THE USER SUPPLIED (UNIT) INITIAL CONDITION FUNCTION(S)
C AT THE COLLOCATION POINTS.
C-----
C K = ORDER OF PIECEWISE POLYNOMIAL SPACE.
C A = BASIS FUNCTION VALUES GENERATED BY INITIAL.
C RHS = TEMPORARY STORAGE USED TO RETURN INITIAL CONDITION COEFFICIENT
C VALUES.
C XT = USER DEFINED PIECEWISE POLYNOMIAL BREAKPOINTS.
C XC = PIECEWISE POLYNOMIAL BREAKPOINT SEQUENCE GENERATED BY INITIAL.
C PW = COLLOCATION POINTS GENERATED BY INITIAL.
C PM = STORAGE FOR BAND MATRIX USED TO GENERATE INITIAL
C COEFFICIENT VALUES.
C IPIV = PIVOT INFORMATION FOR LINEAR EQUATION SOLVER DECB-SOLB.
C ILEFT = POINTERS TO BREAKPOINT SEQUENCE GENERATED BY INITIAL.
C-----
C PACKAGE ROUTINES CALLED.. BSPLVD,COLPNT,DECB,INTERV,SOLB
C USER ROUTINES CALLED.. UNIT
C CALLED BY... PDECOL
C FORTRAN FUNCTIONS USED.. MAX0,MIN0
C-----
DIMENSION A(K,3,1),RHS(1),X(1),XT(1),XC(1),PW(1),IPIV(1),ILEFT(1)
COMMON /SIZES/ NINT,KORD,NCC,NPDE,NPTS,NEQN,IER
COMMON /GEAR9/ EPSJ,R0,ML,MU,LDUM(3),NOW
MFLAG = -2
IER = 0
C-----
C SET UP THE PIECEWISE POLYNOMIAL SPACE BREAKPOINT SEQUENCE.
C-----
KRPT = KORD - NCC
DO 10 I=1,KORD
  XT(NCPTS+I) = X(NINT+I)

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10  Y*(I) = X(I)
    CJ  I=2,NINT
    I1 = (I-2)*KRPT + KORD
    DO 20 J=1,KRPT
20  XT(I1+J) = X(I)
C SET UP COLLOCATION POINTS ARRAY XC.
C-----
    CALL COLPNT(X, XC, XT)
C-----
C GENERATE THE ILEFT ARRAY. STORE THE BASIS FUNCTION VALUES IN THE
C ARRAY A. THE ARRAY A IS DIMENSIONED (KORD,3,NCPTS) AND A(K,J,I)
C CONTAINS THE VALUE OF THE (J-1)-ST DERIVATIVE (J = 1,2,3) OF THE K-TH
C NONZERO BASIS FUNCTION (K = 1, ..., KORD) AT THE I-TH COLLOCATION
C POINT (I = 1, ..., NCPTS). SET UP RHS FOR INTERPOLATING THE INITIAL
C CONDITIONS AT THE COLLOCATION POINTS. SET THE INTERPOLATION MATRIX
C INTO THE Banded MATRIX PM.
C-----
    DO 30 I=1,N04
30  PM(I) = 0.
    DO 40 J=1,NCPTS
    CALL INTERV(XT,NCPTS,XC(I),ILEFT(I),MFLAG)
    CALL BSPLVDIAT,KORD,XC(I),ILEFT(I),A(1,1,1),3)
    I1 = NPDE - (I-1)
    CALL UINIT(XC(I),RHS(I1+1),NPDE)
    ICOL = ILEFT(I) - I - 1
    J1 = MAX(1,I-2-NCPTS)
    J2 = MIN(KORD,KORD+I-2)
    DO 40 J=J1,J2
    J1 = I1 + NEON * (NPDE * (ICOL + J) - 1)
    DO 40 J=J1,NPDE
40  PM(J,J1+1) = A(J,1,1)
C LU DECOMPOSE THE MATRIX PM.
C-----
    CALL DECB (NEON,NEON,ML,MU,PW,IPIV,IER)
    IF ( IER .NE. 0 ) RETURN
C SOLVE THE LINEAR SYSTEM PM*Z = RHS. THIS GIVES THE BASIS FUNCTION
C COEFFICIENTS FOR THE INITIAL CONDITIONS.
C-----
    CALL SOLB (NEON,NEON,ML,MU,PW,RHS,IPIV)
    RETURN
END
SUBROUTINE COLPNT(X, XC, XT)
C-----
C COLPNT IS CALLED ONLY ONCE BY INITIAL TO DEFINE THE REQUIRED COLLOCA-
C TION POINTS WHICH ARE TO BE USED WITH THE USER SELECTED PIECEWISE
C POLYNOMIAL SPACE. THE COLLOCATION POINTS ARE CHOSEN SUCH THAT THEY
C ARE EITHER THE POINTS AT WHICH THE PIECEWISE POLYNOMIAL SPACE BASIS
C FUNCTIONS ATTAIN THEIR UNIQUE MAXIMUM VALUES, OR, THE GAUSS-LEGENDRE
C QUADATURE POINTS WITHIN EACH PIECEWISE POLYNOMIAL SPACE SUBINTERVAL,
C DEPENDING UPON THE SPACE BEING USED AND THE DESIRE OF THE USER.
C-----
C X = USER DEFINED PIECEWISE POLYNOMIAL BREAKPOINTS.
C XC = COLLOCATION POINTS DEFINED BY COLPNT.
C XT = PIECEWISE POLYNOMIAL BREAKPOINT SEQUENCE.
C-----
C PACKAGE ROUTINES CALLED.. BSPLVD, INTERV
C USER ROUTINES CALLED.. NONE
C CALLED BY.. INITIAL
C FORTRAN FUNCTIONS USED.. NONE
C-----
    DIMENSION RHO(40),X(1),XC(1),XT(1)
    COMMON /SIZES/ NINT,KORD,NCC,NPDE,NCPTS,NEON,IQUAD
    COMMON /OPTION/ NOGAUS,MAXDER

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DATA ILEFT/O/
C IF THE VARIABLE NOGAUS IN THE COMMON BLOCK /OPTION/ IS SET .EQ.
C THE USE OF THE GAUSS-LEGENDRE POINTS IS PROHIBITED FOR ALL CASES.
C NOGAUS IS CURRENTLY SET .EQ. 0 BY A DATA STATEMENT IN THE BLOCK DATA.
C THE USER MAY CHANGE THIS AS DESIRED.
C-----
      IF ( NCC .NE. 2 .OR. NOGAUS .EQ. 1 ) GO TO 200
C-----
C COMPUTE THE COLLOCATION POINTS TO BE AT THE GAUSS-LEGENDRE POINTS IN
C EACH PIECEWISE POLYNOMIAL SPACE SUBINTERVAL. THE ARRAY RHO IS SET TO
C CONTAIN THE GAUSS-LEGENDRE POINTS FOR THE STANDARD INTERVAL (-1,1).
C-----
      IPTS = KORD - 2
      GO TO (10,20,30,40,50,60,70,80,90,100,110,120,130,140,150,160,170,
      * 180), IPTS
10  RHO(1) = 0.
      GO TO 190
20  RHO(2) = .577350269189626E-00
      RHO(1) = - RHO(2)
      GO TO 190
30  RHO(3) = .774596669241483E-00
      RHO(1) = - RHO(3)
      RHO(2) = 0.
      GO TO 190
40  RHO(3) = .339981043584856E-00
      RHO(2) = - RHO(3)
      RHO(4) = .861136311594053E-00
      RHO(1) = - RHO(4)
      GO TO 190
50  RHO(4) = .538469310105683E-00
      RHO(2) = - RHO(4)
      RHO(5) = .906179845938684E-00
      RHO(1) = - RHO(5)
      RHO(3) = 0.
      GO TO 190
60  RHO(4) = .238619186083197E-00
      RHO(3) = - RHO(4)
      RHO(5) = .661209386466265E-00
      RHO(2) = - RHO(5)
      RHO(6) = .932469514203152E-00
      RHO(1) = - RHO(6)
      GO TO 190
70  RHO(5) = .405845151377397E-00
      RHO(3) = - RHO(5)
      RHO(6) = .74153118559394E-00
      RHO(2) = - RHO(6)
      RHO(7) = .949107912342759E-00
      RHO(1) = - RHO(7)
      RHO(4) = 0.
      GO TO 190
80  RHO(5) = .183434642495650E-00
      RHO(4) = - RHO(5)
      RHO(6) = .525532409916329E-00
      RHO(3) = - RHO(6)
      RHO(7) = .796668477413627E-00
      RHO(2) = - RHO(7)
      RHO(8) = .960289856497536E-00
      RHO(1) = - RHO(8)
      GO TO 190
90  RHO(5) = 0.
      RHO(6) = .324253423403809E-00
      RHO(7) = .613371432700590E-00
      RHO(8) = .83603107328636E-00
      RHO(9) = .968160239507826E-00
      DO 95 I=1,4

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95  RHO(1) = -RHO(10-1)
   GO TO 190
100  RHO( 6) = .118874338981831E-00
   RHO( 7) = .433395394129247E-00
   RHO( 8) = .674039568299024E-00
   RHO( 9) = .865063366888884E-00
   RHO(10) = .973906528517172E-00
   DO 105 I=1,5
   RHO(1) = -RHO(11-1)
105  GO TO 190
110  RHO( 6) = .0
   RHO( 7) = .269543155952315E-00
   RHO( 8) = .519096129206812E-00
   RHO( 9) = .730152005574049E-00
   RHO(10) = .887062599788095E-00
   RHO(11) = .978228658146057E-00
   DO 115 I=1,5
   RHO(1) = -RHO(12-1)
115  GO TO 190
120  RHO( 7) = .125233408511459E-00
   RHO( 8) = .36783198998180E-00
   RHO( 9) = .587317954286617E-00
   RHO(10) = .759902674194305E-00
   RHO(11) = .904117256370475E-00
   RHO(12) = .881560634246719E-00
   DO 125 I=1,6
   RHO(1) = -RHO(13-1)
125  GO TO 190
130  RHO( 7) = .0
   RHO( 8) = .230458315955135E-00
   RHO( 9) = .48492751036447E-00
   RHO(10) = .642349339440340E-00
   RHO(11) = .801578090733310E-00
   RHO(12) = .91759839922978E-00
   RHO(13) = .984183054718588E-00
   DO 135 I=1,6
   RHO(1) = -RHO(14-1)
135  GO TO 190
140  RHO( 8) = .108054948707344E-00
   RHO( 9) = .319112368927890E-00
   RHO(10) = .515248636358154E-00
   RHO(11) = .687292904811685E-00
   RHO(12) = .827201315069765E-00
   RHO(13) = .928434883663574E-00
   RHO(14) = .96628380666812E-00
   DO 145 I=1,7
   RHO(1) = -RHO(15-1)
145  GO TO 190
150  RHO( 8) = .0
   RHO( 9) = .201194093997435E-00
   RHO(10) = .394151347077563E-00
   RHO(11) = .570972172608539E-00
   RHO(12) = .72417731360170E-00
   RHO(13) = .848206583410427E-00
   RHO(14) = .93273392400706E-00
   RHO(15) = .987992518020485E-00
   DO 155 I=1,7
   RHO(1) = -RHO(16-1)
155  GO TO 190
160  RHO( 9) = .950125098376374E-01
   RHO(10) = .281603550779259E-00
   RHO(11) = .45801677657227E-00
   RHO(12) = .61787624402644E-00
   RHO(13) = .755404408355003E-00
   RHO(14) = .865631202387832E-00
   RHO(15) = .944575023073233E-00

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RHO(16) = .959400934981650E-00
DO 65 I=1,8
  J(1) = -RHO(17-1)
165 GO TO 190
170 RHO( 9) = .0
  RHO(10) = .178484181495848E-00
  RHO(11) = .351231763453876E-00
  RHO(12) = .512690537086477E-00
  RHO(13) = .657671159216691E-00
  RHO(14) = .781514003896801E-00
  RHO(15) = .880239153726986E-00
  RHO(16) = .950675521768768E-00
  RHO(17) = .990575475314417E-00
DO 175 I=1,8
  RHO(1) = -RHO(18-I)
175 GO TO 190
180 RHO(10) = .847750130417353E-01
  RHO(11) = .251886225691506E-00
  RHO(12) = .411751161462843E-00
  RHO(13) = .559770831073948E-00
  RHO(14) = .691687043060353E-00
  RHO(15) = .803704958972523E-00
  RHO(16) = .832602466497556E-00
  RHO(17) = .95823949571396E-00
  RHO(18) = .991585168420931E-00
DO 185 I=1,9
  RHO(1) = -RHO(19-I)
185 GO TO 190
C-----
C COMPUTE THE GAUSS-LEGENDRE COLLOCATION POINTS IN EACH SUBINTERVAL.
C-----
190 DO 195 I=1,NINT
  FAC = ( X(I-1) - X(1) ) * .5
  DO 195 J=1,IPTS
    NCOT = IPTS * (I-1) + J + 1
    XC(KNOT) = X(1) + FAC * ( RHO(J) + 1. )
    XC(1) = X(1)
    XC(NCPTS) = X(MINT+1)
    RETURN
C-----
C COMPUTE THE COLLOCATION POINTS TO BE AT THE POINTS WHERE THE BASIS
C FUNCTIONS ATTAIN THEIR MAXIMA. A BISECTION METHOD IS USED TO FIND
C THE POINTS TO MACHINE PRECISION. THIS PROCESS COULD BE SPEEDED UP
C BY USING A SECANT METHOD IF DESIRED.
C-----
200 ITOP = NCPTS - 1
  MFLAG = -2
  XC(1) = X(1)
  XC(NCPTS) = X(MINT+1)
  DO 240 I=2,ITOP
    XOLD = 1.E+20
    XL = XT(1)
    XR = XT(I+KORD)
    XNEW = .5 * (XL + XR)
    IF( XOLD .EQ. XNEW ) GO TO 240
    CALL INTERV(XT,NCPTS,XNEW,ILEFT,MFLAG)
    CALL BSPLVD(XT,KORD,XNEW,ILEFT,RHO,2)
    DO 220 J=1,KORD
      IF( 1.EQ. J + ILEFT - KORD ) GO TO 230
      CONTINUE
      XVAL = RHO(KORD+J)
      IF( XVAL .EQ. 0.0 ) XR = XNEW
      IF( XVAL .GT. 0.0 ) XL = XNEW
      IF( XVAL .LT. 0.0 ) XR = XNEW
      XOLD = XNEW
      GO TO 210
210
220
230
240 XC(I) = XR

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      RETURN
      EV
      SUBROUTINE BSPLVD ( XT, K, X, ILEFT, VNIKK, NDERIV )
      C-----
      C THIS SUBROUTINE IS PART OF THE B-SPLINE PACKAGE FOR THE STABLE
      C EVALUATION OF ANY B-SPLINE BASIS FUNCTION OR DERIVATIVE VALUE.
      C SEE REFERENCE BELOW.
      C
      C CALCULATES THE VALUE AND THE FIRST NDERIV-1 DERIVATIVES OF ALL
      C B-SPLINES WHICH DO NOT VANISH AT X. THE ROUTINE FILLS THE TWO-
      C DIMENSIONAL ARRAY VNIKK(J,IDERIV). J=IDERIV, ..., K WITH NONZERO
      C VALUES OF B-SPLINES OF ORDER K+1-IDERIV. IDERIV=NDERIV, ..., 1, BY
      C REPEATED CALLS TO BSPLVN.
      C
      C XT = PIECEWISE POLYNOMIAL BREAKPOINT SEQUENCE.
      C K = ORDER OF THE PIECEWISE POLYNOMIAL SPACE.
      C X = POINT AT WHICH THE B-SPLINE IS TO BE EVALUATED.
      C ILEFT = POINTER TO THE BREAKPOINT SEQUENCE.
      C VNIKK = TABLE OF B-SPLINE VALUES AND DERIVATIVES.
      C NDERIV = DETERMINES NUMBER OF DERIVATIVES TO BE GENERATED.
      C
      C REFERENCE
      C
      C DEBOOR, C.. PACKAGE FOR CALCULATING WITH B-SPLINES, SIAM J.
      C NUMER. ANAL., VOL. 14, NO. 3, JUNE 1977, PP. 441-472.
      C
      C PACKAGE ROUTINES CALLED... BSPLVN
      C USER ROUTINES CALLED... NONE
      C CALLED BY... COLPNT,INITAL,VALUES
      C FORTRAN FUNCTIONS USED... FLOAT,NMAXO
      C-----
      DIMENSION XT(1),VNIKK(K,NDERIV)
      DIMENSION A(20,20)
      KO = K + 1 - NDERIV
      CALL BSPLVN(XT,KO,1,X,ILEFT,VNIKK(NDERIV,NDERIV))
      IF (NDERIV.LE. 1) GO TO 120
      IDERIV = NDERIV
      DO 20 I=2,NDERIV
        IDERVM = IDERIV-1
        DO 10 J=IDERIV,K
          VNIKK(J-1,IDERVM) = VNIKK(J,IDERIV)
        IDERIV = IDERVM
        CALL BSPLVN(XT,0.2,X,ILEFT,VNIKK(IDERIV,IDERIV))
      20 CONTINUE
      DO 40 I=1,K
        DO 30 J=1,K
          A(I,J) = 0.
        A(I,1) = 1.
      KMD = K
      DO 110 M=2,NDERIV
        KMD = KMD - 1
        FKMD = FLOAT(KMD)
        I = ILEFT
        J = K
      50 JN1 = J-1
        IPKMD = I + KMD
        DIFF = XT(IPKMD) - XT(I)
        IF (JN1.EQ. 0) GO TO 80
        IF (DIFF.EQ. 0.) GO TO 70
        DO 60 L=1,J
          A(L,J) = (A(L,J) - A(L,J-1))/DIFF*FKMD
        J = JN1
        I = I - 1
      GO TO 50
      80 IF (DIFF.EQ. 0.) GO TO 90
        A(1,1) = A(1,1)/DIFF*FKMD
      90

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90 DO 110 I=1,K
   V = 0.
   ( LOW = MAXO(I,M)
     DO 100 J=JLOW,K
100   V = A(I,J)*VNIKK(J,M) + V
110   VNIKK(I,M) = V
120 RETURN
END
SUBROUTINE BSPLVN ( XT, JHIGH, INDEX, X, ILEFT, VNIKK )
-----
C THIS SUBROUTINE IS PART OF THE B-SPLINE PACKAGE FOR THE STABLE
C EVALUATION OF ANY B-SPLINE BASIS FUNCTION OR DERIVATIVE VALUE.
C SEE REFERENCE BELOW.
C
C CALCULATES THE VALUE OF ALL POSSIBLY NONZERO B-SPLINES AT THE
C POINT X OF ORDER MAX(JHIGH,(J+1)(INDEX-1)) FOR THE BREAKPOINT SEQ-
C UENCE XT. ASSUMING THAT XT(ILEFT) .LE. X .LE. XT(ILEFT+1), THE ROUT-
C INE RETURNS THE B-SPLINE VALUES IN THE ONE DIMENSIONAL ARRAY VNIKK.
C
C FOR DEFINITIONS OF CALLING ARGUMENTS SEE ABOVE AND BSPLVD.
C
C REFERENCE
C
C DEBOOR, C., PACKAGE FOR CALCULATING WITH B-SPLINES, SIAM J.
C NUMER. ANAL., VOL. 14, NO. 3, JUNE 1977, PP. 441-472.
C
C PACKAGE ROUTINES CALLED.. NONE
C USER ROUTINES CALLED.. NONE
C CALLED BY.. BSPLVD
C FORTRAN FUNCTIONS USED.. NONE
C
-----
DIMENSION XT(1),VNIKK(1)
DIMENSION DELTAM(20),DELTAP(20)
DATA J/1,DELTAM/20*0.E-00,DELTAP/20*0.E-00/
DO 10 J = 1
   VNIKK(1) = 1.
   IF (J .GE. JHIGH) GO TO 40
   20 IPU = ILEFT+J
     DELTAP(J) = XT(IPU) - X
     IMJP1 = ILEFT-J+1
     DELTAM(J) = X - XT(IMJP1)
     VMPREV = 0.
     JPI = J+1
     DO 30 L=1,J
        JPIML = JPI-L
        VM = VNIKK(L)/(DELTAP(L) + DELTAM(JPIML))
        VNIKK(L) = VM*DELTAP(L) + VMPREV
        VMPREV = VM*DELTAM(JPIML)
        VNIKK(JPI) = VMPREV
        J = JPI
   IF (J .LT. JHIGH) GO TO 20
40 RETURN
END
SUBROUTINE INTERV ( XT, LXT, X, ILEFT, MFLAG )
-----
C THIS SUBROUTINE IS PART OF THE B-SPLINE PACKAGE FOR THE STABLE
C EVALUATION OF ANY B-SPLINE BASIS FUNCTION OR DERIVATIVE VALUE.
C SEE REFERENCE BELOW.
C
C COMPUTES LARGEST ILEFT IN (1,LXT) SUCH THAT XT(ILEFT) .LE. X. THE
C PROGRAM STARTS THE SEARCH FOR ILEFT WITH THE VALUE OF ILEFT THAT WAS
C RETURNED AT THE PREVIOUS CALL (AND WAS SAVED IN THE LOCAL VARIABLE
C ILO) TO MINIMIZE THE WORK IN THE COMMON CASE THAT THE VALUE OF X ON
C THIS CALL IS CLOSE TO THE VALUE OF X ON THE PREVIOUS CALL. SHOULD
C THIS ASSUMPTION NOT BE VALID, THEN THE PROGRAM LOCATES ILO AND IHI

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C SUCH I XT(ILO) .LE. X .LT. XT(IMI) AND, ONCE THEY ARE FOUND, IS
C BISECTION TO FIND THE CORRECT VALUE FOR ILEFT. MFLAG IS AN ERROR FLAG.
C
C FOR DEFINITIONS OF CALLING ARGUMENTS SEE ABOVE AND BSPLVD.
C
C REFERENCE
C
C DEBOR, C., PACKAGE FOR CALCULATING WITH B-SPLINES, SIAM J.
C NUMER. ANAL., VOL. 14, NO. 3, JUNE 1977, PP. 441-472.
C
C PACKAGE ROUTINES CALLED... NONE
C USER ROUTINES CALLED... NONE
C CALLED BY... COLPNT, INITIAL VALUES
C FORTRAN FUNCTIONS USED... NONE
C-----
C DIMENSION XT(LXT)
C IF(MFLAG.EQ.-2) ILO = 1
C IMI = ILO + 1
C IF (IMI .LT. LXT) GO TO 20
C IF (X .GE. XT(LXT)) GO TO 110
C IF (LXT .LE. 1) GO TO 90
C ILO = LXT - 1
C GO TO 21
C 20 IF (X .GE. XT(IMI)) GO TO 40
C 21 IF (X .GE. XT(ILO)) GO TO 100
C-----
C NOW X .LT. XT(IMI). FIND LOWER BOUND.
C-----
C 30 ISTEP = 1
C 31 IMI = ILO
C ILO = IMI - ISTEP
C IF (ILO .LE. 1) GO TO 35
C IF (X .GE. XT(ILO)) GO TO 50
C ISTEP = ISTEP*2
C GO TO 31
C 35 ILO = 1
C IF (X .LT. XT(1)) GO TO 90
C GO TO 50
C-----
C NOW X .GE. XT(ILO). FIND UPPER BOUND.
C-----
C 40 ISTEP = 1
C 41 ILO = IMI
C IMI = ILO + ISTEP
C IF (IMI .GE. LXT) GO TO 45
C IF (X .LT. XT(IMI)) GO TO 50
C ISTEP = ISTEP*2
C GO TO 41
C 45 IF (X .GE. XT(LXT)) GO TO 110
C IMI = LXT
C-----
C NOW XT(ILO) .LE. X .LT. XT(IMI). NARROW THE INTERVAL.
C-----
C 50 MIDDLE = (ILO + IMI)/2
C IF (MIDDLE .EQ. ILO) GO TO 100
C-----
C NOTE.. IT IS ASSUMED THAT MIDDLE = ILO IN CASE IMI = ILO+1.
C-----
C IF (X .LT. XT(MIDDLE)) GO TO 53
C ILO = MIDDLE
C GO TO 50
C 53 IMI = MIDDLE
C GO TO 50
C-----
C SET OUTPUT AND RETURN.
C-----

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90 KFLAG = -1
   IL = 1
   IF (N) = 1
100 MF = 0
   ILEFT = ILO
   RETURN
110 KFLAG = 1
   ILEFT = LAT
   RETURN
   END
SUBROUTINE STIFIB (NO, Y, YMAX, ERROR, SAVE1, SAVE2, SAVE3,
                  PM, IDIV, WORK, IWORK)
C-----
C STIFIB PERFORMS ONE STEP OF THE INTEGRATION OF AN INITIAL VALUE
C PROBLEM FOR A SYSTEM OF ORDINARY DIFFERENTIAL EQUATIONS OF THE FORM,
C  $A(Y,T) \cdot (DY/DT) = G(Y,T)$ , WHERE  $Y = (Y(1), Y(2), \dots, Y(N))$ .
C STIFIB IS FOR USE WHEN THE MATRICES A AND DG/DY HAVE BANDED OR NEARLY
C BANDED FORM. THE DEPENDENCE OF A(Y,T) ON Y IS ASSUMED TO BE WEAK.
C-----
C REFERENCE
C HINDMARSH, A.C., PRELIMINARY DOCUMENTATION OF GEARIB, SOLUTION
C OF IMPLICIT SYSTEMS OF ORDINARY DIFFERENTIAL EQUATIONS WITH
C BANDED JACOBIANS, LAWRENCE LIVERMORE LAB, UCID-30130, FEBRUARY
C 1976.
C COMMUNICATION WITH STIFIB IS DONE WITH THE FOLLOWING VARIABLES..
C Y AN NO BY LMAX ARRAY CONTAINING THE DEPENDENT VARIABLES
C AND THEIR SCALED DERIVATIVES. LMAX IS 13 FOR THE ADAMS
C METHODS AND 6 FOR THE GEAR METHODS. LMAX - 1 = MAXDER
C IS THE MAXIMUM ORDER AVAILABLE. SEE SUBROUTINE COSET.
C Y(1..N) CONTAINS THE J-TH DERIVATIVE OF Y(1), SCALED BY
C  $H^{(J+1)/FACTOIAL(J)}$  (J = 0, 1, ..., NO).
C NO A CONSTANT INTEGER. GE. N. USED FOR DIMENSIONING PURPOSES.
C T THE INDEPENDENT VARIABLE. T IS UPDATED ON EACH STEP TAKEN.
C H THE STEP SIZE TO BE ATTEMPTED ON THE NEXT STEP.
C H IS ALTERED BY THE ERROR CONTROL ALGORITHM DURING THE
C PROBLEM. H CAN BE EITHER POSITIVE OR NEGATIVE, BUT ITS
C SIGN MUST REMAIN CONSTANT THROUGHOUT THE PROBLEM.
C THE MINIMUM AND MAXIMUM ABSOLUTE VALUE OF THE STEP SIZE
C TO BE USED FOR THE STEP THESE MAY BE CHANGED AT ANY
C TIME, BUT WILL NOT TAKE EFFECT UNTIL THE NEXT H CHANGE.
C EPS THE RELATIVE ERROR BOUND. SEE DESCRIPTION IN PDECOL.
C N THE UNIT ROUNDOFF OF THE MACHINE.
C MF THE NUMBER OF FIRST-ORDER DIFFERENTIAL EQUATIONS.
C KFLAG THE METHOD FLAG. SEE DESCRIPTION IN PDECOL.
C A COMPLETION CODE WITH THE FOLLOWING MEANINGS..
C 0 THE STEP WAS SUCCESSFUL.
C -1 THE REQUESTED ERROR COULD NOT BE ACHIEVED
C WITH  $ABS(H) \geq HMIN$ .
C -2 THE REQUESTED ERROR IS SMALLER THAN CAN
C BE HANDLED FOR THIS PROBLEM.
C -3 CORRECTOR CONVERGENCE COULD NOT BE
C ACHIEVED FOR  $ABS(H) \geq HMIN$ .
C -4 SINGULAR A-MATRIX ENCOUNTERED.
C ON A RETURN WITH KFLAG NEGATIVE, THE VALUES OF T AND
C THE Y ARRAY ARE AS OF THE BEGINNING OF THE LAST
C STEP, AND H IS THE LAST STEP SIZE ATTEMPTED.
C JUSTART AN INTEGER USED ON INPUT AND OUTPUT.
C ON INPUT, IT HAS THE FOLLOWING VALUES AND MEANINGS..
C 0 PERFORM THE FIRST STEP.
C .GT.0 TAKE A NEW STEP CONTINUING FROM THE LAST.
C .LT.0 TAKE THE NEXT STEP WITH A NEW VALUE OF
C H, EPS, N, AND/OR MF.
C ON EXIT, JUSTART IS NO, THE CURRENT ORDER OF THE METHOD.

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C YMA1 AN ARRAY OF N ELEMENTS WITH WHICH THE ESTIMATED LOCAL
C ERRORS IN Y ARE COMPARED.
C ERROR AN ARRAY OF N ELEMENTS. ERROR(1)/TQ(2) IS THE ESTIMATED
C ONE-STEP ERROR IN Y(1).
C SAVE1,SAVE2,SAVE3 THREE WORKING STORAGE ARRAYS, EACH OF LENGTH N.
C PW A BLOCK OF LOCATIONS USED FOR THE CHORD ITERATION
C MATRIX SEE DESCRIPTION IN PDECOL.
C IPIV AN INTEGER ARRAY OF LENGTH N FOR PIVOT INFORMATION.
C ML,MLU THE LOWER AND UPPER HALF BANDWIDTHS, RESPECTIVELY, OF
C THE CHORD ITERATION MATRIX. SEE DESCRIPTION IN PDECOL.
C WORK,WORK1 WORKING ARRAYS WHICH ARE USED TO PASS APPROPRIATE
C ARRAYS TO OTHER SUBROUTINES.
C
C PACKAGE ROUTINES CALLED.. COSET,DIFFUN,PSETIB,RES,SOLB
C USER ROUTINES CALLED.. NONE
C CALLED BY.. PDECOL
C FORTRAN FUNCTIONS USED.. ABS,AMAX1,AMIN1,FLOAT
C
C DIMENSION Y(N0,1),YMAX(N0),ERROR(N0),SAVE1(N0),SAVE2(N0),
C SAVE3(N0),P4(1),IPIV(1),KORK(1),IWORK(1)
C COMMON /SIZES/ NINT,KORD,ACC,NPOE,SCRIPTS,NEON,LOUAD
C COMMON /ISTART/ IWT,IW2,IW3,IW4,IW5,IW6,IW7,IW8,IW9,IW10,IW11,
C IW12,IW13,IW14,IW15,IW16,IW17,IW18
C COMMON /GEAR1/ T,M,HMIN,HMAX,EPS,UROUND,N,MF,AFLAG,JSTART
C COMMON /GEAR2/ EPSJ,RO,ML,MM,NMT,NOML,NOW
C COMMON /GEAR3/ HUSED,NQUSED,NSTEP,NFE,NJE
C COMMON /OPTION/ NOGAUS,MAXDER
C DIMENSION EL(13),TQ(4)
C DATA EL(2)/1.,.OLDLO/1.,.TQ(1)/0.,.IER/0/
C KFLAG = 0
C TOLD = T
C IF (JSTART .GT. 0) GO TO 200
C IF (JSTART .NE. 0) GO TO 120
C
C ON THE FIRST CALL, THE ORDER IS SET TO 1 AND THE INITIAL YDOT IS
C CALCULATED. RMAX IS THE MAXIMUM RATIO BY WHICH H CAN BE INCREASED
C IN A SINGLE STEP. IT IS INITIALLY 1.E4 TO COMPENSATE FOR THE SMALL
C INITIAL H, BUT THEN IS NORMALLY EQUAL TO 10. IF A FAILURE
C OCCURS (IN CORRECTOR CONVERGENCE OR ERROR TEST), RMAX IS SET AT 2
C FOR THE NEXT INCREASE.
C
C H0 = 1
C IER = 0
C CALL DIFFUN (N, T, Y, SAVE1, IER, PW, IPIV, WORK, IWORK)
C IF (IER .NE. 0) GO TO 685
C DO 110 I = 1, N
C 110 Y(I,2) = H*SAVE1(I)
C METH = MF/10
C MITER = MF - 10*METH
C L = 2
C IDOUB = 3
C RMAX = 1.E+04
C RC = 0.
C CRATE = 1.
C EPSOLD = EPS
C HOLD = H
C MOLD = MF
C NOLD = N
C NSTEP = 0
C NSTEPJ = 0
C NFE = 0
C NJE = 1
C IRET = 3
C GO TO 130
C
C IF THE CALLING PROGRAM WOULD WANT TO CALL TO SET

```

C THE COEFFICIENTS OF THE METHOD. IF THE CALLER HAS CHANGED  
C N, E, OR METH, THE CONSTANTS E, EDN, EUP, AND BND MUST BE RE-  
C E IS A COMPARISON FOR ERRORS OF THE CURRENT ORDER NO. EUP IS  
C TO TEST FOR INCREASING THE ORDER, EDN FOR DECREASING THE ORDER.  
C BND IS USED TO TEST FOR CONVERGENCE OF THE CORRECTOR ITERATES.  
C IF THE CALLER HAS CHANGED M, Y MUST BE RESCALED.  
C IF M OR METH HAS BEEN CHANGED, IDOUB IS RESET TO L + 1 TO PREVENT  
C FURTHER CHANGES IN H FOR THAT MANY STEPS.

120 IF (MF .EQ. MFOLD) GO TO 150

MEO = METH,  
MIO = MITER,  
METH = MF/10  
MITER = MF - 10\*METH  
MFOLD = MF  
IF (MITER .NE. MIO) IWEVAL = MITER  
IF (METH .EQ. MEO) GO TO 150  
IDOUB = L + 1  
IRET = 1

130 CALL COSET (METH, NQ, EL, TQ)  
LMAX = MAXDER + 1  
RC = RC-EL(1)/OLDLO

OLDLO = EL(1)  
140 FN = FDATE(N)  
EDN = FN-(TQ(1)-EPS)\*\*2  
E = FN-(TQ(2)-EPS)\*\*2  
EUP = FN-(TQ(3)-EPS)\*\*2  
BND = FN-(TQ(4)-EPS)\*\*2

GO TO (160, 170, 200), IRET  
150 IF ((EPS .EQ. EPSOLD) .AND. (N .EQ. NOLD)) GO TO 160  
EPSOLD = EPS  
NOLD = N  
IRET = 1

GO TO 140  
160 IF (H .EQ. HOLD) GO TO 200  
RH = H/HOLD  
H = HOLD  
IREDO = 3

GO TO 175  
170 RH = AMAX1(RH, HMIN/ ABS(H))  
175 RH = AMIN1(RH, HMAX/ ABS(H), RMAX)  
R1 = 1.

DO 180 J = 2, L  
R1 = R1+RH  
DO 180 I = 1, N  
Y(I, J) = Y(I, J)+R1

180 H = H+RH  
RC = RC+RH  
IDOUB = L + 1  
IF (IREDO .EQ. 0) GO TO 690

C THIS SECTION COMPUTES THE PREDICTED VALUES BY EFFECTIVELY  
C MULTIPLYING THE Y ARRAY BY THE PASCAL TRIANGLE MATRIX.  
C RC IS THE RATIO OF NEW TO OLD VALUES OF THE COEFFICIENT H\*EL(1).  
C WHEN RC DIFFERS FROM 1 BY MORE THAN 30 PERCENT, OR THE CALLER HAS  
C CHANGED MITER, IWEVAL IS SET TO MITER TO FORCE PW TO BE UPDATED.  
C IN ANY CASE, PW IS UPDATED AT LEAST EVERY 40-TH STEP.  
C PW IS THE CHORD ITERATION MATRIX A - H\*EL(1)\*(DG/DY).

200 IF (ABS(RC-1.) .GT. 0.3) IWEVAL = MITER  
IF (NSTEP .GE. NSTEPJ+40) IWEVAL = MITER  
T = T + H  
DO 210 J1 = 1, NQ  
DO 210 J2 = J1, NQ  
J = (NQ + J1) - J2  
DO 210 I = 1, N

```

210      Y(I,J) = Y(I,J) + Y(I,J+1)
C-----
C UP TO J-CORRECTOR ITERATIONS ARE TAKEN. A CONVERGENCE TEST IS
C MADE ON THE R.M.S. NORM OF EACH CORRECTION, USING BND, WHICH
C IS DEPENDENT ON EPS. THE SUM OF THE CORRECTIONS IS ACCUMULATED
C IN THE VECTOR ERROR(I). THE Y ARRAY IS NOT ALTERED IN THE CORRECTOR
C LOOP. THE UPDATED Y VECTOR IS STORED TEMPORARILY IN SAVE1.
C THE UPDATED -H*YDOT IS STORED IN SAVE2.
C-----
220 DO 230 I = 1, N
    SAVE2(I) = Y(I,2)
230   ERROR(I) = 0.
    M = 0
    CALL RES (T, H, Y, SAVE2, SAVE3, NPDE, NCPTS, WORK(IW1), IWORK,
    * WORK, WORK(IW14), WORK(IW15), WORK(IW16), WORK(IW3), WORK(IW9))
    NFE = NFE + 1
    IF (IWEAL .LE. 0) GO TO 350
C-----
C IF INDICATED, THE MATRIX PW IS REEVALUATED BEFORE STARTING THE
C CORRECTOR ITERATION. IWEAL IS SET TO 0 AS AN INDICATOR
C THAT THIS HAS BEEN DONE. PW IS COMPUTED AND PROCESSED IN PSETIB.
C-----
    IWEAL = 0
    RC = 1.
    NFE = NFE + 1
    NSTEPJ = NSTEP
    CON = -H*EL(1)
    CALL PSETIB (Y, PW, NO, CON, WITER, IER, WORK(IW1), IWORK,
    * WORK(IW3), WORK(IW9), SAVE2, IP1V, YMAX, WORK(IW11), WORK(IW12),
    * WORK(IW13), WORK(IW16), WORK(IW14), WORK(IW15), WORK, NPDE)
    IF (IER .NE. 0) GO TO 420
C-----
C COMPUTE THE CORRECTOR ERROR, R SUB M, AND SOLVE THE LINEAR SYSTEM
C WITH THAT AS RIGHT-HAND SIDE AND PW AS COEFFICIENT MATRIX.
C USING THE LU DECOMPOSITION OF PW.
C-----
350 CALL SOLB (NO, N, ML, MU, PW, SAVE3, IP1V)
370 D = 0.
DO 380 I = 1, N
    ERROR(I) = ERROR(I) + SAVE3(I)
D = D + (SAVE3(I)/YMAX(I))*2
SAVE1(I) = Y(I,1) + EL(1)*ERROR(I)
380   SAVE2(I) = Y(I,2) + ERROR(I)
C-----
C TEST FOR CONVERGENCE. IF M.GT.0, AN ESTIMATE OF THE CONVERGENCE
C RATE CONSTANT IS STORED IN CRATE, AND THIS IS USED IN THE TEST.
C-----
400 IF (M .NE. 0) CRATE = AMAX1(.9*CRATE,D/D1)
    IF (10*AMIN1(1,EO/2.*CRATE)) .LE. BND) GO TO 450
    D1 = D
    M = M + 1
    IF (M .EQ. 3) GO TO 410
    CALL RES(T, H, SAVE1, SAVE2, SAVE3, NPDE, NCPTS, WORK(IW1), IWORK,
    * WORK, WORK(IW14), WORK(IW15), WORK(IW16), WORK(IW3), WORK(IW9))
    GO TO 350
C-----
C THE CORRECTOR ITERATION FAILED TO CONVERGE IN 3 TRIES.
C IF THE MATRIX PW IS NOT UP TO DATE, IT IS REEVALUATED FOR THE
C NEXT TRY. OTHERWISE THE Y ARRAY IS RETRACTED TO ITS VALUES
C BEFORE PREDICTION, AND H IS REDUCED, IF POSSIBLE. IF NOT, A
C NO-CONVERGENCE EXIT IS TAKEN.
C-----
410 NFE = NFE + 2
    IF (IWEAL .EQ. -1) GO TO 440
420 T = TOLD
    RMAX = 2.

```

```

DO 430 J1 = 1,NQ
(
  430 J2 = J1,NQ
    J = (NQ + J1) - J2
    DO 430 I = 1,N
      Y(I,J) = Y(I,J) - Y(I,J+1)
    430 IF (ABS(H) .LE. HMIN*1.00001) GO TO 680
    RH = .25
    IREDO = 1
    GO TO 170
  440 IAEVAL = MITER
    GO TO 220
)
C THE CORRECTOR HAS CONVERGED. IAEVAL IS SET TO -1 TO SIGNAL
C THAT WE MAY NEED UPDATING ON SUBSEQUENT STEPS. THE ERROR TEST
C IS MADE AND CONTROL PASSES TO STATEMENT 500 IF IT FAILS.
C
450 IAEVAL = -1
NFE = NFE + M
D = 0.
DO 460 I = 1,N
  460 D = D + (ERROR(I)/YMAX(I))**2
  IF (D .GT. E) GO TO 500
C
C AFTER A SUCCESSFUL STEP, UPDATE THE Y ARRAY.
C CONSIDER CHANGING H IF IDOUB = 1. OTHERWISE DECREASE IDOUB BY 1.
C IF IDOUB IS THEN 1 AND NO .LT. MAXDER THEN ERROR IS SAVED FOR
C USE IN A POSSIBLE ORDER INCREASE ON THE NEXT STEP.
C IF A CHANGE IN H IS CONSIDERED, AN INCREASE OR DECREASE IN ORDER
C BY ONE IS CONSIDERED ALSO. A CHANGE IN H IS MADE ONLY IF IT IS BY A
C FACTOR OF AT LEAST 1.1. IF NOT, IDOUB IS SET TO 10 TO PREVENT
C TESTING FOR THAT MANY STEPS.
C
KFLAG = 0
IREDO = 0
NSTEP = NSTEP + 1
HUSED = H
HUSED = NO
DO 470 J = 1,L
  DO 470 I = 1,N
    Y(I,J) = Y(I,J) + EL(J)*ERROR(I)
  470 IF (IDOUB .EQ. 1) GO TO 520
  IDOUB = IDOUB - 1
  IF (IDOUB .GT. 1) GO TO 700
  IF (L .EQ. LMAX) GO TO 700
  DO 490 I = 1,N
    490 Y(I,LMAX) = ERROR(I)
  GO TO 700
C
C THE ERROR TEST FAILED. KFLAG KEEPS TRACK OF MULTIPLE FAILURES.
C RESTORE T AND THE Y ARRAY TO THEIR PREVIOUS VALUES, AND PREPARE
C TO TRY THE STEP AGAIN. COMPUTE THE OPTIMUM STEP SIZE FOR THIS OR
C ONE LOWER ORDER.
C
500 KFLAG = KFLAG + 1
T = TOLD
DO 510 J1 = 1,NQ
  DO 510 J2 = J1,NQ
    J = (NQ + J1) - J2
    DO 510 I = 1,N
      Y(I,J) = Y(I,J) - Y(I,J+1)
    510 RMAX = 2.
    IF (ABS(H) .LE. HMIN*1.00001) GO TO 660
    IF (KFLAG .LE. -3) GO TO 640
    IREDO = 2
    PR3 = 1.-E+20
    GO TO 540

```

```

C-----
C REGARDS OF THE SUCCESS OR FAILURE OF THE STEP, FACTORS
C PR1, PR2, AND PR3 ARE COMPUTED, BY WHICH H COULD BE DIVIDED
C AT ORDER NO - 1, ORDER NO, OR ORDER NO + 1, RESPECTIVELY.
C IN THE CASE OF FAILURE, PR3 = 1.E20 TO AVOID AN ORDER INCREASE.
C THE SMALLEST OF THESE IS DETERMINED AND THE NEW ORDER CHOSEN
C ACCORDINGLY. IF THE ORDER IS TO BE INCREASED, WE COMPUTE ONE
C ADDITIONAL SCALED DERIVATIVE.
C-----
520 PR3 = 1.E+20
   IF (L.EQ. LMAX) GO TO 540
   D1 = 0.
   DO 530 I = 1,N
     D1 = D1 + ((ERROR(I) - Y(I,LMAX))/YMAX(I))**2
   EN3 = .5/ FLOAT(L+1)
   PR3 = ((D1/EUP)**EN3)*1.4 + 1.4E-06
540 EN2 = .5/ FLOAT(L)
   PR2 = ((D1/E)**EN2)*1.2 + 1.2E-06
   PR1 = 1.E+20
   IF (NQ.EQ. 1) GO TO 560
   D = 0.
   DO 550 I = 1,N
     D = D + (Y(I,L)/YMAX(I))**2
   EN1 = .5/ FLOAT(NQ)
   PR1 = ((D/EDN)**EN1)*1.3 + 1.3E-06
560 IF (PR2.LE. PR3) GO TO 570
   IF (PR3.LT. PR1) GO TO 590
   GO TO 580
570 IF (PR2.GT. PR1) GO TO 580
   NEWQ = NQ
   RH = 1./PR2
   GO TO 620
580 NEWQ = NQ - 1
   RH = 1./PR1
   GO TO 620
590 NEWQ = L
   RH = 1./PR3
   IF (RH.LT. 1.1) GO TO 610
   DO 600 I = 1,N
     Y(I,NEWQ+1) = ERROR(I)*EL(L)/ FLOAT(L)
   GO TO 630
610 ILOCB = 10
   GO TO 700
620 IF ((KFLAG.EQ. 0) .AND. (RH.LT. 1.1)) GO TO 610
C-----
C IF THERE IS A CHANGE OF ORDER, RESET NQ, L, AND THE COEFFICIENTS.
C IN ANY CASE H IS RESET ACCORDING TO RH AND THE Y ARRAY IS RESCALED.
C THEN EXIT FROM 690 IF THE STEP WAS OK, OR REDO THE STEP OTHERWISE.
C-----
   IF (NEWQ.EQ. NQ) GO TO 170
630 NQ = NEWQ
   L = NQ + 1
   IRET = 2
   GO TO 130
C-----
C CONTROL REACHES THIS SECTION IF 3 OR MORE FAILURES HAVE OCCURRED.
C IT IS ASSUMED THAT THE DERIVATIVES THAT HAVE ACCUMULATED IN THE
C Y ARRAY HAVE ERRORS OF THE WRONG ORDER. HENCE THE FIRST
C DERIVATIVE IS RECOMPUTED, AND THE ORDER IS SET TO 1. THEN
C H IS REDUCED BY A FACTOR OF 10, AND THE STEP IS RETRIED.
C AFTER A TOTAL OF 7 FAILURES, AN EXIT IS TAKEN WITH KFLAG = -2.
C-----
640 IF (KFLAG.EQ. -7) GO TO 670
   RH = .1
   RH = AMAX1(HMIN/ ABS(H),RH)
   U = -U

```

```

IF = 0
C DIFFUN (N, T, Y, SAVE1, IER, PW, IPIV, WORK, IWORK) (
IF ( IER.NE. 0 ) GO TO 685
NJE = NJE + 1
DO 650 I = 1,N
650 Y(I,2) = H*SAVE1(I)
I*EVAL = MITER
IDUB = 10
IF (NQ.EQ. 1) GO TO 200
NQ = 1
L = 2
IRET = 3
GO TO 130
-----
C ALL RETURNS ARE MADE THROUGH THIS SECTION. H IS SAVED IN HOLD
C TO ALLOW THE CALLER TO CHANGE H ON THE NEXT STEP.
C
660 KFLAG = -1
GO TO 700
670 KFLAG = -2
GO TO 700
680 KFLAG = -3
GO TO 700
685 KFLAG = -4
GO TO 700
690 RNAX = 10.
700 HOLD = H.
JSTART = NQ
RETURN
END
* SUBROUTINE GFUN (T,C,UOOT,NPDE,NCPTS,A,BC,DBDU,DBDUX,DZDT,
XC,UVAL,ILEFT)
-----
C CALLING ARGUMENTS ARE DEFINED BELOW AND IN PDECOL.
C
C SUBROUTINE GFUN COMPUTES THE FUNCTION UOOT=G(C,T). THE RIGHT-
C HAND SIDE OF THE SEMI-DISCRETE APPROXIMATION TO THE ORIGINAL
C SYSTEM OF PARTIAL DIFFERENTIAL EQUATIONS AND UPDATES THE BOUNDARY
C CONDITION INFORMATION.
C
C PACKAGE ROUTINES CALLED.. EVAL
C USER ROUTINES CALLED.. BNDRY,F
C CALLED BY.. DIFFUN,PSETIB,RES
C FORTRAN FUNCTIONS USED.. NONE
C
DIMENSION C(NPDE,NCPTS),UOOT(NPDE,NCPTS)
DIMENSION A(1),BC(NPDE,NPDE,4),XC(1),UVAL(NPDE,3),ILEFT(1)
DIMENSION DZDT(NPDE),DBDU(NPDE,NPDE),DBDUX(NPDE,NPDE)
COMMON /SIZES/ NINT,KORD,IDUM(4),IQDAD
DO 10 K=1,4
DO 10 J=1,NPDE
DO 10 I=1,NPDE
BC(I,J,K) = 0.0
10 CONTINUE
C
C UPDATE THE LEFT BOUNDARY VALUES. SAVE LEFT BOUNDARY CONDITION
C INFORMATION IN THE FIRST 2*NPDE*NPDE LOCATIONS OF BC.
C
C NOTE.. UVAL(K,1) = U(K), UVAL(K,2) = UX(K), AND UVAL(K,3) = UXX(K).
C
CALL EVAL(1,NPDE,C,UVAL,A,ILEFT)
CALL BNDRY(T,XC(1),UVAL,UVAL(1,2),DBDU,DBDUX,DZDT,NPDE)
CALL F(T,XC(1),UVAL,UVAL(1,2),UVAL(1,3),UOOT,NPDE)
ILIM = KORD + 2
DO 30 K=1,NPDE
BC(K,K,1) = 1.

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      DBDU(K,K) .EQ. 0.0 .AND. DBDUX(K,K) .EQ. 0.0 ) GO ( 30
      DO 20 J=1,NPDE
        BC(K,J,2) = A(ILIM) * DBDUX(K,J)
        BC(K,J,1) = DBDU(K,J) - BC(K,J,2)
      20 CONTINUE
      30 CONTINUE
C-----
C MAIN LOOP TO FORM RIGHT SIDE OF ODES AT THE COLLOCATION POINTS.
C-----
      ILIM = NCPTS - 1
      DO 40 I=2,ILIM
        CALL EVAL(I,NPDE,C,UVAL,A,ILEFT)
        CALL F(T,XC(I),UVAL,UVAL(1,2),UVAL(1,3),YDOT(1,1),NPDE)
      40 CONTINUE
C-----
C UPDATE THE RIGHT BOUNDARY VALUES. SAVE THE RIGHT BOUNDARY CONDITION
C INFORMATION IN THE LAST 2*NPDE-NPDE LOCATIONS IN BC.
C-----
      CALL EVAL(NCPTS,NPDE,C,UVAL,A,ILEFT)
      CALL F(T,XC(NCPTS),UVAL,UVAL(1,2),UVAL(1,3),YDOT(1,NCPTS),NPDE)
      CALL BNDRY(T,AC(NCPTS),UVAL,UVAL(1,2),DBDU,DBDUX,DZDT,NPDE)
      ILIM = NCPTS - 3 * KORD - KORD - 1
      DO 60 K=1,NPDE
        BC(K,K,4) = 1.
        IF( DBDU(K,K) .EQ. 0.0 .AND. DBDUX(K,K) .EQ. 0.0 ) GO TO 60
        YDOT(K,NCPTS) = DZDT(K)
        DO 50 J=1,NPDE
          BC(K,J,3) = A(ILIM) * DBDUX(K,J)
          BC(K,J,4) = DBDU(K,J) - BC(K,J,3)
        50 CONTINUE
        60 CONTINUE
      END
      SUBROUTINE EVAL(ICPT,NPDE,C,UVAL,A,ILEFT)
C-----
C CALLING ARGUMENTS ARE DEFINED BELOW AND IN PDECOL.
C-----
C SUBROUTINE EVAL EVALUATES U(K), UX(K), AND UXX(K), K=1 TO NPDE,
C AT THE COLLOCATION POINT WITH INDEX ICPT USING THE VALUES OF
C THE BASIS FUNCTION COEFFICIENTS IN C AND THE BASIS FUNCTION VALUES
C STORED IN A. THE RESULTS ARE STORED IN UVAL AS FOLLOWS..
C UVAL(K,1) = U(K), UVAL(K,2) = UX(K), AND UVAL(K,3) = UXX(K).
C-----
C PACKAGE ROUTINES CALLED.. NONE
C USER ROUTINES CALLED.. NONE
C CALLED BY.. GFUN,PDECOL,PSETIB
C FORTRAN FUNCTIONS USED.. NONE
C-----
      DIMENSION C(NPDE,1),UVAL(NPDE,3),A(1),ILEFT(1)
      COMMON /SIZES/ NINT,KORD,IDUM(5)
      IK = ILEFT(ICPT) - KORD
      IC = 3*KORD-(ICPT-1)
      DO 10 M=1,3
        ICC = IC + KORD*(M-1)
        DO 10 J=1,NPDE
          UVAL(J,M) = 0.
          DO 10 I=1,KORD
            UVAL(J,M) = UVAL(J,M) + C(J,I+IK)*A(I+ICC)
          10 CONTINUE
        RETURN
      END
      SUBROUTINE DIFFUN (N, T, Y, YDOT, IER, PW, IPIV, WORK, IWORK)
C-----
C CALLING ARGUMENTS ARE DEFINED BELOW AND IN PDECOL.
C-----

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C THIS ROUTINE COMPUTES YDOT = A(Y,T) + G(Y,T) BY USE OF
C THE ROUTINES GFUN, ADDA, DECB, AND SOLB.
C
C PACKAGE ROUTINES CALLED.. ADDA,DECB,GFUN,SOLB
C USER ROUTINES CALLED.. NONE
C CALLED BY.. STIFIB
C FORTRAN FUNCTIONS USED.. NONE
C-----
      DIMENSION Y(N),YDOT(N),PA(1),IPIV(1),WORK(1),IWORK(1)
      COMMON /GEAR9/ EPSJ,R0,ML,MU,NM1,NOML,NOW
      COMMON /SIZES/ NINT,KORD,NPDE,NCPDS,NEQN,IQUAD
      COMMON /ISTART/ I1,I2,I3,I4,I5,I6,I7,I8
      CALL GFUN (T, Y, YDOT, NPDE, NCPDS, WORK(I1), WORK(I14),
      *          WORK(I15), WORK(I16), WORK(I13), WORK(I19), IWORK)
      DO 10 I = 1,NOM
      *          PM(I) = 0.
      *
      *          NO = NM1 + 1
      *          CALL ADDA (PA, NO, WORK(I1), IWORK, WORK, NPDE)
      *          CALL DECB (NO, N, ML, MU, PA, IPIV, IER)
      *          IF ( IER.NE. 0 ) RETURN
      *          CALL SOLB (NO, N, ML, MU, PA, YDOT, IPIV)
      *          RETURN
      *          END
      *
      *          SUBROUTINE ADDA(PA,NO,A,LEFT,BC,NPDE)
      *
      *          CALLING ARGUMENTS ARE DEFINED BELOW AND IN PDECOL AND STIFIB.
      *
      *          SUBROUTINE ADDA ADDS THE MATRIX A TO THE MATRIX STORED IN PA IN
      *          BAND FORM. PA IS STORED BY DIAGONALS WITH THE LOWERMOST DIAGONAL
      *          STORED IN THE FIRST COLUMN OF THE ARRAY.
      *
      *          PACKAGE ROUTINES CALLED.. NONE
      *          USER ROUTINES CALLED.. NONE
      *          CALLED BY.. DIFFUN,PSETIB
      *          FORTRAN FUNCTIONS USED.. NONE
      *
      *          DIMENSION PA(N0,1),A(1),LEFT(1),BC(NPDE,NPDE,4)
      *          COMMON /SIZES/ NINT,KORD,NCC,NPD,NCPDS,NEQN,IQUAD
      *
      *          C ADD THE BOUNDARY CONDITION PORTIONS OF THE A MATRIX TO PA ( THE FIRST
      *          C AND LAST BLOCK ROWS).
      *
      *          ICOL = (LEFT(1) + IQUAD - 1) * NPDE
      *          DO 10 I=1,NPDE
      *             IBOT = NEQN - NPDE + I
      *             DO 10 J=1,NPDE
      *                INO = ICOL + J - I
      *                PA(I,INO) = PA(I,INO) + BC(I,J,1)
      *                PA(I,INO+NPDE) = PA(I,INO+NPDE) + BC(I,J,2)
      *                PA(IBOT,INO-NPDE) = PA(IBOT,INO-NPDE) + BC(I,J,3)
      *                PA(IBOT,INO) = PA(IBOT,INO) + BC(I,J,4)
      *
      *          10 CONTINUE
      *
      *          C UPDATE THE REMAINING ROWS OF PA BY ADDING THE APPROPRIATE VALUES
      *          C IN A TO PA.
      *
      *          INO = NCPDS - 1
      *          DO 20 I=2,INO
      *             I1 = (I-1) * NPDE
      *             I2 = (I-1) * KORD + 3
      *             ICOL = ILEFT(1) - I + IQUAD - 1
      *             DO 20 J=1,KORD
      *                J1 = (ICOL+J) * NPDE
      *                J2 = I2 + J
      *                DO 20 JJ=1,NPDE

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DO 30 I=1,NPDE
  30 ( YDUXX(I,J) = ( DFDUXX(I,J) - SAVE2(I+10) ) * RINV (
    X(J) = UC
  40 CONTINUE
    RETURN
  END
SUBROUTINE INTERP (TOUT, Y, NO, Y0)
C-----
C CALL'G ARGUMENTS ARE DEFINED BELOW AND IN STIFB
C
C SUBROUTINE INTERP COMPUTES INTERPOLATED VALUES OF THE DEPENDENT
C VARIABLE Y AND STORES THEM IN Y0. THE INTERPOLATION IS TO THE
C POINT T = TOUT, AND USES THE NORDSTROM HISTORY ARRAY Y, AS FOLLOWS..
C
C NO
C Y0(I) = SUM Y(I,J+1)*S**J.
C J=0
C
C WHERE S = -(T-TOUT)/H.
C
C PACKAGE ROUTINES CALLED.. NONE
C USER ROUTINES CALLED.. NONE
C CALLED BY.. PDECOL
C FORTRAN FUNCTIONS USED.. NONE
C-----
C
C DIMENSION Y0(NO),Y(NO,1)
C COMMON /GEAR/ T,H,DUMMY(4),N,ICUMMY(2),JSTART
C DO 10 I = 1,N
  10 Y0(I) = Y(I,1)
  L = JSTART + 1
  S = (TOUT - T)/H
  S1 = 1.
  DO 30 J = 2,L
    S1 = S1*S
    DO 20 I = 1,N
      20 Y0(I) = Y0(I) + S1*Y(I,J)
    30 CONTINUE
  RETURN
END
SUBROUTINE COSET (METH, NO, EL, TQ)
C-----
C COSET IS CALLED BY THE INTEGRATOR AND SETS COEFFICIENTS USED THERE.
C THE VECTOR EL, OF LENGTH NO + 1, DETERMINES THE BASIC METHOD.
C THE VECTOR TQ, OF LENGTH 4, IS INVOLVED IN ADJUSTING THE STEP SIZE
C IN RELATION TO TRUNCATION ERROR. ITS VALUES ARE GIVEN BY THE
C PERTST ARRAY.
C
C THE VECTORS EL AND TQ DEPEND ON METH AND NO.
C THE MAXIMUM ORDER, MAXDER, OF THE METHODS AVAILABLE IS CURRENTLY
C 12 FOR THE ADAMS METHODS AND 5 FOR THE BDF METHODS. MAXDER DEFAULTS
C TO 5 UNLESS THE USER SETS MAXDER TO SOME OTHER LEGITIMATE VALUE
C THROUGH THE COMMON BLOCK /OPTION/. SEE PDECOL FOR ADDITIONAL DETAILS.
C LMAX = MAXDER + 1 IS THE NUMBER OF COLUMNS IN THE Y ARRAY (SEE STIFB
C AND THE VARIABLE C, Y, OR WORK(IW10) IN PDECOL.
C
C THE COEFFICIENTS IN PERTST NEED BE GIVEN TO ONLY ABOUT
C ONE PERCENT ACCURACY. THE ORDER IN WHICH THE GROUPS APPEAR BELOW
C IS.. COEFFICIENTS FOR ORDER NO - 1. COEFFICIENTS FOR ORDER NO.
C COEFFICIENTS FOR ORDER NO + 1. WITHIN EACH GROUP ARE THE
C COEFFICIENTS FOR THE ADAMS METHODS, FOLLOWED BY THOSE FOR THE
C BDF METHODS.
C
C REFERENCE
C GEAR, C.W., NUMERICAL INITIAL VALUE PROBLEMS IN ORDINARY
C DIFFERENTIAL EQUATIONS, PRENTICE-HALL, ENGLEWOOD CLIFFS,
C N. J., 1971.

```

```

C PACKING ROUTINES CALLED.. NONE
C USER ROUTINES CALLED.. NONE
C CALLED BY.. STIFB
C FORTRAN FUNCTIONS USED.. FCAT
C
C DIMENSION PERST(12,2,3),EL(13),TO(4)
C DATA PERST / 1,1,2,3,1,1,2,3,158,07407,.01391,.002182,
C * .0002945,.00033492,.000003692,.000003524,
C * 1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,
C * 2,12,124,37,89,53,33,70,08,87,97,106,9,
C * 126,7,147,4,168,8,191,0,
C * 2,0,4,5,7,33,10,42,13,7,1,1,1,1,1,1,1,1,1,1,
C * 12,0,24,0,37,89,53,33,70,08,87,97,106,9,
C * 126,7,147,4,168,8,191,0,1,
C * 3,0,6,0,9,167,12,5,1,1,1,1,1,1,1,1,1,1,1,1, /
C
C GO TO (1,2),METH
C 1 GO TO 101,102,103,104,105,106,107,108,109,110,111,112),NO
C 2 GO TO 1201,222,203,204,225),NO
C
C THE FOLLOWING COEFFICIENTS SHOULD BE DEFINED TO MACHINE ACCURACY.
C FOR A GIVEN ORDER NO, THEY CAN BE CALCULATED BY USE OF THE
C GENERATING POLYNOMIAL  $L(T)$ , WHOSE COEFFICIENTS ARE  $EL(1)$ ..
C  $L(T) = EL(1) + EL(2)*T + \dots + EL(NO+1)*T^{NO}$ .
C FOR THE IMPLICIT ADAMS METHODS,  $L(T)$  IS GIVEN BY
C  $OL/DY = (T+1)*(T+2)* \dots *(T+NO-1)/K$ ,  $L(-1) = 0$ ,
C WHERE  $K = FACTORIAL(NO-1)$ .
C FOR THE BDF METHODS,
C  $L(T) = (T+1)*(T+2)* \dots *(T+NO)/K$ ,
C WHERE  $K = FACTORIAL(NO)*(1 + 1/2 + \dots + 1/NO)$ .
C
C THE ORDER IN WHICH THE GROUPS APPEAR BELOW IS..
C IMPLICIT ADAMS METHODS OF ORDERS 1 TO 12,
C BDF METHODS OF ORDERS 1 TO 5.
C
101 EL(1) = 1.0E-00
GO TO 900
102 EL(1) = 0.5E-00
EL(3) = 0.5E-00
GO TO 900
103 EL(1) = 4.166666666667E-01
EL(3) = 0.75E-00
EL(4) = 1.6666666666667E-01
GO TO 900
104 EL(1) = 0.375E-00
EL(3) = 9.1666666666667E-01
EL(4) = 3.333333333333E-01
EL(5) = 4.1666666666667E-02
GO TO 900
105 EL(1) = 3.486111111111E-01
EL(3) = 1.0416666666667E-00
EL(4) = 4.861111111111E-01
EL(5) = 1.0416666666667E-01
EL(6) = 8.333333333333E-03
GO TO 900
106 EL(1) = 3.298611111111E-01
EL(3) = 1.1416666666667E-00
EL(4) = 0.625E-00
EL(5) = 1.770833333333E-01
EL(6) = 0.025E-00
EL(7) = 1.3888888888889E-03
GO TO 900
107 EL(1) = 3.1559193121693E-01
EL(3) = 1.225E-00
EL(4) = 7.518518518518E-01
EL(5) = 2.552083333333E-01

```

EL(1) = 4.881111111111111E-02  
 EL(2) = 4.881111111111111E-03  
 EL(3) = 1.9841269841270E-04  
 GO TO 900  
 108 EL(1) = 3.0422453703704E-01  
 EL(2) = 1.2664285714286E-00  
 EL(3) = 8.6851851851852E-01  
 EL(4) = 3.3576388888889E-01  
 EL(5) = 7.7777777777778E-02  
 EL(6) = 1.0644148148148E-02  
 EL(7) = 7.9365079365079E-04  
 EL(8) = 2.4801587301587E-05  
 GO TO 900  
 109 EL(1) = 2.9486800044092E-01  
 EL(2) = 1.3589285714286E-00  
 EL(3) = 9.7655121280423E-01  
 EL(4) = 0.4171875E-00  
 EL(5) = 1.1135416666667E-01  
 EL(6) = 0.01875E-00  
 EL(7) = 1.9345238095238E-03  
 EL(8) = 1.1160714285714E-04  
 EL(9) = 2.7557319223986E-06  
 GO TO 900  
 110 EL(1) = 2.8697544642857E-01  
 EL(2) = 1.414841269841E-00  
 EL(3) = 1.072156084658E-00  
 EL(4) = 4.9856701940035E-01  
 EL(5) = 0.1484375E-00  
 EL(6) = 2.9060570987654E-02  
 EL(7) = 3.7202380952381E-03  
 EL(8) = 2.9968584656085E-04  
 EL(9) = 1.3778653611993E-05  
 EL(10) = 2.7557319223986E-07  
 GO TO 900  
 111 EL(1) = 2.801895964394E-01  
 EL(2) = 1.4644841269841E-00  
 EL(3) = 1.171514502646E-00  
 EL(4) = 5.7935619003527E-01  
 EL(5) = 1.8832286155203E-01  
 EL(6) = 4.1430362654321E-02  
 EL(7) = 6.2111441798942E-03  
 EL(8) = 6.2520667989418E-04  
 EL(9) = 4.0417401528513E-05  
 EL(10) = 1.5156525573193E-06  
 EL(11) = 2.5052108385442E-08  
 GO TO 900  
 112 EL(1) = 2.7426554003160E-01  
 EL(2) = 1.5099386724387E-00  
 EL(3) = 1.2602711640212E-00  
 EL(4) = 6.5923418209877E-01  
 EL(5) = 2.3045800264550E-01  
 EL(6) = 5.5637248105233E-02  
 EL(7) = 9.4394841269841E-03  
 EL(8) = 1.1192745669312E-03  
 EL(9) = 9.0939153439153E-05  
 EL(10) = 4.8225308641975E-06  
 EL(11) = 1.5031265031265E-07  
 EL(12) = 2.0876756987868E-09  
 GO TO 900  
 201 EL(1) = 1.0E-00  
 GO TO 900  
 202 EL(1) = 6.6666666666667E-01  
 EL(3) = 3.3333333333333E-01  
 GO TO 900  
 203 EL(1) = 5.4545454545455E-01  
 EL(3) = EL(1)

```

204 205 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223 224 225 226 227 228 229 230 231 232 233 234 235 236 237 238 239 240 241 242 243 244 245 246 247 248 249 250 251 252 253 254 255 256 257 258 259 260 261 262 263 264 265 266 267 268 269 270 271 272 273 274 275 276 277 278 279 280 281 282 283 284 285 286 287 288 289 290 291 292 293 294 295 296 297 298 299 300 301 302 303 304 305 306 307 308 309 310 311 312 313 314 315 316 317 318 319 320 321 322 323 324 325 326 327 328 329 330 331 332 333 334 335 336 337 338 339 340 341 342 343 344 345 346 347 348 349 350 351 352 353 354 355 356 357 358 359 360 361 362 363 364 365 366 367 368 369 370 371 372 373 374 375 376 377 378 379 380 381 382 383 384 385 386 387 388 389 390 391 392 393 394 395 396 397 398 399 400 401 402 403 404 405 406 407 408 409 410 411 412 413 414 415 416 417 418 419 420 421 422 423 424 425 426 427 428 429 430 431 432 433 434 435 436 437 438 439 440 441 442 443 444 445 446 447 448 449 450 451 452 453 454 455 456 457 458 459 460 461 462 463 464 465 466 467 468 469 470 471 472 473 474 475 476 477 478 479 480 481 482 483 484 485 486 487 488 489 490 491 492 493 494 495 496 497 498 499 500 501 502 503 504 505 506 507 508 509 510 511 512 513 514 515 516 517 518 519 520 521 522 523 524 525 526 527 528 529 530 531 532 533 534 535 536 537 538 539 540 541 542 543 544 545 546 547 548 549 550 551 552 553 554 555 556 557 558 559 560 561 562 563 564 565 566 567 568 569 570 571 572 573 574 575 576 577 578 579 580 581 582 583 584 585 586 587 588 589 590 591 592 593 594 595 596 597 598 599 600 601 602 603 604 605 606 607 608 609 610 611 612 613 614 615 616 617 618 619 620 621 622 623 624 625 626 627 628 629 630 631 632 633 634 635 636 637 638 639 640 641 642 643 644 645 646 647 648 649 650 651 652 653 654 655 656 657 658 659 660 661 662 663 664 665 666 667 668 669 670 671 672 673 674 675 676 677 678 679 680 681 682 683 684 685 686 687 688 689 690 691 692 693 694 695 696 697 698 699 700 701 702 703 704 705 706 707 708 709 710 711 712 713 714 715 716 717 718 719 720 721 722 723 724 725 726 727 728 729 730 731 732 733 734 735 736 737 738 739 740 741 742 743 744 745 746 747 748 749 750 751 752 753 754 755 756 757 758 759 760 761 762 763 764 765 766 767 768 769 770 771 772 773 774 775 776 777 778 779 780 781 782 783 784 785 786 787 788 789 790 791 792 793 794 795 796 797 798 799 800 801 802 803 804 805 806 807 808 809 810 811 812 813 814 815 816 817 818 819 820 821 822 823 824 825 826 827 828 829 830 831 832 833 834 835 836 837 838 839 840 841 842 843 844 845 846 847 848 849 850 851 852 853 854 855 856 857 858 859 860 861 862 863 864 865 866 867 868 869 870 871 872 873 874 875 876 877 878 879 880 881 882 883 884 885 886 887 888 889 890 891 892 893 894 895 896 897 898 899 900 901 902 903 904 905 906 907 908 909 910 911 912 913 914 915 916 917 918 919 920 921 922 923 924 925 926 927 928 929 930 931 932 933 934 935 936 937 938 939 940 941 942 943 944 945 946 947 948 949 950 951 952 953 954 955 956 957 958 959 960 961 962 963 964 965 966 967 968 969 970 971 972 973 974 975 976 977 978 979 980 981 982 983 984 985 986 987 988 989 990 991 992 993 994 995 996 997 998 999 1000

```

```

32 LR = ML
D( ) NR = 1.N1
  IF (LR .NE. N) LR = LR + 1
  MX = NR
  XM = ABS(B(NR,1))
  IF (ML .EQ. 0) GO TO 42
  DO 40 I = NP,LR
    IF (ABS(B(I,1)) .LE. XM) GO TO 40
    MX = I
    XM = ABS(B(I,1))
  CONTINUE
40 IPIV(NR) = MX
42 IF (MX .EQ. NR) GO TO 60
  DO 50 I = 1,LL
    XX = B(NR,I)
    B(NR,I) = B(MX,I)
    B(MX,I) = XX
  IF (XM .EQ. 0.) GO TO 100
  B(NR,1) = 1./XM
  IF (ML .EQ. 0) GO TO 90
  XM = -B(NR,1)
  KK = MIN0(N-NR,LL-1)
  DO 80 I = NP,LR
    J = LL + I - NR
    XX = B(I,1)*XM
    B(NR,J) = XX
  DO 70 II = 1,KK
    B(II,II) = B(II,II+1) + XX*B(NR,II+1)
  B(II,LL) = 0.
70 CONTINUE
80 NR = N
90 CONTINUE
92 IF (B(N,1) .EQ. 0.) GO TO 100
  BIN(1) = 1./B(N,1)
  RETURN
100 IER = NR
  RETURN
  END

```

SUBROUTINE SOLB (NDIM, N, ML, MU, B, Y, IPIV)

C SUBROUTINES DECB AND SOLB FORM A TWO SUBROUTINE PACKAGE FOR THE  
C DIRECT SOLUTION OF A SYSTEM OF LINEAR EQUATIONS IN WHICH THE  
C COEFFICIENT MATRIX IS REAL AND BANDED.

C C SOLUTION OF  $A \cdot X = C$  GIVEN LU DECOMPOSITION OF A FROM DECB.  
C Y = RIGHT-HAND VECTOR C, OF LENGTH N, ON INPUT,  
C = SOLUTION VECTOR X ON OUTPUT.

C ALL THE ARGUMENTS ARE INPUT ARGUMENTS.

C THE OUTPUT ARGUMENT IS Y.

C PACKAGE ROUTINES CALLED... NONE  
C USER ROUTINES CALLED... NONE  
C CALLED BY... DIFFUN,INITAL,STIFIB  
C FORTRAN FUNCTIONS USED... MINO

DIMENSION B(NDIM,1),Y(N),IPIV(N)

IF (N .EQ. 1) GO TO 60

N1 = N - 1

LL = ML + MU + 1

IF (ML .EQ. 0) GO TO 32

DO 30 NR = 1,N1

IF (IPIV(NR) .EQ. NR) GO TO 10

J = IPIV(NR)

XX = Y(NR)

Y(NR) = Y(J)

```

      ( J ) = XX
10      . = MINO(N-NR,ML)
      DO 20 I = 1,KK
20      Y(NR+I) = Y(NR+I) + Y(NR)*B(NR,LL+I)
30      CONTINUE
32      LL = LL + 1
34      Y(N) = Y(N)*B(N,1)
      KK = 0
      DO 50 NB = 1,N1
      NR = N - NB
      IF (KK .NE. LL) KK = KK + 1
      DP = 0.
      IF (LL .EQ. 0) GO TO 50
      DO 40 I = 1,KK
40      DP = DP + B(NR,I+1)*Y(NR+I)
50      Y(NR) = (Y(NR) - DP)*B(NR,1)
60      Y(1) = Y(1)*B(1,1)
      RETURN
      END
C      PROGRAM EXAMP:(UGOUT,TAPE3=UGOUT)
      COMMON /ENDPT/ XLEFT
      COMMON /GEAR0/ DTUSED,NQ,NSTEPS,NF,NJ
      DIMENSION U(2,31),XBKPT(31),SCTCH(10),WORK(5000),IWORK(500)

C      INITIALIZE PARAMETERS AND PDECOL CALLING ARGUMENTS
C
      NPDE = 2
      NINT = 30
      NPTS = NINT + 1
      KORD = 4
      NCC = 2
      TO = 0.0
      TOUT = 1.E-3
      DT = 1.E-7
      EPS = 1.E-4
      NF = 21
      INDEX = 1
      IWORK(1) = 5000
      IWORK(2) = 500
      DX = 1. / FLOAT(NPTS-1)
      DO 10 I=1,NPTS
      XBKPT(I) = FLOAT(I-1) * DX
10      CONTINUE
      XLEFT = XBKPT(1)

C      CALL THE PACKAGE TO INTEGRATE TO TIME T = TOUT
C
20      CALL PDECOL(TO,TOUT,DT,XBKPT,EPS,NINT,KORD,NCC,NPDE,NF,INDEX,
      .      WORK,IWORK)
C      CHECK FOR EXECUTION ERRORS
C
      IF ( INDEX .NE. 0 ) GO TO 70
C      OUTPUT PERFORMANCE DATA AND COMPUTED SOLUTION VALUES
C
      WRITE(3,30) TOUT,DTUSED,NSTEPS
30      FORMAT(//10X,3HT=,E10.3,7H DT=,E10.3,15H TOTAL STEPS=,I5)
      CALL VALUES(XBKPT,U,SCTCH,NPDE,NPTS,NPTS,0,WORK)
      DO 60 K=1,NPDE
60      K=1,NPDE
      WRITE(3,40) K
40      FORMAT(//10X,13HPDE COMPONENT,12/)
      WRITE(3,50) (U(K,I),I=1,NPTS)
50      FORMAT(10X,5E12.4)
60      CONTINUE

```

```

C SET N1 OUTPUT TIME AND CONTINUE THE INTEGRATION IF TOUT .LT. I
C OTHERS ., TERMINATE THE PROBLEM
C

```

```

      TOUT = TOUT + 10.
      IF ( TOUT .LT. 11. ) GO TO 20
      70 SITE(3,80) INDEX = 13
      80 FORMAT(10X,7HINDEX= ,13)
      STOP
      END
      SUBROUTINE F(T,X,U,UX,FVAL,NPDE)
      DIMENSION U(NPDE), UX(NPDE), UX(NPDE), FVAL(NPDE)
      FVAL(1) = U(2)*U(2)*UX(1) - U(1)*U(2) - U(1)**2 + 10.
      * FVAL(2) = U(1)*U(2)*UX(1)
      * FVAL(2) = U(1)*U(1)*UX(2) + U(1)*U(2) - U(2)**2
      * FVAL(2) = UX(1) + 2.*U(1)*UX(1)*UX(2)
      RETURN
      END

```

```

      SUBROUTINE BNDY(T,X,U,UX,DBDU,DBDU,DBDU,DBDU,DBDU,DBDU,DBDU,DBDU)
      DIMENSION U(NPDE), UX(NPDE), DZDT(NPDE)
      DIMENSION DBDU(NPDE,NPDE), DBDU(NPDE,NPDE)
      COMMON /ENDPT/ KLEFT
      IF ( X .NE. KLEFT ) GO TO 10

```

```

      DBDU(1,1) = 1.
      DBDU(1,2) = 0.
      DBDU(2,1) = 0.
      DBDU(2,2) = 1.
      DBDU(1,1) = 0.
      DBDU(1,2) = 0.
      DBDU(2,1) = 0.
      DBDU(2,2) = 0.
      DZDT(1) = 0.
      DZDT(2) = 0.
      RETURN

```

```

10 DBDU(1,1) = U(2) * COS( U(1) * U(2) )
   DBDU(1,2) = U(1) * COS( U(1) * U(2) )
   DBDU(2,1) = U(2) * SIN( U(1) * U(2) )
   DBDU(2,2) = U(1) * SIN( U(1) * U(2) )
   DBDU(1,1) = 1.
   DBDU(1,2) = 0.
   DBDU(2,1) = 0.
   DBDU(2,2) = 1.
   DZDT(1) = 0.
   DZDT(2) = 0.
   RETURN

```

```

      END
      SUBROUTINE UINIT(X,U,NPDE)
      DIMENSION U(1)

```

```

C SET INITIAL CONDITIONS. NOTE THAT PI = 4.*ATAN(1.)
C

```

```

      U(1) = .5 * ( X + 1. )
      U(2) = 4.*ATAN(1.)
      RETURN
      END

```

```

      SUBROUTINE DERIV(T,X,U,UX,DFDU,DFDU,DFDU,DFDU,DFDU,DFDU,DFDU,DFDU)
      DIMENSION U(NPDE), UX(NPDE), UX(NPDE), DFDU(NPDE,NPDE)
      DIMENSION DFDU(NPDE,NPDE), DFDU(NPDE,NPDE), DFDU(NPDE,NPDE)
      DFDU(1,1) = -U(2) - 2.*U(1)
      DFDU(1,2) = 2.*U(2)*UX(1) - U(1) + 2.*UX(2)*UX(1)
      DFDU(2,1) = 2.*U(1)*UX(2) + U(2) + 2.*UX(1)*UX(2)
      DFDU(2,2) = U(1) - 2.*U(2)

```

```

      DFDU(1,1) = 2.*U(2)*UX(2)
      DFDU(1,2) = 2.*U(2)*UX(1)
      DFDU(2,1) = 2.*U(1)*UX(2)

```

```

C      ( UAX(2,2) = 2.*U(1)-UAX(1)
      DFUXX(1,1) = U(2)-U(2)
      DFUXX(1,2) = 0.
      DFUXX(2,1) = 1.
      DFUXX(2,2) = U(1)-U(1)
      RETURN
      END
C      PROGRAM SINE1,TSIN,TSTOUT,TAPE2=TSIN,TAPE3=TSTOUT)
C      THIS PROGRAM WAS WRITTEN TO BE RUN IN SINGLE PRECISION ON A CDC7600.
      DIMENSION XPLT(25),UPLT(2,2,404),SCRATCH(10)
      DIMENSION ERRAX(2),UEXACT(2),XBKPT(404),ID(80)
      DIMENSION KORD(30000),IWORK(1000)
      COMMON /GEAR/ MUDED,NQUDED,NS,NF,NJ
      COMMON /ISTAR/ I=1,IM2,IM3,IDUM(15)
      COMMON /PARAMS/ PI
C      READ IN AND INITIALIZE PDECOL CALLING ARGUMENTS AND PARAMETERS.
      PI = 4.*ATN(1.0)
      READ(2,10) (ID(I),I=1,80)
      10 FORMAT(80A1)
      READ(2,20) NPDE,MF,TOUT
      20 FORMAT(2I3,E10.3)
      WRITE(3,10) (ID(I),I=1,80)
      WRITE(3,30) NPDE,MF,TOUT
      30 FORMAT(//8H NPDE = ,I2,5X,5HMF = ,I2,
      * 5X,26H RESULTS ARE FOR TIME T = ,E10.3//
      * 27H KORD NINT EPS
      * 1X,30H ERROR(1)
      * 1X,30H ERROR(2) )
      40 READ(2,50) KORD,NINT,EPS
      50 FORMAT(2I3,E10.3)
      IF(KORD.LE.0) STOP
      NPTS = NINT + 1
      NCC = 2
      IWORK(1) = 30000
      IWORK(2) = 1000
      DT = 0.0
      DT = EPS * .5
      INDEX = 1
      NCPTS1 = KORD*NINT - NCC*(NINT-1) - 1
      DX = 1. / FLOAT(NINT)
      DO 60 I=1,NPTS
        XBKPT(I) = FLOAT(I-1)*DX
      60 CONTINUE
      CALL PDECOL(TO,TOUT,DT,XBKPT,EPS,NINT,KORD,NCC,NPDE,MF,INDEX,
      * WGRK,IWORK)
C      CHECK FOR ERRORS AND THEN COMPUTE AN ESTIMATE FOR THE OVERALL
C      MAXIMUM ERROR FOR EACH PDE COMPONENT. OUTPUT RESULTS.
      IF(INDEX.NE.0) WRITE(3,70) INDEX
      70 FORMAT(7H INDEX=,I3)
      DO 80 I=1,NPDE
        80 ERRMX(I) = 0.
      C      WORK(IW3) TO WORK(IW3+NCPTS1) CONTAINS THE COLLOCATION POINTS.
      DO 100 J=1,NCPTS1
        DX = (WORK(IW3+J)-WORK(IW3+J-1))/24.
        DO 90 I=2,24
          90 XPLT(I) = FLOAT(I-1)*DX + WORK(IW3+J-1)
          XPLT(1) = WORK(IW3+J-1)
          XPLT(25) = WORK(IW3+J)
          DO 100 I=1,25
            CALL VALUES(XPLT(I),UPLT(1,1,1),SCRATCH,1,1,0,WORK)
            CALL TRUSOL(XPLT(I),TOUT,UEXACT)
            DO 100 K=1,NPDE
              UPLT(K,2,1) = ABS( UPLT(K,1,1) - UEXACT(K) )
              IF(UPLT(K,2,1) .GT. ERRMX(K)) ERRMX(K) = UPLT(K,2,1)
            100 CONTINUE
            WRITE(3,110) KORD,NINT,EPS,NS,NF,NJ,NQUDED,(ERRMX(I),I=1,NPDE)

```

```

110 FQ=AT(14,16,1E14,3,1X,416,2E15,3)
CQ ) 40
END
C SUBROUTINES FOR SINE PROBLEM.
SUBROUTINE FIT,X,U,UX,UXX,FVAL,NPDE)
COMMON /PARAMS/ PI
FVAL = UXX + PI*PI-SIN(PI*X)
RETURN
END
SUBROUTINE BNDRY(T,X,U,UX,DBDU,DBDUX,DZDT,NPDE)
DBDU = 1.
DBDUX = 0.
DZDT = 0.
RETURN
END
SUBROUTINE UINIT(X,U,NPDE)
DIMENSION U(NPDE)
CALL TRUSOL(X,0.,U)
RETURN
END
SUBROUTINE DERIV(T,X,U,UX,UXX,DFDU,DFDUX,DFDUXX,NPDE)
DFDUXX = 1.
DFDUX = 0.
DFDU = 0.
RETURN
END
SUBROUTINE TRUSOL(X,T,U,UXACT)
COMMON /PARAMS/ PI
UXACT = 1. + SIN(PI*X) * ( 1. - EXP(-PI*PI*T) )
RETURN
END
SINE PROBLEM INPUT DATA
1 21 .1
3 1 .01
3 3 .001
3 10 .00003
3 20 .00001
3 60 .000001
3 100 .000001
4 1 .01
4 2 .001
4 4 .00001
4 8 .000001
4 16 .0000001
5 1 .0001
5 2 .00001
5 4 .000001
5 8 .00000001
6 1 .0001
6 2 .000001
6 4 .00000001
-1
PROGRAM TWO(TSIN,ISTOUT,TAPE2=TSIN,TAPE3=ISTOUT)
C THIS PROGRAM WAS WRITTEN TO BE RUN IN SINGLE PRECISION ON A CDC7600.
DIMENSION APLT(25),UPLT(2,2,404),SCRICH(10)
DIMENSION ERRMX(2),UEXACT(2),X8KPY(404),ID(80)
DIMENSION WORK(30000),IMORK(1000)
COMMON /GEAR/ HUSED,NOUSED,NS,NF,NJ
COMMON /ISTART/ IM1,IM2,IM3,IDUM(15)
COMMON /PARAMS/ PI
C READ IN AND INITIALIZE PDECOL CALLING ARGUMENTS AND PARAMETERS.
PI = 4. * ATAN(1.0)
READ(2,10) (ID(I),I=1,80)
10 FORMAT(80A1)
READ(2,20) NPDE,MF,TOUT
20 FORMAT(2I3,E10,3)

```



```

      DFDUX(1,1) = 0.
      DFDUX(1,2) = 0.
      DFDUX(2,1) = 0.
      DFDUX(2,2) = 0.
      RETURN
10 CONTINUE
      DZDT(1) = 0.
      DZDT(2) = 0.
      DEDU(1,1) = 3.
      DEDU(1,2) = 0.
      DEDU(2,1) = -5.4598150033142E+01
      DEDU(2,2) = 0.
      DEDU(1,1) = 1.
      DEDU(1,2) = 0.
      DEDU(2,1) = 0.
      DEDU(2,2) = 5.
      RETURN
      END
      SUBROUTINE UINIT(X,U,NPDE)
      DIMENSION U(NPDE)
      CALL TRUSOL(X,0.,U)
      RETURN
      END
      SUBROUTINE DERIV(T,X,U,UX,UXX,DFDU,DFDUX,DFDUXX,NPDE)
      DIMENSION U(NPDE),UX(NPDE),UXX(NPDE),DFDU(NPDE),DFDUX(NPDE),DFDUXX(NPDE)
      DFDU(1,1) = 16.*X*T - 2.*T - 16.*(U(2)-1.0)
      DFDU(1,2) = UXX(1) - 16.*(U(1)-1.0)
      DFDU(2,1) = 4.
      DFDU(2,2) = 0.
      DFDUX(1,1) = UX(2)
      DFDUX(1,2) = UX(1)
      DFDUX(2,1) = 1.
      DFDUX(2,2) = 0.
      DFDUXX(1,1) = U(2) - 1.0
      DFDUXX(1,2) = 0.
      DFDUXX(2,1) = 0.
      DFDUXX(2,2) = 1.
      RETURN
      END
      SUBROUTINE TRUSOL(X,T,U,UEXACT)
      DIMENSION UEXACT(2)
      UEXACT(1) = 10.*X*T*EXP(-4.*X) + 1.0
      UEXACT(2) = 1.*X*X + 1.0
      RETURN
      END
      TWO PDE PROBLEM INPUT DATA
      2 21 1.0
      3 3 .0027
      3 6 .00064
      3 12 .00016
      3 24 .00004
      3 48 .00001
      3 96 .0000025
      4 1 .015
      4 2 .0013
      4 4 .0007
      4 8 .00004
      4 16 .0000025
      4 32 .000000015
      4 64 .000000001
      5 1 .0035
      5 2 .0001
      5 4 .000003
      5 8 .00000006
      5 16 .000000001

```

```

6 1 .0046
6 2 .013
6 4 .0000013
6 8 .00000001
-1
C PROGRAM BURG(TSIN,TSOUT,TAPE2=TSIN,TAPE3=TSOUT)
C THIS PROGRAM WAS WRITTEN TO BE RUN IN SINGLE PRECISION ON A CDC7600.
C DIMENSION XPLT(25),UPLT(2,2,404),SCRATCH(10)
C DIMENSION ERRMX(2),UEXACT(2),XBKPT(404),ID(80)
C DIMENSION WORK(30000),IWORK(1000)
C COMMON /GEAR/ MUUSED,NOUSED,NS,NF,NJ
C COMMON /ISTART/ IM1,IM2,IM3,IDUM(15)
C COMMON /PARAMS/ PI
C READ IN AND INITIALIZE PDECOL CALLING ARGUMENTS AND PARAMETERS.
PI = 4. * ATAN(1.0)
READ(2,10) (ID(I),I=1,80)
10 FORMAT(80A1)
READ(2,20) NPDE,MF,TOUT
20 FORMAT(2I3,E10.3)
WRITE(3,10) (ID(I),I=1,80)
WRITE(3,30) NPDE,MF,TOUT
30 FORMAT(/,8H NPDE = ,I2,5X,5HMF = ,I2,
* 5X,26H RESULTS ARE FOR TIME T = ,E10.3//
* 27H KORD NINT EPS ERROR(2) )
* 1X,30H ERROR(1)
40 READ(2,50) KORD,NINT,EPS
50 FORMAT(2I3,E10.3)
IF(KORD.LE.0) STOP
NPTS = NINT + 1
NCC = 2
IWORK(1) = 30000
IWORK(2) = 1000
T0 = 0.0
DT = EPS * .5
INDEX = 1
DT = EPS * .5
INDEX = 1
NCPST1 = KORD*NINT - NCC*(NINT-1) - 1
DX = 1. / FLOAT(NINT)
DO 60 I=1,NPTS
XBKPT(I) = FLOAT(I-1)*DX
60 CONTINUE
CALL PDECOL(T0,TOUT,DT,XBKPT,EPS,NINT,KORD,NCC,NPDE,MF,INDEX,
* WORK,IWORK)
C CHECK FOR ERRORS AND THEN COMPUTE AN ESTIMATE FOR THE OVERALL
C MAXIMUM ERROR FOR EACH PDE COMPONENT. OUTPUT RESULTS.
IF(INDEX.NE.0) WRITE(3,70) INDEX
70 FORMAT(7H INDEX=,I3)
DO 80 I=1,NPDE
80 ERRMX(I) = 0.
C WORK(IW3) TO WORK(IW3+NCPST1) CONTAINS THE COLLOCATION POINTS.
DO 100 J=1,NCPST1
DX = (WORK(IW3+J)-WORK(IW3+J-1))/24.
DO 90 I=2,24
90 XPLT(I) = FLOAT(I-1)*DX + WORK(IW3+J-1)
XPLT(1) = WORK(IW3+J-1)
XPLT(25) = WORK(IW3+J)
DO 100 I=1,25
CALL VALUES(XPLT(I),UPLT(1,1,I),SCRATCH,1,1,1,0,WORK)
CALL TRUSOL(XPLT(I),TOUT,UEXACT)
DO 100 K=1,NPDE
UPLT(K,2,1) = ABS( UPLT(K,1,1) - UEXACT(K) )
IF(UPLT(K,2,1) .GT. ERRMX(K)) ERRMX(K) = UPLT(K,2,1)
100 CONTINUE
WRITE(3,110) KORD,NINT,EPS,NS,NF,NJ,NOUSED,(ERRMX(I),I=1,NPDE)
110 FORMAT(14,I6,1E14,3,1X,4I6,2E15,3)

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GO TO 40
EX
C SUBROUTINE FOR BURGERS EQUATIONS PROBLEM.
SUBROUTINE FIT(X,U,UX,UXX,FVAL,NPDE)
FVAL = .003*UXX - U*UX
RETURN
END
SUBROUTINE BNDRY(T,X,U,UX,DBOU,DBDUX,DZOT,NPDE)
IF( X.NE. 0. ) GO TO 10
DZOT = 0.
DBOU = 1.
DBDUX = 0.
RETURN
10 CONTINUE
DZOT = 0.
DBOU = 0.
DBDUX = 0.
RETURN
END
SUBROUTINE UNIT(X,U,NPDE)
DIMENSION U(NPDE)
CALL TRUSOL(X,O.,U)
RETURN
END
SUBROUTINE DERIV(T,X,U,UX,DFDU,DFDUX,DFDUXX,NPDE)
DFDUXX = .003
DFDUX = -U
DFDU = -UX
RETURN
END
SUBROUTINE TRUSOL(X,T,UEXACT)
ANUIN = 1./ .006
A = -(X-.5 + 4.95*T)*ANUIN*.1
B = -(X-.5 + 0.75*T)*ANUIN*.5
C = -(X-.375) * ANUIN
A = EXP(A)
B = EXP(B)
C = EXP(C)
UEXACT = (.1*A + .5*B + C) / (A + B + C)
RETURN
END
BURGERS EQUATION PROBLEM INPUT DATA
1 21 .8
3 50 .003
3100 .0003
3200 .0001
3400 .00004
4 20 .004
4 40 .001
4 80 .0001
4160 .00001
5 15 .004
5 30 .001
5 60 .00003
5100 .000003
6 10 .01
6 20 .001
6 40 .00003
6 80 .000001
-1
C PROGRAM WAVE(TSIN,TSTOUT,TAPE2=TSIN,TAPE3=TSTOUT)
C THIS PROGRAM WAS WRITTEN TO BE RUN IN SINGLE PRECISION ON A CDC7600.
DIMENSION XPLI(25),UPLI(2,2,404),SCRCH(10)
DIMENSION ERMA(2),UEXACT(2),XBKPT(404),ID(80)
DIMENSION WORK(30000),IWORK(1000)
COMMON /GEAR0/ HUSED,NQUSED,NS,NF,NJ

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COMMON /ISTART/ IW1,IW2,IW3,IOWM(15)
C ( JH /PARAMS/ PI
C READ I... AND INITIALIZE PDECOL CALLING ARGUMENTS AND PARAMETERS.
PI = 4. * ATAN(1.0)
READ(2,10) (ID(I),I=1,80)
10 FORMAT(80A1)
READ(2,20) NPDE,MF,TOUT
20 FORMAT(2I3,E10.3)
WRITE(3,10) (ID(I),I=1,80)
WRITE(3,30) NPDE,MF,TOUT
30 FORMAT(//8H NPDE = ,I2,5X,SHWF = ,I2,
* 5X,25H RESULTS ARE FOR TIME T = ,E10.3//
* 27H KORD NINT EPS .24HINSTEPS NRES NJAC NO .
* 1X,30H ERROR(1)
40 READ(2,50) KORD,NINT,EPS
50 FORMAT(2I3,E10.3)
IF(KORD.LE.0) STOP
NPTS = NINT + 1
NCC = 2
IWORK(1) = 30000
IWORK(2) = 1000
T0 = 0.0
DT = EPS * .5
INDEX = 1
NCPTS1 = KORD*NINT - NCC*(NINT-1) - 1
DX = 1. / FLOAT(NINT)
DO 60 I=1,NPTS
XBKPT(I) = FLOAT(I-1)*DX
60 CONTINUE
CALL PDECOL(T0,TOUT,DT,XBKPT,EPS,NINT,KORD,NCC,NPDE,MF,INDEX,
* WORK,IWORK)
C CHECK FOR ERRORS AND THEN COMPUTE AN ESTIMATE FOR THE OVERALL
C MAXIMUM ERROR FOR EACH PDE COMPONENT. OUTPUT RESULTS.
IF(INDEX.NE.0) WRITE(3,70) INDEX
70 FORMAT(7H INDEX=,I3)
DO 80 I=1,NPDE
80 ERRMX(I) = 0.
C WORK(IW3) TO WORK(IW3+NCPTS1) CONTAINS THE COLLOCATION POINTS.
DO 100 J=1,NCPTS1
DX = (WORK(IW3+J)-WORK(IW3+J-1))/24.
DO 90 I=2,24
90 XPLT(I) = FLOAT(I-1)*DX + WORK(IW3+J-1)
XPLT(1) = WORK(IW3+J-1)
XPLT(25) = WORK(IW3+J)
DO 100 I=1,25
CALL VALUES(XPLT(I),UPLT(1,1,1),SCRATCH,1,1,1,0,WORK)
CALL TRUSOL(XPLT(I),TOUT,UEXACT)
DO 100 K=1,NPDE
UPLT(K,2,1) = ABS( UPLT(K,1,1) - UEXACT(K) )
IF(UPLT(K,2,1) .GT. ERRMX(K)) ERRMX(K) = UPLT(K,2,1)
100 CONTINUE
WRITE(3,110) KORD,NINT,EPS,NS,NF,NJ,NQUSED,(ERRMX(I),I=1,NPDE)
110 FORMAT(14,I6,1E14,3,I8,4I6,2E15,3)
GO TO 40
END
C SUBROUTINES FOR WAVE PROPAGATION PROBLEM.
SUBROUTINE F(T,X,U,UX,FVAL,NPDE)
FVAL = -UX
RETURN
END
SUBROUTINE BNDRY(T,X,U,UX,DBDU,DBDUX,DZDT,NPDE)
COMMON /PARAMS/ PI
IF( X.NE. 0. ) GO TO 10
DZDT = -2.*PI*COS(-4.*PI*T)
DBDU = 1.
DBDUX = 0.

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      10 CALL UINIT
      DZDT = 0.
      DBDU = 0.
      DBDUX = 0.
      RETURN
    END
    SUBROUTINE UINIT(X,U,NPDE)
      DIMENSION U(NPDE)
      CALL TRUSOL(X,0.,U)
      RETURN
    END
    SUBROUTINE DERIVE(T,X,U,UX,UXX,DFDU,DFDUX,DFDUXX,NPDE)
      DFDU = 0.
      DFDUX = -1.
      DFDUXX = 0.
      RETURN
    END
    SUBROUTINE TRUSOL(X,T,U,UEXACT)
      COMMON /PARAMS/ PI
      UEXACT = 1.0 + .5*SIN(4.*PI*(X-T))
      RETURN
    END
    WAVE PROPAGATION PROBLEM INPUT DATA
    1 11 .5
    3 20 .001
    3 40 .001
    3 80 .00001
    3 160 .000001
    3 320 .0000001
    4 5 .001
    4 10 .0001
    4 20 .00001
    4 40 .000001
    4 80 .00000001
    4 1601 .00000E-10
    5 3 .001
    5 6 .0004
    5 12 .000002
    5 24 .00000001
    5 481 .00000E-10
    6 2 .001
    6 4 .0001
    6 8 .00000001
    6 16 .000000001
    6 321 .00000E-10

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IF NO.LE.O.CR.MAP.LE.O.OR.MAR.LE.2)GO TO 610
IF NO.GE.MAP.CR.MAR.GE.100)GO TO 610
IF MAP.GT.NOPTS.CR.MAP.GE.100)GO TO 610
IF NOPTS.LE.O.OR.NOVS.LE.O.OR.NO.GT.610)GO TO 610
IF NO.LE.O.OR.NOVS.GT.NO)GO TO 610
IF NO.GT.NOVS)GO TO 610
GO TO 615
610 CONTINUE
WRITE(KFILE,611)
611 FORMAT(//,' $$$ PROBLEMS WITH ARGUMENT LIST. PLOT NOT PERFORMED.',
1)
ARITE(KFILE,612)MAD,MAP,MAR,NOPTS,NOV,NO
612 FORMAT(1X,618)
RETURN
C
500 NSUB = AMAX + EPS
MEXP1 = MEXP
SUBT = NSUB
DO 520 N = MAD,MAP,MAR
DO 520 L = NO,NOVS
520 Y(N,L) = Y(N,L) - SUBT
615 CONTINUE
AMAX = -1.0E+20
AMIN = 1.0E+20
DO 620 N = MAD,MAP,MAR
DO 620 L = NO,NOVS
IF(Y(N,L).GT.AMAX)AMAX = Y(N,L)
IF(Y(N,L).LT.AMIN)AMIN = Y(N,L)
620 CONTINUE
IPT = 0
JPT = 0
FAC = ABS(AMAX-AMIN)
IF(FAC.GT.1.0E-16)GO TO 630
WRITE(KFILE,623)
629 FORMAT(//,' $$$ (AMAX - AMIN) IS LESS THAN 1.0E-16, PLOT NOT',
1,' PERFORMED.',/)
RETURN
C
C AMIN---AMAX IS THE RANGE FROM THE SMALLEST TO THE LARGEST VALUE
C PLOTTED. IF AMIN AND AMAX ARE BOTH POSITIVE OR BOTH NEGATIVE, THE
C PROCEDURE FOR SELECTING THE SCALE IS CONTAINED IN STATEMENTS 630
C TO 702. IF THE RANGE INCLUDES ZERO, HOWEVER, THE SCALE IS FOUND
C IN STATEMENTS 800 TO 860.
C
630 DMAX = AMAX
DMIN = AMIN
FAC = AMAX-AMIN
IF(FAC.LT.ZERO)GO TO 800
FAC = ONE
IF(AMAX.GE.ZERO.AND.AMIN.GE.ZERO)GO TO 632
DMAX = -AMIN
DMIN = -AMAX
FAC = -ONE
C
C STATEMENTS 632-663 SCALE DOWN (OR UP) AMAX, AMIN, AND THE DATA (BY
C THE FACTOR TEN**MEXP) SO THAT AMAX FALLS IN THE RANGE 1 TO 19.....
C
632 IF(DMAX-ONE)635,660,640
635 DMAX = DMAX*1.0E+03
IPT = IPT + 1
GO TO 632
640 IF(DMAX-19.E+00)660,660,661
650 DMAX = DMAX/TEN
JPT = JPT + 1
GO TO 640
660 MEXP = JPT - IPT

```

```

IF MEXP.EQ.0)GO TO 663
F( TEN**MEXP)
  NMAX = AMAX*FX
  NMIN = AMIN*FX
DO 661 N = MAD,MAP,VAR
DO 661 L = NO,NOVS
  YIN(L) = YIN(L)*FX
661 CONTINUE
663 CONTINUE
  IF FAC.LT.ZERO)GO TO 652
  DMAX = AMAX
  DMIN = AMIN
  GO TO 655
652 DMAX = -AMIN
  DMIN = -DMAX
655 CONTINUE
C
C STEPS UP TO 670 DECIDE 'M', THE NUMBER OF DIGITS PAST THE DECIMAL
C POINT IN ALL THE SCALE POINTS. LATER, MT = 3 + M, WHERE MT IS
C THE NUMBER OF DIGITS IN THE SCALE POINT, COUNTING THE DECIMAL
C AND 2 DIGITS TO ITS LEFT.
C
  IF (SUB.NE.0)FACN = FACN*TEN**(MEXP)
  M = 0
  MT = 3
  NMAX = DMAX + EPS*FACN
  NMIN = DMIN - EPS*FACN
  EYOP = NMIN
  IF (DMIN.EQ.ZERO)NMIN = -1
  INDEX(1,3) = NMIN
  NMIN = NMIN + 1
  NMAX = NMAX - NMIN
  IF (NMIN-1)665,670,670
665 M = M + 1
  DMIN = (DMIN-EYOP)*TEN - EPS*FACN
  DMAX = (DMAX-EYOP)*TEN + EPS*FACN
  NMAX = DMAX
  NMIN = DMIN
  MT = 3 + M
  IF (MT.GT.8)GO TO 500
  INDEX(1,MT) = NMIN
  ENOP = NMIN
  GO TO 662
670 INDEX(1,MT) = INDEX(1,MT) + 1
C
C STEPS UP TO 680 FIX THE LAST DIGITS OF THE SCALE POINTS---THAT IS,
C FIX INDEX(L,M) FOR ALL L, FOR THE LARGEST M USED (MT). NRAN IS THE
C NUMBER OF SCALE POINTS TO BE USED.....
C
  NRAN = NRAN + 1
  IF (MT.EQ.7).AND.(NRAN.GT.17))GO TO 500
  IF (MT.CO.8).AND.(NRAN.GT.15))GO TO 500
  IPT = INDEX(1,MT)
  DO 680 MTA = 2,NRAN
  IPT = IPT + 1
  IPT = INDEX(MTA,MT) = IPT
680
C
C STEPS 685-690 CORRECT THE LAST DIGIT OF THE SCALE POINTS, IF NE-
C CCESSARY, AND DETERMINE ALL OTHER DIGITS TO THE RIGHT OF THE DE-
C CIMAL---THAT IS, FIX INDEX(L,M) FOR ALL L AND ALL M>3.....
C
  IF (M-1)693,685,685
685 DO 690 LT = 1,M
  NLT = MT - LT
  IPT = INDEX(1,NT)
  DO 690 MTA = 1,NRAN

```

```

IPT = INDEX(MTA,NT+1)
INDEX(MTA,NT) = IPT
IF (IPT-10) 690, 688, 688
688 INDEX(MTA,NT) = INDEX(MTA,NT) + 1
INDEX(MTA,NT+1) = INDEX(MTA,NT+1) - 10
IPT = INDEX(MTA,NT+1)
IF (IPT-10) 690, 689, 689
689 INDEX(MTA,NT+1) = INDEX(MTA,NT+1) - 10
INDEX(MTA,NT) = INDEX(MTA,NT) + 1
690 CONTINUE

C STEPS 693-703 FIX THE DIGITS TO THE LEFT OF THE DECIMAL---THAT IS,
C INDEX(L,M) FOR ALL L AND ALL M<3---AND FIX XSCALE WHICH IS THE EX-
C ACT NUMERICAL VALUE OF THE SCALE POINT.....
693 DO 703 MTA = 1, NRAN
INDEX(MTA,1) = 0
IPT = INDEX(MTA,3)
IF (IPT-10) 693, 698, 698
698 IPT = IPT - 10
INDEX(MTA,1) = 1
INDEX(MTA,2) = IPT
XSCALE(MTA) = ZERO
DO 700 M = 3, MT
DIFF = INDEX(MTA,M)
700 XSCALE(MTA) = XSCALE(MTA) + DIFF*(TEN**(3-M))
703 INDEX(MTA,3) = 0

C STEPS UP TO 702 REORDER XSCALE AND INDEX IF BOTH AMAX AND
C AMIN ARE NEGATIVE. HERETOFORE, EVERYTHING IS THE SAME FOR
C NEGATIVE NUMBERS AS FOR POSITIVE ONES, BECAUSE ABSOLUTE VAL-
C UES WERE USED. IN ORDER TO PRINT THE SMALL ('MORE NEGATIVE')
C VALUES ON THE LEFT AND THE LARGER ONES ON THE RIGHT, AS WITH
C POSITIVE NUMBERS, THE SCALE HAS TO BE INVERTED.....
IF (FAC.GT.ZERO) GO TO 702
NT = NRAN/2
DO 701 MTA = 1, NT
EMOP = -XSCALE(MTA)
XSCALE(MTA) = -XSCALE(NRAN-MTA+1)
XSCALE(NRAN-MTA+1) = EMOP
DO 701 LT = 1, MT
N = INDEX(MTA,LT)
INDEX(MTA,LT) = INDEX(NRAN-MTA+1,LT)
INDEX(NRAN-MTA+1,LT) = N
701 CONTINUE
MTA = NRAN - NT
IF (MTA.NE.NT) XSCALE(MTA) = -XSCALE(MTA)
702 CONTINUE
GO TO 704
.....

C THE FOLLOWING SECTION (THROUGH 800) SETS THE SCALE FOR THE CASE
C WHEN AMIN IS NEGATIVE AND AMAX IS POSITIVE. STATEMENTS THROUGH
C 825 FIND NRAN, THE NUMBER OF SCALE POINTS TO BE USED, DIFF, THE
C NUMERICAL DIFFERENCE BETWEEN CONSECUTIVE SCALE POINTS, DECIDE
C WEXP (WHICH HAS THE SAME FUNCTION AS ABOVE), AND SCALE THE DATA
C WITH MEKP, IF NECESSARY.
.....
800 DMIN = ABS(AMIN)
DMAX = ABS(AMAX)
IF (DMIN.GT.DMAX) DMIN = DMAX
DO 801 N = 1, 41

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MEXP = 21 - N
DIF = TEN**MEXP
IF (AMAX.GT.DMIN)GO TO 810
801 CONTINUE
RETURN
810 DIFF = DMIN
DMIN = ABS(AMIN)
DIF = ABS(AMAX)
NRAN = 1
DO 815 N = 1,40
  PN = N
  DUFF = PN-DIFF
  IF(DUFF.GT.DMIN.AND.DUFF.GT.DMAX)GO TO 816
  IF(DUFF.LE.DMAX)NRAN = NRAN + 1
  IF(DUFF.LE.DMIN)NRAN = NRAN + 1
  EMOP = DUFF
815 CONTINUE
RETURN
816 IF(NRAN.GE.41)GO TO 820
  JPT = 1
  B4C = DIFF/TEN
  DO 817 N = 1,40
    PN = N
    DUFF = PN-B4C
    IF(DUFF.GT.DMAX.AND.DUFF.GT.DMIN)GO TO 818
    IF(DUFF.LE.DMAX)JPT = JPT + 1
    IF(DUFF.LE.DMIN)JPT = JPT + 1
    ETA1 = DUFF
817 CONTINUE
818 IF((NRAN.EQ.2.AND.JPT.GT.19).OR.(NRAN.EQ.3.AND.JPT.GT.15))GO
  1 TO 820
  DIFF = B4C
  NRAN = JPT
  EMOP = ETA1
  MEXP = MEXP - 1
820 CONTINUE
  M = 1
  IF(EMOP.LT.99.0E+00.AND.EMOP.GT..99E-02)GO TO 830
  FX = TEN**MEXP
  DIF = DIFF*FX
  AMAX = AMAX*FX
  AMIN = AMIN*FX
  DO 825 N = MAD,MAP,MAR
    DO 825 L = NO,NOVS
      YIN(L) = YIN(L)*FX
825 CONTINUE
GO TO 831
830 IF(EMOP.LT.0)M = -MEXP
  MEXP = 0
831 CONTINUE
C
C STEPS THROUGH 860 FIX MT, INDEX, AND XSCALE. THE FIRST THREE DI-
C GITS OF INDEX ARE FIXED BEFORE STATEMENT 855.....
C
  EMOP = ZERO
  DMIN = ABS(AMIN)
  MTA = 0
839 IF(EMOP.GT.DMIN)GO TO 840
  DMAX = EMOP
  EMOP = EMOP + DIFF
  MTA = MTA + 1
  IF(MTA.GT.100)RETURN
GO TO 839
840 XSCALE(1) = -DMAX
DO 842 N = 2,NRAN

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AD-A139 604

MATHEMATICAL MODELLING OF WAVEGUIDING TECHNIQUES AND  
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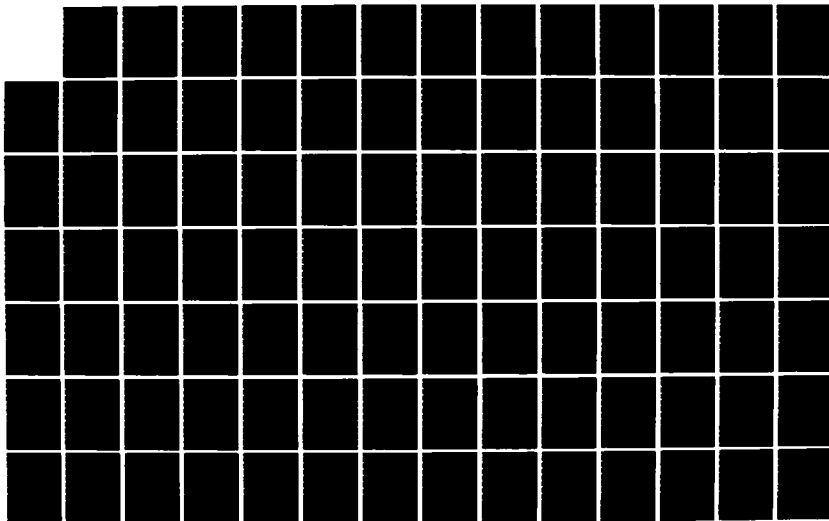
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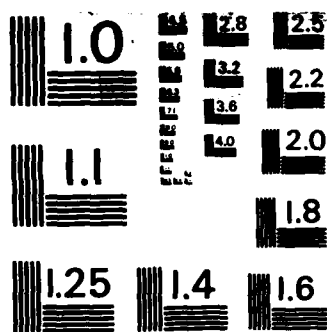
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F/G 9/2

NL





MICROCOPY RESOLUTION TEST CHART  
NATIONAL BUREAU OF STANDARDS-1963-A



```

D( 45 L = NC.NOVS
745 Y(...L) = Y(N,L) + SUBT
WRITE(KFILE,746)EMOP
746 FORMAT('BX,*IT WAS NECESSARY TO SUBTRACT*,F8.1,* FROM THE*,
1* VARIABLES PLOTTED TO MAKE THE SCALE MEANINGFUL*')
WRITE(KFILE,747)
747 FORMAT('4IX,* VALUES OF Y(N,L) RETURNED ARE ORIGINAL VALUES*')
750 CONTINUE
IF(MEXP1.EQ.0)GO TO 760
FX = TEN*(-MEXP1)
DO 755 N = MAD,MAR,MAR
DO 755 L = NO.NOVS
755 Y(N,L) = Y(N,L)*FX
760 RETURN
END

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Program Listing

- ONEDSN -

(Reference: Vol. I, Section IV.A.2)

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C-----
C PROGRAM ONEDSN(INPUT,OUTPUT)
C-----
C ONEDSN SOLVES THE ONE-DIMENSIONAL STEADY STATE DISCRETE ORDINATE
C FORM OF THE TRANSPORT EQUATION IN SLAB GEOMETRY FOR THE CASES OF
C ISOTROPIC AND SCREENED-RUTHERFORD SCATTERING WITH SURFACE OR
C DISTRIBUTED PARTICLE SOURCES. OTHER SCATTERING MODELS CAN BE
C INCORPORATED BY SUBSTITUTING AN APPROPRIATE EXPRESSION FOR THE
C ANGULAR REDISTRIBUTION FUNCTION ARRAY "SCAT(K,L,M)" IN SUBROUTINE
C "INPUTS". THE NUMBER OF DISCRETE ORDINATES CAN BE VARIED UP TO A
C MAXIMUM OF 12 FORWARD DIRECTIONS (24 DISCRETE ORDINATES IN ALL).
C UP TO 100 CELLS ARE ALLOWED FOR THE SPATIAL DISCRETIZATION. THE
C DIAMOND DIFFERENCE ITERATION SCHEME IS USED. NO CONVERGENCE
C ACCELERATION SCHEME IS EMPLOYED, SO THAT EACH ITERATION
C CORRESPONDS TO A PHYSICAL ORDER OF SCATTERING.
C-----
C COMMON/A/SCAT(24,24),AMU(24),SURF(24),A(12,2),B(12,2),EFLUX(101),
C +,CUR(24),S(100,24),SDISTR(100,24),CFLUX(100,24),EF(100),X(101),
C +WGT(24),ERR,ZERO,ETA(2),NX,NANG,NPOS,NSCAT,NORDS,MT,
C +KSRC,SIG(2),ALBEDO(2),NINT,DX
C-----
C ITMAX      = MAXIMUM NO. OF ITERATIONS
C NORDS      = 1 IF PRINT OUT OF ORDERS-OF-SCATTERING FLUX IS
C             DESIRED. NO PRINTOUT FOR OTHER VALUES OF N.
C NSCAT      = 0 IF ISOTROPIC SCATTERING IS ASSUMED
C             = 1 IF SCREENED-RUTHERFORD OR OTHER NON-ISOTROPIC
C               SCATTERING LAW IS ASSUMED
C DX         = WIDTH OF X-CELL
C SIG        = 1/COLLISION MEAN FREE PATH
C ALBEDO     = PROBABILITY OF A PARTICLE BEING SCATTERED (NOT
C             ABSORBED).
C ERR        = FRACTIONAL ERROR IN THE SOURCE FUNCTION AT WHICH
C             ITERATIONS ARE TERMINATED.
C ZERO       = LEVEL AT WHICH ERRORS IN THE CALCULATED SOURCE ARE
C             IGNORED
C NX         = NO. OF X-CELLS
C NPOS       = NO. OF POSITIVE DIRECTIONS (=NANG/2)
C ETA        = SCREENING PARAMETER USED WHEN SCREENED-
C             RUTHERFORD SCATTERING IS ASSUMED (NSCAT=1);
C             ALSO ANISOTROPY PARAMETER WHEN NON-ISOTROPIC
C             SCATTERING OTHER THAN SCREENED-RUTHERFORD IS
C             ASSUMED.
C KSRC       = 0 IF INITIAL CONDITION IS SURFACE SOURCE ONLY
C             = 1 IF NON-ZERO INITIAL DISTRIBUTED SOURCE ONLY
C             = 2 IF BOTH SURFACE AND DISTRIBUTED SOURCES ARE
C               PRESENT INITIALLY
C NCASE      = PARAMETER TO SELECT FORM OF SURFACE SOURCE
C             = 1, COSINE SOURCE
C             = 2, (COSINE)**2 SOURCE
C             = 3, (COSINE)**3 SOURCE
C NINT       = INDEX OF FIRST X-CELL BEYOND MATERIAL INTERFACE.
C             IF NO MATERIAL INTERFACE PRESENT, READ IN NINT = 0.
C-----
C-----DERIVED PARAMETERS AND BOUNDARY CONDITIONS
C SCAT(K,L,M) = SCATTERING KERNEL
C             (K = INDEX OF INITIAL DIRECTION DISCRETE ORDINATE
C             L = INDEX OF FINAL DIRECTION DISCRETE ORDINATE
C             M = INDEX OF MATERIAL LAYER)
C NANG       = TOTAL NO. OF DISCRETE ORDINATES (ANGLES)

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C      Y 'J)      * DIRECTION COSINES OF POSITIVE DIRECTIONS
C      (K)      * INCIDENT ANGULAR FLUX AT X = 0.
C              ( X IS THE SPATIAL COORDINATE; X = 0 IS THE LEFT
C              BOUNDARY)
C
C      SDISTR(I,K) = INITIAL DISTRIBUTED SOURCE AT CENTER OF ITH CELL
C              IN KTH DIRECTION
C
C      -----CALCULATED QUANTITIES-----
C      CFLUX(I,K) = ANGULAR FLUX AT CENTER OF ITH CELL IN KTH DIRECTION
C      EF(I,K) = ANGULAR FLUX AT CELL EDGE
C      EFLUX(I) = SCALAR FLUX AT CELL EDGE
C      CUR(K) = PARTICLE CURRENT IN KTH DIRECTION
C      S(I,K) = SCATTERED SOURCE FUNCTION
C      QUT = TOTAL EMERGENT CURRENT FROM SLAB
C
C      -----READ INPUT PARAMETERS-----
C      READ 555,ITMAX,NORDS,NSCAT,KSRC
C
C      SUBROUTINE 'INPUTS'
C      1) READS IN REMAINING INPUT PARAMETERS
C      2) INITIALIZES GAUSS QUADRATURE ANGLES AND WEIGHTS
C      3) COMPUTES THE SCATTERING PROBABILITY FUNCTION
C      4) INITIALIZES THE SOURCE FUNCTION
C
C      CALL INPUTS
C
C      -----PRINT OUT DERIVED CONSTANTS, BOUNDARY CONDITIONS, ETC.-----
C      PRINT 502
C      *AMU* IS ARRAY OF GAUSS QUADRATURE COSINES
C      PRINT 500,(AMU(J),J=1,NPOS)
C      PRINT 505
C      *SURF* IS ARRAY OF SURFACE SOURCE VALUES AT DISCRETE ORDINATES
C      PRINT 500,(SURF(J),J=1,NPOS)
C
C      *SUM* IS SOURCE NORMALIZATION FACTOR = INTEGRAL OF SOURCE OVER
C      ANGLE
C      SUM=0.
C      DO 100 J=1,NPOS
C      SUM=SUM+SURF(J)*WGT(J)
C      *A,B* ARE CONSTANTS THAT ARE USED REPETITIVELY IN THE ITERATIONS
C      DO 100 M=1,INT
C      A(J,M)=2.*AMU(J)/(2.*AMU(J)+SIG(M)*DX)
C      100 B(J,M)=DX/(2.*AMU(J)+SIG(M)*DX)
C      PRINT 514,SUM
C
C      -----CONVERT FROM CURRENT SOURCE TO FLUX SOURCE-----
C      DO 10 J=1,NPOS
C      10 SURF(J)=SURF(J)/AMU(J)
C
C      -----ZERO OUT THE FLUX-----
C      DO 11 K=1,NANG
C      DO 11 I=1,NX
C      CFLUX(I,K)=0.
C      NXPN=NX+1
C
C      DO 12 I=1,NXP
C      12 X(I)=(I-1)*DX
C      NANG=2*NPOS
C      NANGP=NANG+1
C
C      -----MAIN ITERATION LOOP-----
C      DO 350 IT=1,ITMAX
C      OUT=TRAN=BACK=0.
C
C      PRINT OUT ORDERS OF SCATTERING SCALAR FLUX VALUES IF DESIRED
C      IF(NORDS.EQ.1)CALL INTPRINT

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C      ( - FLUXES(TRAN,1,0,1,0)
C      CALL FLUXES(BACK,-1,NANGP,-1,NXP)
C      OUT=TRAN+BACK
C-----CALCULATE NEW SOURCE-----
C      EMAX=0.
C      M=1
C      DO 200 I=1,NX
C      IF(I.GE.NINT)M=2
C      DO 200 K=1,NANG
C      SUM=SDISTR(I,K)
C      DO 150 L=1,NANG
C      150 SUM=SUM+SCAT(K,L,M)*CFUX(I,L)*NGT(L)*ALBEDO(M)*SIG(M)
C      IF(ABS(SUM).LT.ZERO)GO TO 200
C      ERROR=ABS((S(I,K)-SUM)/SUM)
C      IF(ERROR.GT.EMAX)EMAX=ERROR
C      200 S(I,K)=SUM
C      IF(EMAX.LT.ERR)GO TO 400
C      350 CONTINUE
C-----FINAL PRINTOUT-----
C      400 PRINT 511
C      PRINT 512,IT,EMAX,OUT
C      PRINT 513,IT,TRAN,BACK
C-----FORMAT STATEMENTS-----
C      500 FORMAT(1X,12E11.4)
C      502 FORMAT(1X,AMU(J)= X-COSINES*)
C      505 FORMAT(1X,SURF(J), SURFACE SOURCE*)
C      507 FORMAT(1X,A(J), B(J)*
C      509 FORMAT(1X,ANGULAR FLUXES, DIRECTION*,13)
C      511 FORMAT(//)
C      512 FORMAT(1X,NUMBER OF ITERATIONS*,13.5X,MAXIMUM ERROR*,E12.5,2X,
C      1-TOTAL EMERGENT CURRENT*,E12.5)
C      513 FORMAT(1X,CURRENT CONTRIBUTION FOR ITERATION NO.*,19/1X,
C      1-TRANSMISSION*,2X,E13.6/1X,BACKSCATTER*,1X,E13.6)
C      514 FORMAT(1X,TOTAL INCIDENT CURRENT *,E12.5)
C      555 FORMAT(4IS)
C      STOP
C      END
C      SUBROUTINE INPUTS
C      COMMON/SCAT(24,24,2),AMU(24),SURF(24),A(12,2),B(12,2),EFLUX(101)
C      *CUR(24),S(100,24),SDISTR(100,24),CFUX(100,24),EP(100),X(101),
C      *NGT(24),ERR,ZERO,ETA(2),NX,NANG,NPOS,NSCAT,NORDS,MT,
C      *KSRC,SIG(2),ALBEDO(2),NINT,DX
C      DATA SURF,S,SDISTR/24*0.,4800*0./
C-----READ IN INPUT PARAMETERS-----
C      C
C      C      DX = WIDTH OF X-CELL
C      C      SIG = 1./COLLISION MEAN FREE PATH
C      C      ALBEDO = PROBABILITY OF A PARTICLE BEING SCATTERED (NOT
C      C      ABSORBED).
C      C      ERR = FRACTIONAL ERROR IN THE SOURCE FUNCTION AT WHICH
C      C      ITERATIONS ARE TERMINATED.
C      C      ZERO = LEVEL AT WHICH ERRORS IN THE CALCULATED SOURCE ARE
C      C      IGNORED
C      C      NX = NO. OF X-CELLS
C      C      NPOS = NO. OF POSITIVE DIRECTIONS (=NANG/2)
C      C      ETA = SCREENING PARAMETER USED WHEN SCREENED-
C      C      RUTHERFORD SCATTERING IS ASSUMED (NSCAT=1);
C      C      ALSO ANISOTROPY PARAMETER WHEN NON-ISOTROPIC

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C SCATTERING OTHER THAN SCREENED-RUTHERFORD IS
C ASSUMED.
C SE PARAMETER TO SELECT FORM OF SURFACE SOURCE (
C ANGULAR DEPENDENCE
C = 1. COSINE SOURCE
C = 2. (COSINE)**2 SOURCE
C = 3. (COSINE)**3 SOURCE
C NINT INDEX OF FIRST X-CELL BEYOND MATERIAL INTERFACE.
C IF NO MATERIAL INTERFACE PRESENT, READ IN NINT = 0.

READ 100.DX,ERR,ZERO
READ 10.NX,NPOS,NCASE,NINT
MT=2
IF(NINT.EQ.0)MT=1
DO 1 M=1,MT
1 READ 100.SIG(M),ALBEDO(M),ETA(M)
IF(DX.EQ.0.)DX=.1
IF(ERR.EQ.0.)ERR=.001
IF(ZERO.EQ.0.)ZERO=.0001
NANG=2*NPOS
PRINT 501.DX,ERR,ZERO,NX,NANG,NPOS,NSCAT,NSRC
DO 14 M=1,MT
14 PRINT 506.SIG(M),ALBEDO(M)
IF(NSCAT.EQ.0)PRINT 502
IF(NSCAT.EQ.1)PRINT 503,(ETA(M),M=1,MT)
PRINT 505,NCASE,NINT
IF(NINT.EQ.0)NINT=NX+10
C COMPUTE GAUSS QUADRATURE ANGLES AND HEIGHTS
C
C N1=NANG+1
C CALL GRULE(NANG,AMU,WGT)
C DO 15 J=1,NPOS
C AMU(N1-J)=AMU(J)
C WGT(N1-J)=WGT(J)
15
C COMPUTE SCATTERING PROBABILITY FUNCTION
C
C ISOTROPIC SCATTERING
C PROB=0.5
C DO 21 K=1,NANG
C DO 21 L=1,NANG
C DO 21 M=1,MT
C IF(NSCAT.EQ.0) GO TO 21
C SCREENED-RUTHERFORD SCATTERING
C
C AT THIS POINT THE EXPRESSION FOR "PROB", THE SCATTERING
C PROBABILITY(ANGULAR REDISTRIBUTION) FUNCTION CAN BE CHANGED
C ACCORDING TO OTHER SCATTERING MODELS. ANISOTROPIC SCATTERING
C FUNCTIONS OF THE FORM "PROB = 0.5*(1.+ETA(M)*AMU(K)*AMU(L))"
C COULD BE USED, WHERE ETA, THE ANISOTROPY PARAMETER, MAY LIE IN THE
C RANGE 0.0 TO 0.5 (SEE P.205, VOL. 1)
C
C PROB=ETA(M)*(1.+5*ETA(M))*(1.+ETA(M)-AMU(K)*AMU(L))/(ETA(M)*(ETA
C 1(M)+2.*(1.-AMU(K)*AMU(L))+(AMU(K)-AMU(L))**2)**1.5)
21 SCAT(K,L,M)=PROB
C
C SOURCE INITIALIZATION
C IF(NSRC.EQ.1) GO TO 35
C SURFACE (COSINE CURRENT) SOURCE

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DO 30 J=1,NPOS
  C TO (31,32,33),NCASE
  31 IF(2,AMU(J)
  C TO 30
  32 FACTR=3,AMU(J)*AMU(J)
  C TO 30
  33 FACTR=4,AMU(J)*AMU(J)*AMU(J)
  C TO 30
  30 SURF(J)=FACTR
  IF(KSRC.EQ.0)GO TO 40
  35 CONTINUE
C
C CALCULATE SPATIALLY DISTRIBUTED SOURCE FUNCTION, IF APPLICABLE.
C IF THERE IS NO SPATIALLY DISTRIBUTED SOURCE, ZERO OUT THE
C "SDISTR" ARRAY.
C
DO 36 I=1,NX
DO 36 K=1,NANG
  36 SDISTR(I,K)=0.
  40 CONTINUE
C
C-----FORMAT STATEMENTS-----
  10 FORMAT(10I3)
  100 FORMAT(6F10,0)
  501 FORMAT(1X,DX,PF,3,2X,ERR=,
  1F9.6,2X,ZERO,10,7,2X,NH=,13,2X,NANG=,13,2X,NPOS=,13,
  22X,NSCAT=,13,2X,NSRC=,13)
  502 FORMAT(/1X,ISOTROPIC SCATTERING*)
  503 FORMAT(/1X,SCREENED-RUTHERFORD SCATTERING, ETA=,2E12.5)
  505 FORMAT(1X,NCASE = ,13,2X,NINT = ,13)
  506 FORMAT(1X,SIG=,E12.5,2X,ALBEDO=,E12.5)
C
RETURN
END
SUBROUTINE FLUXES(OUT,JS,NANGP,IS,NXP)
C
C THIS SUBROUTINE PERFORMS THE DIAMOND DIFFERENCE ITERATION SCHEME
C OF EQUATIONS 11A, 11B ON P. 198, VOL. I.
C
COMMON/A/SCAT(24,24,2),AMU(24),SURF(24),A(12,2),B(12,2),EFLUX(101)
+.CUR(24),S(100,24),SDISTR(100,24),CFUX(100,24),EF(100),X(101),
+WGT(24),ERR,ZERO,ETA(2),NX,NANG,NPOS,NSCAT,NORDS,MT,
+KSRC,SIG(2),ALBEDO(2),NINT,DX
C
IF(JS.NE.1)GO TO 10
NXP1=NXP+1
DO 15 I=1,NXP1
  15 EFLUX(I)=0.
  10 DO 100 J=1,NPOS
    JR=J+JS+NANGP
    EF(J)=SURF(JR)
    IF(15.EQ.-1)EFLUX(NXP1)=EFLUX(NXP1)+WGT(J)*EF(J)
    DO 90 I=1,NX
      IF(15.EQ.-1)GO TO 20
      M=1
      IF(1.GE.NINT)M=2
      GO TO 21
    20 M=2
    IF(1.LT.NINT)M=1
    21 IR=I+IS+NXP
    IF(15.EQ.1)EFLUX(IR)=EFLUX(IR)+WGT(J)*EF(J)
    CFUX(IR,JR)=A(J,M)*EF(J)+B(J,M)*S(IR,JR)
    P=2.-CFUX(IR,JR)
    EF(J)=P-EF(J)
    IF(15.EQ.-1)EFLUX(IR)=EFLUX(IR)+WGT(J)*EF(J)
  90 CONTINUE

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**Program Listing**

**- BEAMSRC -**

**(Reference: Vol. I, Section IV.A.2)**

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C
C
C      PROGRAM BEAMSRC(INPUT,OUTPUT,TAPE7,TAPER)
C
C      BEAMSRC CALCULATES, BY MEANS OF THE METHOD OF DISCRETE ORDINATES,
C      THE FLUX DUE TO A ONCE SCATTERED SOURCE OF ELECTRONS FROM
C      MONODIRECTIONAL BEAM INCIDENT ON A SLAB. BOTH SCREENED RUTHERFORD
C      AND ISOTROPIC SCATTERING ARE TREATED. WHEN SCREENED
C      RUTHERFORD SCATTERING IS ASSUMED, EXTENDED TRANSPORT CORRECTED
C      CROSS SECTIONS ARE EMPLOYED. NEGATIVE FLUX FIXUP IS AND COARSE
C      MESH REBALANCE CONVERGENCE ACCELERATION ARE INCLUDED.
C
C      COMMON SCAT(12,12,2),AMU(12),A(12,2),B(12,2),EFLUX( 51),ESRC
C      +,CUR(12),S( 51,12),SOISTR( 50),CFLUX( 50,12),EF( 50),X( 51),
C      +WGT(12),DX,SIG(2),ALBEDO(2),ERR,ZERO,ETA(2),NX,NANG,NPOS,
C      +KSRG,INTRFC,NLAY,E,Z(2),AA(2),IPRNT,COSIN,ENTAB(100),DS(2),
C      +DLSTR(13,2),ELAMPR(2),LMAX,PL(13,12),ISO,IEDGE,ISKIP,DELE(50),
C      +EXCII(2),STPR(2),DE,ITER(2),NGRPS,NG,ITMAX,IT,MATL(2),CSUM,
C      +CMESH(2,10),FBAL(10),INBL,SUM1,NCRS,STPD,ELAMO
C
C      DIMENSION BCUR(12),EDPOS(100),CONT(100),FJ(51),BJ(51),TOTJ(51)
C      DIMENSION EFOLD(51)
C      DATA EFOLD/51*0./
C      DATA EDPOS,CONT/200*0./
C
C-----INPUT PARAMETERS-----
C
C      ITMAX      * MAXIMUM NO. OF ITERATIONS
C      ISO        * 0/1. SCREENED RUTHERFORD/ISOTROPIC SCATTERING
C      IEDGE      * 0/1. EDGE ANGULAR FLUX COMPUTATION, NO/YES
C      ISKIP      * FLUX PRINTOUT AT EVERY ISKIP-TH SPATIAL POSITION
C      INBL       * 0/1. COARSE MESH REBALANCE OPTION, YES/NO
C      NCRS       * NUMBER OF COARSE MESH INTERVALS
C      NCRSKP     * COARSE MESH REBALANCE(CMR) SKIP PARAMETER
C                * (PERFORM CMR EVERY NCRSKP ITERATIONS)
C      DX         * WIDTH OF X-CELL(CM-CM**2)
C      NGRPS      * NUMBER OF ENERGY GROUPS
C      IPRNT      * PRINTOUT PARAMETER,0/1/2/3 = FINAL SCALAR FLUX ONLY/
C                * ITERATIONS/ANGULAR FLUX/FULL DEBUG PRINTOUT
C      SIG(M)     * TOTAL INTERACTION CROSS SECTION FOR MATERIAL M.
C                * UNITS ARE (CM**2/CM)
C      ALBEDO(M)  * PROBABILITY OF A PARTICLE BEING SCATTERED (NOT
C                * ABSORBED) FOR MATERIAL M.
C      ERR        * FRACTIONAL ERROR IN THE SOURCE FUNCTION AT WHICH
C                * ITERATIONS ARE TERMINATED.
C      ZERO       * LEVEL AT WHICH ERRORS IN THE CALCULATED SOURCE ARE
C                * IGNORED
C      NX         * NO. OF X-CELLS
C      NPOS       * NO. OF POSITIVE DIRECTIONS
C      ETA(M)     * SCREENING PARAMETER FOR MATERIAL M USED WHEN
C                * SCREENED RUTHERFORD SCATTERING IS ASSUMED
C      INTRFC     * INDEX OF MATERIAL BOUNDARY INTERFACE
C      E          * ELECTRON KINETIC ENERGY(MEV)
C      DE         * ENERGY GROUP WIDTH (MEV)
C      COSIN      * COSINE OF SOURCE BEAM ANGLE OF INCIDENCE
C      Z(M)       * COSINE OF SOURCE BEAM ANGLE OF INCIDENCE
C      AA(M)      * ATOMIC WEIGHT OF MATERIAL M
C
C-----DERIVED PARAMETERS-----
C
C      SCAT(X,L,M) * SCATTERING KERNEL FOR MATERIAL M
C      NANG       * TOTAL NO. OF DISCRETE ORDINATES (ANGLES)-2*NPOS
C      AMU(J)      * DIRECTION COSINES OF POSITIVE DIRECTIONS

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C      SOSTR(1)      = DISTRIBUTED SOURCE AT CENTER OF ITH CELL
C      (          )  (ACTUALLY SOSTR(1,K) WHERE K DENOTES THE K-1 K
C      DIRECTION. THE SOSTR(1) ARRAYS ARE STORED IN K
C      RECORDS OF LENGTH NX ON SCRATCH FILE TAPE7).
C-----CALCULATED QUANTITIES-----
C      CELUX(1,K)    = ANGULAR FLUX AT CENTER OF ITH CELL IN KTH DIRECTION
C      EFLUX(1,K)    = ANGULAR FLUX AT CELL EDGE
C      SFLUX(1,K)    = SCALAR FLUX AT CELL EDGE
C      CUR(1,K)      = PARTICLE CURRENT IN KTH DIRECTION
C      S(1,K)        = SCATTERED SOURCE FUNCTION
C      OUT           = TOTAL EMERGENT CURRENT FROM SLAB
C      ETA(M)        = ATOMIC(RUTHERFORD) SCREENING CONSTANT FOR MATERIAL
C      M.            = IT IS CALCULATED FROM THE MOLIERE FORMULA
C      SGG(L)        = LEGENDRE COEFFICIENTS OF SCATTERING CROSS SECTION
C      (UNCORRECTED)
C      SIGSTR(L)     = LEGENDRE COEFFICIENTS OF TRANSPORT CORRECTED
C      SCATTERING CROSS SECTION
C      SIGA          = ABSORPTION CROSS SECTION(=ABSORB*SGG(1))
C      DLSTR(L,M)    = LEGENDRE COEFFICIENTS OF TRANSPORT CORRECTED
C      SCATTERING PROBABILITY
C      STPMR(M)      = ELECTRON STOPPING POWER FOR MATERIAL M.
C      UNITS ARE (MEV-CM**2/GM)
C      DS(M)         = ELECTRON PATH LENGTH IN MATERIAL M FOR ENERGY
C      GROUP OF WIDTH DE.
C      UNITS ARE (GM/CM**2)
C-----BEGIN CALCULATIONS-----
C      DATA ETAPM,EBACK,TSUM,BSUM/A*0./
C      READ 555,NRUN,ITMAX,NCRPS,IPRNT,ISO,IEDGE,ISKIP,IRBL,NCRS,NCRSKP
C      PRINT 557,NRUN
C      PRINT 556,ITMAX,NCRPS,IPRNT,ISO,IEDGE,ISKIP,IRBL,NCRS,NCRSKP
C
C      SET NG, THE ENERGY GROUP INDEX
C      CALL INPUTS
C      NG=1
C      ECUM=ESRC
C      IF(IPRNT.LT.3)GO TO 60
C-----PRINT OUT DERIVED CONSTANTS, BOUNDARY CONDITIONS, ETC.-----
C      PRINT 511
C      PRINT 502
C      PRINT 500,(AMU(J),J=1,NPOS)
C
C      PRECOMPUTATION OF A AND B ARRAYS (USED SEVERAL TIMES IN SUBROUTINE
C      FLUX
C
C      60 DO 100 J=1,NPOS
C         DO 100 M=1,NLAY
C            A(J,M)=2.*AMU(J)/(2.*AMU(J)+SIG(M)*DX)
C            B(J,M)=DX/(2.*AMU(J)+SIG(M)*DX)
C            IF(IPRNT.LT.3) GO TO 61
C         DO 101 M=1,NLAY
C            PRINT 511
C            PRINT 507,M
C            PRINT 500,(A(J,M),J=1,NPOS)
C            PRINT 511
C            PRINT 508,M
C            PRINT 500,(B(J,M),J=1,NPOS)
C      101
C
C      INITIALIZE DISTRIBUTED SOURCE
C      61 REWIND 7
C         DO 13 K=1,NANG
C            READ(7)(SOSTR(1),I=1,NX)
C            DO 13 II=1,NX
C               S(1,K)=SOSTR(II)
C      13

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C-----Y--ZERO OUT THE CELL CENTER FLUX-----
      DO 11 I=1,NX
      DO 11 J=1,NY
      CFLUX(I,J)=0.
11 CFLUX(I,J)=0.
C
C  COMPUTE COORDINATES, X(I), OF CELL EDGES (NXP OF THEM)
C
      NXP=NX+1
      DO 12 I=1,NXP
      FJ(I)=BU(I)*TOTJ(I)+0.
12 X(I)=(I-1)*DX
      XMIN=0.
      XMAX=X(NXP)
      YMIN=0.
C
C  TOTAL NUMBER OF ANGLES = NANG
C
      NANG=2*NPOS
C
C  REMIND PLOT FILE BUFFER
C
      REMIND 8
      WRITE(8)NRUN,NGRPS,NXP,NANG,(X(I),I=1,NXP)
C
C  ENERGY GROUP (OUTER ITERATION) LOOP
C
1000 CONTINUE
      IX=NX/NCRS
      PRINT 511
C-----MAIN (INNER) ITERATION LOOP-----
      DO 350 IT=1,ITMAX
      IF(NG.LT.3)GO TO 699
      IRBL=0
      IF(IT.LE.10)GO TO 699
      IF((IT/NCRSKIP)*NCRSKIP-IT).NE.0)IRBL=1
      OUT=TRAN=BACK=0.
      CALL FLUXES(TRAN,1,0,1,0)
      CALL FLUXES(BACK,-1,LMAX,-1,NXP)
      IF(IRBL.GT.0)GO TO 715
C
C  PERFORM COARSE MESH REBALANCE
C
      CALL REBAL
      DO 710 J=1,NANG
      DO 710 I=1,NX
      IRR=(I+IX-1)/IX
      CFLUX(I,J)=CFLUX(I,J)*FBAL(IRR)
      TRAN=TRAN+FBAL(IRR)
      BACK=BACK+FBAL(IRR)
      IF(IT.EQ.1)PRINT 885,IT,(FBAL(I),I=1,NCRS)
      715 OUT=TRAN+BACK
C
C-----CALCULATE NEW SOURCE-----
      ENMAX=0.
      REMIND 7
      DO 200 K=1,NANG
      READ(7)(SDISTR(I),I=1,NX)
      DO 200 I=1,NX
      SUM=SDISTR(I)
      M=1
      IF(I.GE.INTRFC)M=2
      DO 150 L=1,NANG
      SUM=SUM+SCAT(K,L,M)*CFLUX(I,L)*WG(L)*ALBEDO(M)*SIG(M)
      IF(ABS(SUM).LT.ZERO)GO TO 200
      200 S(I,K)=SUM

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2200 I=1,NXP
   FLUX(I),EQ.0.0)GO TO 2200
   E=OR-ABS(1.-EFLD(I)/EFLUX(I))
   IF(ERROR.GT.EMAX)EMAX=ERROR
2200 EFLD(I)=EFLUX(I)
   IF(I.PRINT.LT.2) GO TO 83
   PRINT 512,IT,EMAX.OUT,TRAN,BACK
63 IF(EMAX.LT.ERR)GO TO 400
350 CONTINUE
C
C-----FINAL PRINTOUT FOR THE NG-TH ENERGY GROUP-----
400 PRINT 505,IT
   IF(I.PRINT.LT.2)PRINT 512,IT,EMAX.OUT,TRAN,BACK
   IF(I.PRINT.LT.2)PRINT 512,IT,EMAX.OUT,TRAN,BACK
885 FORMAT(1X,ITERATIONS=15,REBALANCE FACTORS=,/(10E12.5))
   ETRAN=ETRAN+TRAN*DELE(MG)
   EBACK=EBACK+BACK*DELE(MG)
   TSUM=TSUM+TRAN
   BSUM=BSUM+BACK
   CSUM=OUT
   IF(I.PRINT.EQ.1)PRINT 561,(CUR(J),J=1,NPOS)
   DO 62 J=1,NPOS
62 BCUR(J)=CUR(LMAX-J)
   IF(I.PRINT.EQ.1)PRINT 562,(BCUR(J),J=1,NPOS)
C
C PRINT CELL CENTER ANGULAR FLUXES
C
   IF(I.PRINT.LT.2)GO TO 64
   K1=-7
395 K1=K1+8
   K2=K1+7
   IF(K2.GT.NANG)K2=NANG
   PRINT 509
   DO 300 I=1,NX,ISKIP
300 PRINT 517,I,(CFLUX(I,K),K=K1,K2)
   IF(K2.LT.NANG)GO TO 395
C COMPUTE EDGE ANGULAR FLUXES(ONLY VALID WHEN DIAMOND DIFF USED)
C (THE SOURCE ARRAY IS BORROWED HERE FOR THE EDGE FLUX COMPUTATION)
64 DO 579 J=1,NANG
   DO 579 I=1,NXP
579 S(I,J)=0.0
   DO 580 J=1,NPOS
   DO 580 I=2,NXP
580 S(I,J)=2.*CFLUX(I-1,J)-S(I-1,J)
   JJ=LMAX-J
   JJ=LMAX-J
   DO 581 I=1,NX
   II=NXP-I
581 S(II,JJ)=2.*CFLUX(II,JJ)-S(II+1,JJ)
   IF(IEDEGE.EQ.0)GO TO 380
   K1=-7
396 K1=K1+8
   K2=K1+7
   IF(K2.GT.NANG)K2=NANG
   PRINT 510
C PRINT CELL EDGE ANGULAR FLUXES
   DO 306 I=1,NXP,ISKIP
306 PRINT 517,I,(S(I,K),K=K1,K2)
   IF(K2.LT.NANG)GO TO 396
380 CONTINUE
C
C COMPUTE ENERGY FLOW
   DO 370 I=1,NXP
   DO 371 J=1,NPOS
   JJ=LMAX-J
   FU(I)=FU(I)+WGT(J)*AMU(J)*S(I,J)

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371 B(1)=BJ(I)+WGT(JJ)*AMU(JJ)*S(I,JJ)
370 I(1)=FJ(I)+BJ(I)
DO 381 I=1,NXP
DO 381 K=1,NANG
381 S(I,K)=0.
C COMPUTE ENERGY DEPOSITION PROFILE CONTRIBUTION FROM SCATTERED
C ELECTRONS
M=1
DO 450 I=1,NX
CONT(I)=0.
IF(I.GE.INTRFC)M=2
DO 449 J=1,NANG
449 CONT(I)=CONT(I)+STPWR(M)*WGT(J)*CFUX(I,J)
450 CONTINUE
PRINT 511
PRINT 515
C THE FLUX ARRAY IS BORROWED HERE TO COMPUTE THE UNSCATTERED
C FLUX CONTRIBUTION. THE SOURCE WAS THE ONCE SCATTERED
C DISTRIBUTED SOURCE RESULTING FROM THE UNSCATTERED
C MONODIRECTIONAL BEAM. THE UNSCATTERED FLUX CONTRIBUTION IS
C COMPUTED FOR THE FIRST(SOURCE) ENERGY GROUP ONLY.
IF(NG.GT.1)GO TO 309
DO 382 I=1,NXP
382 S(I,2)=EFLUX(I)
YMAX=1.25
C
THK=DX*(INTRFC-1)
M=1
SAVE=0.
CC=STPD*ELAMO*COSIN/DX
CD=1./(ELAMO*COSIN)
DO 308 I=1,NX
IF(I.GE.INTRFC)M=2
IF(INTRFC.EQ.NXP)M=1
IF(I.GT.INTRFC)GO TO 307
S(I,1)=SAVE+EXP(-X(I)*SIG(M)/COSIN)
EDPOS(I)=CC*EXP(-CD*X(I))-EXP(-CD*X(I+1))
EFLUX(I)=EFLUX(I)+SAVE
FJ(I)=FJ(I)+COSIN*SAVE
GO TO 308
307 S(I,1)=SAVI+SAVE*EXP(-(X(I)-THK)*SIG(2)/COSIN)
EFLUX(I)=EFLUX(I)+SAVI
308 CONTINUE
PRINT 5663,(EDPOS(I),I=1,NX,ISKIP)
5663 FORMAT(1X,'ENERGY DEPOSITION PROFILE CONTRIBUTION DUE TO UNSCATE
RED FLUX',(10E12.5))
TR=S(NXP,1)*COSIN
BK=0.
WRITE(8)ESRC,TR,BK,YMIN,YMAX,(S(I,1),I=1,NXP)
YMAX=0.
DO 378 I=1,NXP
378 IF(S(I,2).GE.YMAX)YMAX=S(I,2)
WRITE(8)ENTAB(NG),TRAN,BACK,YMIN,YMAX,(S(I,2),I=1,NXP)
ETRAN=ETRAN+COSIN*S(NXP,1)*DELE(1)
C PRINT CELL EDGE SCALAR FLUXES
C
309 PRINT 516,(I,X(I),EFLUX(I),I=1,NXP,ISKIP)
ETL=0.
DO 319 I=1,NX
319 EDPOS(I)=EDPOS(I)+CONT(I)
ETL=ETL+EDPOS(I)
PRINT 1748,ETL
C WRITE PLOT FILE
C

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C      ENID=ENTAB(NG)
C      I=.NG.GT.1)WRITE(8)EMID,TRAN,BACK,YMIN,YMAX,(EFLUX(I),I=1,.P)
C
C      PRINT THE ENERGY DEPOSITION PROFILES
C
      IF(I.PRINT.GE.1)PRINT 563,(CONT(I),I=1,NX,ISKIP)
      IF(I.PRINT.GE.1)PRINT 564,(EDPOS(I),I=1,NX,ISKIP)
      ECUM=ECUM-DELE(NG)
      IF(NG.LT.(NGRPS-5))GO TO 1752
      DO 1750 I=1,NX
1750  EFLUX(I)=0.5*(EFLUX(I)+EFLUX(I+1))*ECUM/DX+EDPOS(I)
      PRINT 1751,(EFLUX(I),I=1,NX)
1751  FORMAT(1X,'TOTAL ENERGY DEPOSITION PROFILE IF CALCULATION STOPS
      & HERE'/(10E12.5))
      ETL=0.
      DO 1749 I=1,NX
1749  ETL=ETL+DX+EFLUX(I)
      PRINT 1748,ETL
1748  FORMAT(1X,'TOTAL ENERGY DEPOSITED (MEV) =',E12.5)
1752  CONTINUE
C
C      DECREMENT THE ELECTRON ENERGY TO THE NEXT GROUP
C
      NG=NG+1
      E=ENTAB(NG)
      IF(NG.LE.NGRPS)GO TO 600
      EFLMIN=1.E10
      EFLMAX=0.
      DO 602 I=1,NXP
602  IF(TOTJ(I).GE.EFLMAX)EFLMAX=TOTJ(I)
      IF(TOTJ(I).LE.EFLMIN)EFLMIN=TOTJ(I)
      WRITE(8)EFLMIN,EFLMAX,(TOTJ(I),I=1,NXP)
      PRINT 570,(TOTJ(I),I=1,NXP)
      PRINT 571,(F(I),I=1,NXP)
      PRINT 572,(B(I),I=1,NXP)
      PRINT 573,ETRAN,EBACK
1753  FORMAT(1X,'TOTAL ENERGY TRANSMISSION=',E12.5,'MEV',1X,
      & 11X,'TOTAL ENERGY BACKSCATTER=',E12.5,'MEV')
1754  FORMAT(1X,'NET CELL EDGE CURRENTS'/(10E12.5))
1755  FORMAT(1X,'FORWARD CELL EDGE CURRENTS'/(10E12.5))
1756  FORMAT(1X,'BACKWARD CELL EDGE CURRENTS'/(10E12.5))
      EDMAX=0.
      DO 601 I=1,NX
601  X(I)=X(I)+5*DX
      EDPOS(I)=EDPOS(I)+1000.
      IF(EDPOS(I).GE.EDMAX)EDMAX=EDPOS(I)
601  CONTINUE
      WRITE(8)NX,EDMAX,(X(I),I=1,NX),(EDPOS(I),I=1,NX)
      GO TO 610
C
C      COMPUTE NEW CROSS SECTIONS, SCATTERING PROBABILITY, AND
C      SCATTERING SOURCE
C
      GO 600 CALL RECALC
      GO TO 1000
C
C-----FORMAT STATEMENTS
500  FORMAT(1X,12E11.4)
502  FORMAT(1X,'ANU(J)= X-COSINES')
505  FORMAT(1X,'TOTAL NUMBER OF ITERATIONS = ',I5)
507  FORMAT(1X,'A - ARRAY FOR MATERIAL NO.',I2)
508  FORMAT(1X,'B - ARRAY FOR MATERIAL NO.',I2)
509  FORMAT(1X,'ANGULAR CENTER FLUXES'/(1X,'CELL NO.))

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511 7  'AT(//)
512 7  'AT(1X,.ITERATION*,13,. MAX. ERROR=,E12.5,
+ 7  'EXIT CURRENT=,E12.5,
+ 7  'TRANSMISSION=,E12.5, BACKSCATTER=,E12.5)
513 7  'FORMAT(1X,.CURRENT CONTRIBUTION FOR ITERATION NO.=,15/1X,
1-TRANSMISSION=,2X,E13.6/1X, BACKSCATTER=,1X,E13.6)
514 7  'FORMAT(1X,.TOTAL INCIDENT CURRENT =,E12.5)
515 7  'FORMAT(1X,.TOTAL SCALAR EDGE FLUX AT X(I)=
+2X,E13.6X,X(I),7X,FLUX(I)=)
516 7  'FORMAT(4(1X,13,2X,E12.5))
517 7  'FORMAT(1X,14,5X,8E12.5)
518 7  'FORMAT(1015)
519 7  'FORMAT(1X,.ITMAX=,15,. (MAXIMUM ALLOWED NUMBER OF INNER ITERATION
+5) //1X,.NGRPS=,15,. (NUMBER OF ENERGY GROUPS) //1X,.IPRNT=,15,. P
+RINTOUT CONTROL (0/1/2/3 = FINAL SCALAR FLUX ONLY/ITERATIONS/ANGUL
+AR FLUX/FULL DEBUG PRINTOUT) //1X,.ISO =,15,. (0/1, SCREENED RUTH
+ERFORD SCATTERING/ISOTROPIC SCATTERING) //1X,.IEEDGE=,15,
+ (0/1, ANGULAR EDGE FLUX COMPUTATION, NO/YES) //
+1X,.ISKIP=,15,. (FLUX PRINTOUT PARAMETER - PRINT EVERY ISKIP-TH C
+ELL) //1X,.IRBL =,15,. (0/1, COARSE MESH REBALANCE, YES/NO) //
+1X,.NCRS =,15,. NUMBER OF COARSE MESH INTERVALS //1X,.NCRSHP =,
+15,. COARSE MESH REBALANCE SKIP PARAMETER)
520 7  'FORMAT(1X,.RUN NUMBER =,14)
521 7  'FORMAT(1X,.ANGULAR EDGE FLUXES //1X,.POINT NO.=)
522 7  'FORMAT(1X,.ANGULAR TRANSMISSION FRACTION (AT GAUSSIAN ORDINATES)=,
1/(10E12.5))
523 7  'FORMAT(1X,.ANGULAR BACKSCATTER FRACTION (AT GAUSSIAN ORDINATES)=,
1/(10E12.5))
524 7  'FORMAT(1X,.ENERGY DEPOSITION PROFILE CONTRIBUTION FOR THIS GROUP=
1/(10E12.5))
525 7  'FORMAT(1X,.CUMULATIVE ENERGY DEPOSITION PROFILE/(10E12.5))
526 7  'FORMAT(1X,.TSUM=.65UM)
527 7  'FORMAT(1X,.X,.TRANSMISSION=,E12.5,5X,.BACKSCATTER=,E12.5)
528 7  'STOP
529 7  'END
530 7  'SUBROUTINE INPUTS
531 7  'COMMON SCAT(12,12,2),AMU(12),A(12,2),B(12,2),EFLUX( 51),ESRC
+CUR(12),S( 51,12),SDISTR( 50),CFUX( 50,12),EF( 50),X( 51),
+MGT(12),DX,SIG(2),ALBEDO(2),ERR,ZERO,ETA(2),NX,NANG,NPOS,
+KSRC,INTRFC,NLAY,E,Z(2),AA(2),IPRNT,COSIN,ENTAB(100),DS(2),
+DLSTR(13,2),ELAMP(2),LMAX,PL(13,12),ISO,IEEDGE,ISKIP,DELE(50),
+EXCIT(2),STPR(2),DE,ITER(2),NGRPS,NG,ITMAX,IT,NATL(2),CSUM,
+CMESH(2,10),FBAL(10),IRBL,SUM1,NCRS,STPO,ELANO
532 7  'DIMENSION THK(2),Y(25)
533 7  'DIMENSION ZM(3),AM(3)
534 7  'DATA ZM,AM/13.,29.,79.,26.98,63.54,197./
535 7  'Y(N) = OUTPUT ARRAY FROM SUBROUTINE LEP
536 7  'PL(N,K) = LEGENDRE POLYNOMIAL OF ORDER (N-1) FOR THE K-TH
GAUSSIAN QUADRATURE ANGLE
537 7  'ZERO OUT DISTRIBUTED SOURCE, CROSS SECTION, ALBEDO
AND SCREENING FACTOR ARRAYS
538 7  'DO 101 J=1,100
539 7  'SDISTR(J)=0.
540 7  'DO 101 I=1,12
541 7  'S(J,I)=0.
542 7  'DO 102 M=1,2
543 7  '102 SIG(M)=ALBEDO(M)=ETA(M)=0.
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C      J 10,NX,NPOS,INTRFC
C      IF THERE IS NO MATERIAL INTERFACE (ONLY ONE SCATTERING
C      MATERIAL), INTRFC IS READ IN AS "0". IT IS THEN SET TO
C      NX+1 (OUTSIDE THE RIGHT BOUNDARY).
C      NLAY IS THE NUMBER OF MATERIAL LAYERS.
C      IF(INTRFC.EQ.0)INTRFC=NX+1
C      NLAY=1
C      IF(INTRFC.LT.NX.AND.INTRFC.GT.1)NLAY=2
C      FILL IN THE ATOMIC NUMBER (Z), ATOMIC WEIGHT (AA), AND MEAN
C      EXCITATION ENERGY(EXCIT) ARRAYS
C      READ 10,(MATL(I),I=1,NLAY)
C      DO 9 I=1,NLAY
C      MM=MATL(I)
C      EXCIT(I)=(9.76*ZM(MM)+58.5/(ZM(MM)**.19))*1.E-6
C      AA(I)=AM(MM)
C      Z(I)=ZM(MM)
C      9 Z(I)=ZM(MM)
C      DX = SPATIAL CELL WIDTH (CM/CM**2)
C      ERR = TOLERANCE FOR ITERATION CONVERGENCE
C      ZERO = "SAFE" VALUE FOR FLOATING POINT ZERO
C      READ 100,DX,ERR,ZERO
C      E = ELECTRON SOURCE ENERGY (GROUP 1) IN MEV
C      DE = ENERGY GROUP WIDTH IN MEV
C      READ 100,E
C      ESRC=E
C      READ COSINE OF INCIDENT BEAM
C      READ 100,COSIN
C      DEFAULT VALUES FOR DX, ERR, ZERO, COSIN
C      IF(DX.EQ.0.)DX=.01
C      IF(ERR.EQ.0.)ERR=.0001
C      IF(ZERO.EQ.0.)ZERO=.00001
C      IF(COSIN.EQ.0.)COSIN=1.0
C      IF SCATTERING IS ISOTROPIC, READ IN TOTAL CROSS
C      SECTION (SIG) AND ALBEDO (ALBEDO) FOR EACH MATERIAL.
C      IF(ISO.GT.0)READ 100,(SIG(M),ALBEDO(M),M=1,NLAY)
C      NANG=2*NPOS
C      PRINT 501,DX,ERR,ZERO,NX,NANG,NPOS
C      COMPUTE GAUSS QUADRATURE ANGLES AND WEIGHTS
C      LMAX=NANG+1
C      CALL GRULE(NANG,AMU,WGT)
C      DO 15 J=1,NPOS
C      AMU(LMAX-J)=AMU(J)
C      WGT(LMAX-J)=WGT(J)
15  IF(IPRNT.LT.1)GO TO 17
C      PRINT 508,(AMU(J),J=1,NPOS)
C      PRINT 509,(WGT(J),J=1,NPOS)

```



```

11=1
X1 0.0
D1 / I=1,NX
M=1
IF(I-INTRFC)33,32,31
31 M=2
GO TO 33
32 XAS=XX
I=INTRFC
33 XX=(I-IT)*DX/ELAMPR(M)*XXS
FAC=ALBEDO(M)/ELAMPR(M)*EXP(-XX/COSIN)
S(I,K)=0.
IF(ISO.EQ.0)GO TO 35
S(I,K)=FAC
GO TO 37
35 DO 38 N=1,LMAX
38 S(I,K)=S(I,K)+.5*FLOAT(2*N-1)*DLSTR(N,M)*PL(N,K)*FAC
37 CONTINUE
36 CONTINUE
DO 813 I=1,NX
SOISTR(I)=0.
DO 813 K=1,NANG
813 SOISTR(I)=SOISTR(I)+S(I,K)*WGT(K)
PRINT 814,(SOISTR(I),I=1,NX)
814 FORMAT(1X,'INITIAL ONCE SCATTERED SOURCE DISTRIBUTION',(10E12.5))
N1=NX-1
SUM=0.5*DX*(SOISTR(1)+SOISTR(NX))
DO 815 I=2,N1
815 SUM=SUM+SOISTR(I)*DX
PRINT 816,SUM
TH=NX*DX
SUM1=ALBEDO(1)*(1.-EXP(TH/ELAMPR(1)))
PRINT 817,SUM1
816 FORMAT(1X,'TRAPEZOIDAL INTEGRATION OF SOURCE FUNCTION',(E12.5))
817 FORMAT(1X,'EXACT INTEGRATION OF SOURCE FUNCTION',(E12.5))
FNORM=SUM1/SUM
DO 818 K=1,NANG
DO 818 I=1,NX
818 S(I,K)=FNORM*S(I,K)
REWIND 7
DO 819 K=1,NANG
819 WRITE(7)(S(I,K),I=1,NX)
40 CONTINUE
C-----FORMAT STATEMENTS-----
100 FORMAT(18I3)
501 FORMAT(1X,'DX=',(E12.5),' CM/CM2 ERR=,'
1F9.6,2X,'ZERO=,'F10.7,2X,'NX=,'13,2X,'NANG=,'13,2X,'NPOS=,'13)
502 FORMAT(1X,'ISOTROPIC SCATTERING')
503 FORMAT(1X,'SCREENED-RUTHERFORD SCATTERING. MATERIAL NO.=',12,
15X,'ETA',(E12.5))
504 FORMAT(1X,'ENERGY OF INCIDENT ELECTRON BEAM = ',F6.3,'MEV/'
11X,'GROUP WIDTH = ',F7.4,'MEV')
505 FORMAT(1X,'COSINE OF INCIDENT BEAM ANGLE = ',E12.5)
506 FORMAT(1X,'MATERIAL NO.=',12/1X,'SLAB THICKNESS = ',E12.5,1X
+,'(CM/CM2)/1X,'SIG(TOTAL CROSS SECTION) = ',E12.5,1X
+,'(CM2/GM)/1X,'SIG(SCATTERING CROSS SECTION) = ',E12.5,
+1X,'(CM2/GM)/1X,'SIG(ABSORPTION CROSS SECTION) = ',E12.5,
+1X,'(CM2/GM)/1X,'ELAMPR(TOTAL MEAN FREE PATH) = ',E12.5,
+1X,'(CM/CM2)/1X,'ALBEDO = ',F6.3)
508 FORMAT(1X,'GAUSSIAN ORDINATES AT WHICH ALL ANGULAR QUANTITIES ARE
1COMPUTED',(1X,10E12.5))
509 FORMAT(1X,'CORRESPONDING GAUSSIAN WEIGHTS',(1X,10E12.5))
C
E=ENTAB(1)

```

```

NG CROSS
C RETURN
END
SUBROUTINE GROUP
C
COMMON SCAT(12,12,2),AMU(12),A(12,2),B(12,2),ELUX( 51),ESRC
+CUR(12),S( 51,12),SDISTR( 50),CFUX( 50,12),EF( 50),X( 51),
+WT(12),DX,SIG(2),ALBEDO(2),ERR,ZERO,ETA(2),NX,NANG,MPOS,
+NSRC,INTRPC,NLAY,E,Z(2),AA(2),IPRNT,COSIN,ENTAB(100),DS(2),
+DISTR(13,2),ELAMPR(2),LMAX,PL(13,12),ISO,EDGE,ISKP,DELE(50),
+EXCIT(2),STPMR(2),DE,ITER(2),NGRPS,NG,ITMAX,IT,MATL(2),CSUM,
+CMESH(2,10),FBAL(10),IRBL,SUM1,NCRS,STPO,ELAMO
C
DIMENSION ER(20),RBETHE(20,3)
C
C S.D.A. RANGE VALUES IN ALUMINUM, COPPER AND GOLD, RESPECTIVELY,
C ARE GIVEN IN THE ARRAY RBETHE FOR 20 ELECTRON ENERGY VALUES
C CORRESPONDING TO THE ARRAY ER(COMPUTED BELOW). THIS ROUTINE
C CALCULATES ENERGY GROUP WIDTHS CORRESPONDING TO EQUAL PATHLENGTH
C INCREMENTS BY INTEGRATING AN APPROXIMATE EXPRESSION FOR THE
C STOPPING POWER.
C
DATA RBETHE/3.519E-4,7.074E-4,1.165E-3,1.716E-3,2.356E-3,3.08E-3,
13.883E-3,4.762E-3,5.714E-3,6.735E-3,7.822E-3,8.974E-3,1.019E-2,
21.148E-2,1.279E-2,1.417E-2,1.561E-2,1.710E-2,1.864E-2,3.641E-2,
34.558E-4,8.949E-4,1.452E-3,2.121E-3,2.892E-3,3.760E-3,4.719E-3,
45.766E-3,6.896E-3,8.108E-3,9.392E-3,1.075E-2,1.218E-2,1.368E-2,
51.524E-2,1.686E-2,1.858E-2,2.029E-2,2.209E-2,2.427E-2,2.794E-2,
61.453E-3,2.268E-3,3.228E-3,4.319E-3,5.537E-3,6.873E-3,8.322E-3,
79.876E-3,1.153E-2,1.328E-2,1.513E-2,1.706E-2,1.908E-2,2.117E-2,
82.335E-2,2.560E-2,2.792E-2,3.031E-2,3.5749E-2/
MM=MATL(1)
ER(1)=.01
DO 9 I=2,19
9 ER(I)=ER(I-1)*.005
ER(20)=.15
DIFT=10.
DO 10 I=1,20
10 DIF=ABS(ER(I)-E)
IF(DIF.GT.DIFT)GO TO 10
DIFT=DIF
ITAG=I
10 CONTINUE
ECUTE=ECUT(1)*1.03
RB=RBETHE(ITAG,MM)
DS(1)=DS(2)=RB/FLOAT(NGRPS)
IPSAVE=IPRNT
IPRNT=-1
SRANGE=0.
H=0.5*DS(1)
EV=E*1000.
DO 100 J=1,NGRPS
100 CALL RKINT(E,H),RETURNS(20,2)
20 ENTAB(J)=E
NG=J
IF(E.LE.ECUTF)GO TO 2
SRANGE=SRANGE+DS(1)
IF(SRANGE.GE.HB)GO TO 2
100 H=DS(1)
2 NGRPS=NG
IPRNT=IPSAVE
PRINT 3,NGRPS
PRINT 160,ECUTF
3 FORMAT(1X,'NUMBER OF ENERGY GROUPS =',I4)
EOLD=ESRC

```

```

5/ 'GE=0.
6/ JLO
DO 150 J=1,NGRPS
CALL RKINT(E,M),RETURNS(25,5)
25 DELE(J)=EOLD-E
SRANGE=SRANGE+M
IF(SRANGE.GE.RB)GO TO 5
150 EOLD=E
GO TO 85
5 DELE(NGRPS)=EOLD-ECUTF
85 PRINT 161,(J,ENTAB(J),DELE(J),J=1,NGRPS)
160 FORMAT(1X,'CUT OFF ENERGY MEV=',E12.5/13X,'EMID=',5X,'DELE=')
161 FORMAT(1X,15.2E12.5)
RETURN
END
SUBROUTINE RKINT(E,M),RETURNS(M,M)
REAL K1,K2,K3,K4
EOLD=E
IF(E.LE.0.0)RETURN N
K1=STP(E)*H
E=EOLD+.5*K1
IF(E.LE.0.0)RETURN N
K2=STP(E)*H
E=EOLD+.5*K2
IF(E.LE.0.0)RETURN N
K3=STP(E)*H
E=EOLD+K3
IF(E.LE.0.0)RETURN N
K4=STP(E)*H
E=EOLD+1./6.*(K1+2.*K2+2.*K3+K4)
IF(E.LE.0.0)RETURN N
RETURN M
END
SUBROUTINE LEP(Y,X,N)
DIMENSION Y(25)
Y(1)=1.
IF(N)1,1.2
1 RETURN
2 Y(2)=X
IF(N-1)1,1.3
3 DO 4 I=2,N
G=X*Y(I)
4 Y(I+1)=G-Y(I-1)*G-(G-Y(I-1))/FLOAT(I)
RETURN
END
SUBROUTINE CROSS
C
C CALCULATION OF SCREENED RUTHERFORD CROSS SECTION LEGENDRE
C COEFFICIENTS (SEE VOL.1, PAGES 209-214)
C
COMMON SCAT(12,12,2),AMU(12),A(12,2),B(12,2),EFLUX( 51),ESRC
+CUR(12),S( 51,12),SDISTR( 50),CFLUX( 50,12),EF( 50),X( 51),
+NGT(12),DX,SIG(2),ALBEDO(2),ERR,ZERO,ETA(2),NX,NANG,NPOS,
+NSRC,INTRFC,NLAY,E,Z(2),AA(2),IPRNT,COSIN,ENTAB(100),DS(2),
+DLSTR(13,2),ELAMPR(2),LMAX,PL(13,12),ISO,LEDGE,ISKIP,DELE(50),
+EXCIT(2),STPRN(2),DE,ITER(2),NGRPS,NG,ITMAX,IT,MATL(2),CSUM,
+CWESH(2,10),FBAL(10),IRBL,SUMI,NCRS,STPO,ELAMO
C
DIMENSION SGG(25),SIGSTR(25),C(25)
DATA INTIME/0/
DATA TPI/6.2831853/
C-----DEFINITION OF PARAMETERS-----
C
C LMAX = ORDER OF LEGENDRE CROSS SECTION EXPANSION

```



```

IF(DO.GT.DMIN)GO TO 1
DO 1=00
1 CONTINUE
IF(LM.EQ.LMAX)GO TO 2
DO 3 L=L1,LMAX
3 C(L)=Z1
2 CONTINUE
C-----COMPUTE UNCORRECTED SCATTERING CROSS SECTION LEGENDRE COEFFICIENTS
30 SGG(L)=SGG(1)-C(L)*ETA(M)*(1.+5*ETA(M))*SGG(1)
CLAST=C(LMAX)
C-----COMPUTE TRANSPORT CORRECTED SCATTERING CROSS SECTION LEGENDRE
C COEFFICIENTS
DO 31 L=1,LMAX
31 SIGSTR(L)=ETA(M)*(1.+5*ETA(M))*(CLAST-C(L))*SGG(1)
C-----COMPUTE TRANSPORT CORRECTED SCATTERING PROBABILITY LEGENDRE
C COEFFICIENTS
DO 50 L=1,LMAX
50 DLSTR(L,M)=SIGSTR(L)/SIGSTR(1)
C-----ABSORPTION CROSS SECTION
SIGA=1./DS(M)
C-----TRANSPORT CORRECTED TOTAL INTERACTION CROSS SECTION
SIG(M)=(SIGSTR(1)*SIGA)
C-----TRANSPORT CORRECTED MEAN FREE PATH
ELAMPR(M)=1./SIG(M)
C-----TRANSPORT CORRECTED ALBEDO
ALBEDO(M)=SIGSTR(1)/(SIGA+SIGSTR(1))
IF(IPRINT.GE.0)PRINT 590,E=EV,Z(M),STPMR(M),DS(M),ALBEDO(M)
COLL=DS(M)/ELAMPR(M)
IF(IPRINT.LT.0)PRINT 590,E=EV,DS(M),ALBEDO(M),COLL
590 FORMAT(1X,E=EV,Z(M),STPMR(M),DS(M),ALBEDO(M),COLL)
1* COLLISIONS = .E12.5)
IF(IPRINT.LE.0)GO TO 100
C-----PRINT DERIVED QUANTITIES
C
75 LMM=LMAX-1
IF(M.EQ.1)PRINT 502,LMM
76 PRINT 501,M,ETA(M),Z(M),AA(M)
PRINT 503
DO 51 L=1,LMAX
LL=L-1
51 PRINT 504,LL,C(L),SGG(L),SIGSTR(L),DLSTR(L,M)
100 CONTINUE
C-----RESTORE E TO MEV UNITS
E=E*EV
C-----FORMATS
501 FORMAT(1X,MATERIAL NUMBER,12.2X,ETA=.E12.5,
15X,Z(ATOMIC NO.)=.F7.3,5X,AA(ATOMIC WGT.)=.F7.3/)
502 FORMAT(1X,THIS IS AN S=.12.CALCULATION)
503 FORMAT(10X,SCREENED RUTHERFORD,5X,TRUE SCATTERING,4X,
+EXTENDED TRANSPORT,3X,TRANSPORT CORRECTED,9X,
+CORRECTION COEFFICIENTS,4X,CROSS SECTION,6X,
+CORRECTED CROSS,4X,SCATTERING PROBABILITY,5X,LE,12X,
+C(L),16X,(CM2/GM)*,8X,SECTION (CM2/GM)*,4X,
+LEGENDRE COEFFICIENTS/)
504 FORMAT(1X,15.5X,E16.9,8X,E16.9,4X,E16.9,8X,E16.9)
510 FORMAT(1X,ISOTROPIC SCATTERING)
590 FORMAT(1X,E=EV,Z(M),STPMR(M),DS(M),ALBEDO(M),COLL)
+1X,(MEV/(CM2/GM)), DS = .E12.5,CM/CM2,1X,ALBEDO=.E12.5)
C
RETURN
END

```

```

C
SUBROUTINE FLUXES(OUT,JS,NANGP,IS,NXP)
C
      DIMENSION IXP(10),IXM(10)
      DATA INTR/0/
      IF(INTR.GT.0)GO TO 19
      INTR=1
      NXP1=NXP+1
      IX=NX/NCRS
      DO 66 IRR=1,NCRS
        IXP(IRR)=IX*IRR
        66 IXM(IRR)=1+(IRR-1)*IX
      19 IF(IRBL.GT.0)GO TO 40
      IF(IX.EQ.-1)GO TO 40
      DO 60 IRR=1,NCRS
        DO 60 LL=1,2
        60 CMESH(LL,IRR)=0.
      40 NEG=0
      C
      C      ZERO OUT NEGATIVE FLUX FIXUP COUNTER FOR THIS ITERATION
      C
      C
      C
      IF(JS.NE.1)GO TO 10
      DO 15 I=1,NXP1
      15 EFUX(I)=0.
      10 DO 100 J=1,NPOS
        IRR=1
        IF(IX.EQ.-1)IRR=NCRS
        JR=J+JS+NANGP
        EF(J)=0.
        IF(IX.EQ.-1)EFUX(NXP1)=EFUX(NXP1)+WGT(J)*EF(J)
        DO 90 I=1,NX
          IF(IX.EQ.-1)GO TO 20
          M=1
          IF(IX.GE.INTRFC)M=2
          GO TO 21
        20 M=2
          IF(IX.LT.INTRFC)M=1
        21 IR=I+IS+NXP
          IF(IX.EQ.1)EFUX(IR)=EFUX(IR)+WGT(J)*EF(J)
          CFUX(IR,JR)=A(J,M)*EF(J)+B(J,M)*S(IR,JR)
          P=2.*CFUX(IR,JR)
          EF(J)=P-EF(J)
          IF(EF(J).GE.0.)GO TO 30
      C
      C      NEGATIVE FLUX FIXUP---SET FLUX TO ZERO AND INCREMENT COUNTER
      C
      NEG=NEG+1
      EF(J)=0.
      30 CONTINUE
      IF(IRBL.GT.0)GO TO 70
      IF(IX.EQ.-1)GO TO 61
      IF(IR.NE.1)IXP(IRR)GO TO 70
      CMESH(1,IRR)=CMESH(1,IRR)+WGT(J)*AMU(JR)*EF(J)
      IRR=IRR+1
      GO TO 70
      61 IF(IR.NE.1)IXM(IRR)GO TO 70
      CMESH(2,IRR)=CMESH(2,IRR)+WGT(J)*AMU(JR)*EF(J)
      IRR=IRR+1

```



```

+DLSTR(13,2),ELAMPR(2),LMAX,PL(13,12),ISO,IEDGE,ISKIP,DELE(SO),
+E' '(2),STPM(2),DE,ITER(2),NGRPS,ND,ITMAX,IT,MATL(2),CSU'
+Q' H(2,10),FBAL(10),IRBL,SUM1,NCRS,STPO,ELAMO

      COMPUTE NEW SCATTERING SOURCE

      DO 102 K=1,NANG
      DO 102 I=1,NX
      M=1
      IF(I,GE,INTRFCM-2)
      102 S(I,K)=(1.-ALBEDO(M))*SIG(M)*CFUX(I,K)

      CHECK SOURCE NORMALIZATION INTEGRAL

      DO 114 I=1,NX
      SDISTR(I)=0.
      DO 114 K=1,NANG
      SDISTR(I)=SDISTR(I)+S(I,K)*WGT(K)
      114 FORMAT(1X,'SCATTERING SOURCE DISTRIBUTION*',(1X,10E12.5))
      SUM=0.5*DX*(SDISTR(1)+SDISTR(NX))
      NM1=NX-1
      DO 115 I=2,NM1
      DO 115 K=1,NANG
      SUM=SUM+SDISTR(I)*DX
      115 SUM=SUM+SDISTR(I)*DX
      PRINT 111,SUM
      111 FORMAT(1X,'INTEGRAL OF SCATTERING SOURCE = ',E16.8)
      PHOM=(SUM1-CSUM)/SUM
      PRINT 117,PHOM
      117 FORMAT(1X,'SOURCE NORMALIZATION FACTOR=',E12.5)
      REWIND 7
      DO 104 K=1,NANG
      DO 104 I=1,NX
      DO 105 I=1,NX
      SDISTR(I)=S(I,K)*PHORM
      105 SDISTR(I)=S(I,K)*PHORM
      104 WRITE(7)(SDISTR(I),I=1,NX)
      DO 124 I=1,NX
      SDISTR(I)=0.
      DO 124 K=1,NANG
      SDISTR(I)=SDISTR(I)+S(I,K)*WGT(K)
      124 SDISTR(I)=SDISTR(I)+S(I,K)*WGT(K)
      IF(IPRINT,GT,0)PRINT 110,(SDISTR(I),I=1,NX)
      SUM1=SUM1-CSUM

      COMPUTE NEW SCATTERING CROSS SECTIONS, LEGENDRE COEFFICIENTS

      CALL CROSS

      COMPUTE NEW SCATTERING PROBABILITY

      DO 100 K=1,NANG
      DO 100 L=1,NADG
      DO 100 M=1,NLAY
      PROB=0.
      DO 101 N=1,LMAX
      DO 101 PROB=PROB+.5*FLOAT(2*N-1)*DLSTR(N,M)*PL(N,K)*PL(N,L)
      101 PROB=PROB+.5*FLOAT(2*N-1)*DLSTR(N,M)*PL(N,K)*PL(N,L)
      100 SCAT(K,L,M)=PROB

      COMPUTE CONSTANTS A AND B

      DO 106 J=1,NPOS
      DO 106 M=1,NLAY
      DENOM=2.*AMU(J)*SIG(M)*DX
      A(J,M)=2.*AMU(J)/DENOM
      106 B(J,M)=DX/DENOM
      RETURN
      END
      SUBROUTINE REBAL

```

```

C      JY'S SUBROUTINE PERFORMS THE COARSE MESH REBALANCE PROCEDURE
C      DIMED IN VOL.1, PAGES 199-201
C
C      DIMENSION A1(100),VV(10),FAC(10),CC(10),SS(10),SOLD(50,12)
C      DIMENSION IMARK(10)
C
C      COMMON SCAT(12,12,2),AMU(12),A(12,2),B(12,2),EFLUX( 51),ESRC
C      +CUR(12),S( 51,12),SDISTR( 50),CFUX( 50,12),EF( 50),X( 51),
C      +WGT(12),DX,SIG(2),ALBEDO(2),ERR,ZERO,ETA(2),NX,NANG,NCRS,
C      +SRC,INTRFC,NLAY,E,Z(2),AA(2),IPRNT,COSIN,ENTAB(100),DS(2),
C      +DLSTH(13,2),ELAMPR(2),LMAX,PL(13,12),ISO,IEGGE,ISKIP,DELE(50),
C      +EXCIT(2),STPHR(2),DE,ITER(2),NCRPS,NG,ITMAX,IT,MATL(2),CSUM,
C      +CMESH(2,10),FBAL(10),IRBL,SUM1,NCRS,STPO,ELANO
C
C      DATA A1/100*0./
C      DATA FAC/10*1./
C      DATA ITS/0/
C      IF(ITS.GT.0)GO TO 10
C      FAC(1)=FAC(NCRS)=.5
C      ITS=1
C      SA=SIG(1)=1.-ALBEDO(1)
C      IX=NX/NCRS
C      10 DO 11 I=1,NCRS
C      11 CC(I)=0.
C      IF(IT.GT.1)GO TO 16
C      REMIND 7
C      DO 15 J=1,NANG
C      15 READ(7)(SOLD(I,J),I=1,NX)
C      DO 14 I=1,NCRS
C      14 SS(I)=0.
C      DO 12 I=1,IX
C      12 IMARK(I)=J
C      DO 23 NC=2,NCRS
C      23 IMARK(NC)=IMARK(NC-1)+IX
C      DO 22 J=1,NANG
C      FC=WGT(J)*FAC(I)*DX*SA
C      DO 24 NC=1,NCRS
C      24 IM=IMARK(NC)
C      CC(NC)=CC(NC)+FC*CFUX(IM,J)
C      IF(IT.GT.1)GO TO 22
C      IF(NG.EQ.1)GO TO 22
C      FC=WGT(J)*DX*FAC(I)
C      DO 25 NC=1,NCRS
C      25 SS(NC)=SS(NC)+FC*SOLD(IM,J)
C      22 CONTINUE
C      12 CONTINUE
C      IF(NG.GT.1)GO TO 17
C      IF(IT.GT.1)GO TO 17
C      X1=0.
C      E1=1.
C      DO 18 I=1,NCRS
C      18 I=1,NCRS
C      X2=1-IX*DX
C      E2=EXP(-X2)/(ELAMPR(1)*COSIN))
C      SS(1)=COSIN*ALBEDO(1)*(E1-E2)
C      X1=X2
C      18 E1=E2
C      17 DO 1 I=1,NCRS
C      17 VV(I)=SS(I)
C      K=(NCRS+1)*(I-1)
C      J=K+1
C      L=K+2
C      A1(J)=CMESH(1,1)-CMESH(2,1)+CC(I)
C      IF(I.LT.NCRS)A1(L)=CMESH(1,1)
C      IF(I.GT.1)A1(K)=CMESH(2,1)
C      1 CONTINUE

```

```

CALL SIMQ(A1,VV,NCRS,KA)
DO 1 I=1,NCRS
4  R(I)=VV(I)
2  IF(KA.GT.0)PRINT 2
  FORMAT(IX,'SOLUTION SUSPECT')
  RETURN
END
SUBROUTINE SIMQ(A,B,N,KS)
C  THIS SUBROUTINE INVERTS A TRIANGULAR MATRIX
C
  DIMENSION A(1),B(1)
  TOL=0.0
  KS=0
  JJ=-N
  DO 65 J=1,N
    JY=J+1
    JJ=JJ+N+1
    BIGA=0
    IT=JJ-J
    DO 30 I=J,N
      IU=IT+1
      IF(ABS(BIGA)-ABS(A(IU)))20,30,30
20  BIGA=A(IU)
      IMAX=I
30  CONTINUE
35  KS=1
  RETURN
40  IT=J+N*(J-2)
  IT=ITMAX-J
  DO 50 K=J,N
    I1=I1+N
    I2=I1+IT
    SAVE=A(I1)
    A(I1)=A(I2)
    A(I2)=SAVE
    A(I2)=SAVE/BIGA
    SAVE=B(IMAX)
    B(IMAX)=B(J)
    B(J)=SAVE/BIGA
    IF(J=N)55,70,55
55  IQS=N*(J-1)
    DO 65 IX=JY,N
      IXJ=IQS+IX
      IT=J-IX
      DO 60 JX=JY,N
        IXJN=(JX-1)+IX
        JXJ=IXJN+IT
60  A(IXJ)=A(IXJ)-(A(IXJ)*A(JXJ))
65  B(IX)=B(IX)-(B(J)*A(IXJ))
70  NY=N-1
    IT=N*N
    DO 80 J=1,NY
      IA=IT-J
      IB=N-J
      IC=N
      DO 80 K=1,J
        B(1B)=B(1B)-A(1A)*B(IC)
      IA=IA-N
      IC=IC-1
80  RETURN
  END
  FUNCTION STP(E)
C  STOPPING POWER CALCULATION
C

```

```

C      DA      ITIME/0/
      IF(.TIME.GT.0)GO TO 1
      ITIME=1
      Z=13.
      A=26.98
      EXCIT=(9.76*Z+58.5/(Z**19))*1.E-6
      PRINT 77,EXCIT
      77 FORMAT(1X,'EXCITATION ENERGY=*,E12.5)
      AVOG=6.025E23
      RO=2.8179E-13
      XL2=ALOG(2.)
      FCC=6.2831853*.511*AVOG*RO*RO
C-----CONVERT ENERGY TO MC**2 UNITS
      1 EMEV=E
C      USE GROUP MIDPOINT ENERGY
      E=EMEV/.511
C
C      COMPUTE STOPPING POWER IN MEV/(GM/CM2)
      BETA=(SQRT(E*(E+2.)))/(E+1.)
      XLG=ALOG(E*(E+2.))/(2.*(EXCIT/.511)**2))
      FM=1.-BETA*(A*BETA+ (.125*(E-(2.*E+1.))*XL2)/(E+1.))**2
      STP=-((FCC*Z/A/BETA**2)*(XLG+FM)
C
      E=EMEV
      RETURN
      END

```

Program Listing

- UNISORC -

(Reference: Vol. I, Section IV.A.2)



```

C      E*(M)      * ATOMIC(RUTHERFORD) SCREENING CONSTANT FOR MATERIAL
C      S00(L)      M. IT IS CALCULATED FROM THE MOLIERE FORMULA
C      * LEGENDRE COEFFICIENTS OF SCATTERING CROSS SECTION
C      (UNCORRECTED)
C      SIGSTR(L)    * LEGENDRE COEFFICIENTS OF TRANSPORT CORRECTED
C      * SCATTERING CROSS SECTION
C      SIGA         * ABSORPTION CROSS SECTION (=ABSORB*SGG(1))
C      DLSTR(L,M)   * LEGENDRE COEFFICIENTS OF TRANSPORT CORRECTED
C      * SCATTERING PROBABILITY
C      STPMR(M)     * ELECTRON STOPPING POWER FOR MATERIAL M.
C      * UNITS ARE (MEV-CM**2/GM)
C      DS(M)        * ELECTRON PATH LENGTH IN MATERIAL M FOR ENERGY
C      * GROUP OF WIDTH DE.
C      * UNITS ARE (GM/CM**2)
C
C-----BEGIN CALCULATIONS-----
C      READ 555,ITMAX,NGRPS,IPRNT,ISO,IEDGE,ISKIP
C      PRINT 556,ITMAX,NGRPS,IPRNT,ISO,IEDGE,ISKIP
C
C      SET NG, THE ENERGY GROUP INDEX
C      NG=1
C      CALL INPUTS
C      IF(IPRNT.LT.3)GO TO 80
C
C-----PRINT OUT DERIVED CONSTANTS, BOUNDARY CONDITIONS, ETC.-----
C      PRINT 511
C      PRINT 502
C      PRINT 500.(AMU(J),J=1,NPOS)
C
C      PRECOMPUTATION OF A AND B ARRAYS (USED SEVERAL TIMES IN SUBROUTINE
C      FLUX
C
C      80 DO 100 J=1,NPOS
C      DO 100 M=1,MAT
C      A(J,M)=2.*AMU(J)/(2.*AMU(J)+SIG(M)*DX)
C      100 B(J,M)=DX/(2.*AMU(J)+SIG(M)*DX)
C      IF(IPRNT.LT.3) GO TO 81
C      DO 101 M=1,MAT
C      PRINT 511
C      PRINT 507,M
C      PRINT 500.(A(J,M),J=1,NPOS)
C      PRINT 511
C      PRINT 508,M
C      101 PRINT 500.(B(J,M),J=1,NPOS)
C
C      INITIALIZE DISTRIBUTED SOURCE
C      81 REMIND 7
C      DO 13 K=1,NANG
C      READ(7)(SDISTR(I),I=1,NX)
C      DO 13 II=1,NX
C      13 S(II,K)=SDISTR(II)
C
C-----ZERO OUT THE CELL CENTER FLUX-----
C      DO 11 K=1,NANG
C      DO 11 I=1,NX
C      11 CFLUX(I,K)=0.
C
C      COMPUTE COORDIANATES, X(I), OF CELL EDGES (NXP OF THEM)
C      NXP=NX+1
C      DO 12 I=1,NXP
C      12 X(I)=(I-1)*DX
C
C      TOTAL NUMBER OF ANGLES = NANG
C      C

```

```

C      NANG=2*NPOS
C      R...IND PLOT FILE BUFFER
C      REWIND 8
C      ENERGY GROUP (OUTER ITERATION) LOOP
C      1000 CONTINUE
C      PRINT 511
C-----MAIN (INNER) ITERATION LOOP-----
DO 350 IT=1,ITMAX
  OUT=TRAN*BACK=0.
  CALL FLUXES(TRAN,1,0,1,0)
  CALL FLUXES(BACK,-1,LMAX,-1,NXP)
  OUT=TRAN*BACK
C-----CALCULATE NEW SOURCE-----
  EMAX=0.
  REWIND 7
  DO 200 K=1,NANG
    READ(7) (SDISTR(I),I=1,NX)
    DO 200 I=1,NX
      SUM=SDISTR(I)
      M=1
      IF(1.GE.INTRFC)M=2
      DO 150 L=1,NANG
        SUM=SUM+SCAT(K,L,M)*CFUX(I,L)*WGT(L)*ALBEDO(M)*SIG(M)
        IF(ABS(SUM).LT.ZERO)GO TO 200
        ERROR=ABS((S(I,K)-SUM)/SUM)
        IF(ERROR.GT.EMAX)EMAX=ERROR
      200 S(I,K)=SUM
        IF(IPRINT.LT.1) GO TO 63
        PRINT 512,IT,EMAX,OUT,TRAN,BACK
      63 IF(EMAX.LT.ERR)GO TO 400
      350 CONTINUE
C-----FINAL PRINTOUT FOR THE NG-TH ENERGY GROUP-----
  400 PRINT 505,IT
    IF(IPRINT.LT.2) GO TO 64
C      PRINT CELL CENTER ANGULAR FLUXES
C
    DO 394 I=1,NX
      DO 394 K=LMAX,12
        CFUX(I,K)=0.0
        K1=-7
        395 K1=K1+8
        K2=K1+7
        IF(K2.GT.NANG)K2=NANG
        PRINT 509
        DO 300 I=1,NX,ISKIP
          PRINT 517,I,(CFUX(I,K),K=K1,K2)
          IF(K2.LT.NANG)GO TO 395
          517 FORMAT(1X,14,5X,8E12.5)
          IF(1.EQ.0)GO TO 380
C      COMPUTE EDGE ANGULAR FLUXES(ONLY VALID WHEN DIAMOND DIFF USED)
C      (THE SOURCE ARRAY IS BORROWED HERE FOR THE EDGE FLUX COMPUTATION)
          DO 579 J=1,NANG
            DO 579 J=1,NXP
              579 S(I,J)=0.0
            DO 580 J=1,NPOS
              DO 580 J=2,NXP
                580 S(I,J)=2.*CFUX(I-1,J)-S(I-1,J)
              DO 581 J=1,NPOS
                JJ=LMAX-J

```

```

DO 591 I=1,NX
  IF (XP=1)
    591 S(I,JJ)=2.*CFUX(IJ,JJ)-S(IJ+1,JJ)
  K1=-7
  396 K1=K1+8
  K2=K1+7
  IF (K2.GT.NANG) K2=NANG
  PRINT 510
  PRINT CELL EDGE ANGULAR FLUXES
  DO 306 I=1,NXP,ISKIP
    306 PRINT 517,I,(S(I,K),K=K1,K2)
    IF (K2.LT.NANG) GO TO 396
  380 CONTINUE
  84 PRINT 511
  PRINT CELL EDGE SCALAR FLUXES
  DO 310 I=1,NXP,ISKIP
    310 PRINT 516,I,(I).EFLUX(I)
  WRITE PLOT FILE
  WRITE(B)(X(I),EFLUX(I),I=1,NXP)
  DECREMENT THE ELECTRON ENERGY TO THE NEXT GROUP
  E=E-DE
  NG=NG+1
  IF (NG.GT.NGRPS) GO TO 600
  COMPUTE NEW CROSS SECTIONS, SCATTERING PROBABILITY, AND
  SCATTERING SOURCE
  CALL RECALC
  GO TO 1000
  500-----FORMAT STATEMENTS
  502 FORMAT(1X,12E11.4)
  505 FORMAT(1X,*,AMU(J)= X-COSINES*)
  507 FORMAT(1X,*,TOTAL NUMBER OF ITERATIONS = *,I5)
  508 FORMAT(1X,*,A - ARRAY FOR MATERIAL NO. *,I2)
  509 FORMAT(1X,*,B - ARRAY FOR MATERIAL NO. *,I2)
  510 FORMAT(1X,*,ANGULAR CENTER FLUXES*/1X,*,CELL NO.*/)
  511 FORMAT(//)
  512 FORMAT(1X,*,ITERATION *,I3,*, MAX. ERROR=*,E12.5,
    +*, EXIT CURRENT=*,E12.5,*, BACKSCATTER=*,E12.5)
  513 FORMAT(1X,*,CURRENT CONTRIBUTION FOR ITERATION NO. *,I5/1X,
    +*, TRANSMISSION=*,2X,E13.6/1X,*,BACKSCATTER=*,1X,E13.6)
  514 FORMAT(1X,*,TOTAL INCIDENT CURRENT =*,E12.5)
  515 FORMAT(1X,*,TOTAL SCALAR EDGE FLUX AT X(I)=/
    +2X,*,1*,8X,*,X(I)*,7X,*,FLUX(I)*)
  516 FORMAT(1X,13,2X,2E12.5)
  555 FORMAT(6I5)
  556 FORMAT(1X,*,ITMAX*,I5,*, (MAXIMUM ALLOWED NUMBER OF INNER ITERATION
    +S)*/1X,*,NGRPS*,I5,*, (NUMBER OF ENERGY GROUPS)*/1X,*,PRINT*,I5,*,P
    +RINTOUT CONTROL,(0/1/2/3 = FINAL SCALAR FLUX ONLY/ITERATIONS/ANGUL
    +AR FLUX/FULL DEBUG PRINTOUT)*/1X,*,ISO *,I5,*, (0/1, SCREENED RUTH
    +ERFORD SCATTERING/ISOTROPIC SCATTERING)*/1X,*,IEEDGE*,I5,
    +*, (0/1, ANGULAR EDGE FLUX COMPUTATION, NO/YES)*/
    +1X,*,ISKIP*,I5,*, (FLUX PRINTOUT PARAMETER - PRINT EVERY ISKIP-TH C
    +ELL)*/)
  510 FORMAT(1X,*,ANGULAR EDGE FLUXES*/1X,*,POINT NO.*/)
  600 STOP

```

```

END
C 9 OUTLINE INPUTS
COMMON SCAT(12,12,2),AMU(12),A(12,2),B(12,2),EFUX(101)
+CUR(12),S(101,12),SDISTR(100),CFUX(100,12),EF(100),X(101),
+WG(12),DX,SIG(2),ALBEDO(2),ERR,ZERO,ETA(2),NA,NANG,NPOS,
+KSRG,INTRFC,MAT,E,Z(2),AA(2),IPRNT,
+DLSTR(13,2),ELAMPR(2),LMAX,PL(13,12),ISO,IEDGE,ISKIP,
+EXCIT(2),STPWR(2),DE,ITER(2),NGRPS,NG,ITMAX,IT
C
C DIMENSION THK(2),Y(25)
C
C Y(N) = OUTPUT ARRAY FROM SUBROUTINE LEP
C PL(N,K) = LEGENDRE POLYNOMIAL OF ORDER (N-1) FOR THE K-TH
C GAUSSIAN QUADRATURE ANGLE
C
C ZERO OUT DISTRIBUTED SOURCE, CROSS SECTION, ALBEDO
C AND SCREENING FACTOR ARRAYS
C
DO 101 J=1,100
SDISTR(J)=0
DO 101 I=1,12
101 S(I,J)=0.
102 SIG(M)=ALBEDO(M)=ETA(M)=0.
C
C FILL IN THE ATOMIN NUMBER(Z), ATOMIC WEIGHT(AA), AND
C MEAN EXCITATION ENERGY(EXCIT) ARRAYS.
C (MATERIAL 1 = ALUMINUM, MATERIAL 2 = GOLD)
C
Z(1)=13.
Z(2)=79.
AA(1)=26.98
AA(2)=197.
EXCIT(1)=183.E-8
EXCIT(2)=797.E-6
C
C NX = NUMBER OF SPATIAL CELLS
C NPOS = NUMBER OF FIRST QUADRANT QUADRATURE ANGLES
C INTRFC = SPATIAL INDEX OF MATERIAL BOUNDARY INTERFACE
C
READ 10,NX,NPOS,INTRFC
C
C IF THERE IS NO MATERIAL INTERFACE (ONLY ONE SCATTERING
C MATERIAL), INTRFC IS READ IN AS "0". IT IS THEN SET TO
C NX+1 (OUTSIDE THE RIGHT BOUNDARY).
C
C MAT IS THE NUMBER OF MATERIALS.
C
IF(INTRFC.EQ.0)INTRFC=NX+1
MAT=1
IF(INTRFC.LT.NX.AND.INTRFC.GT.1)MAT=2
C
C DX = SPATIAL CELL WIDTH (CM/CM**2)
C ERR = TOLERANCE FOR ITERATION CONVERGENCE
C ZERO = "SAFE" VALUE FOR FLOATING POINT ZERO
C
READ 100,DX,ERR,ZERO
C
C E = ELECTRON SOURCE ENERGY (GROUP 1) IN MEV
C DE = ENERGY GROUP WIDTH IN MEV
C
READ 100,E,DE
C
C DEFAULT VALUES FOR DX, ERR, ZERO
C

```



```

C C SCREENED RUTHERFORD SCATTERING
C
C PROB=0.
DO 22 N=1,LMAX
22 PROB=PROB+0.5*FLOAT(2*N-1)*DLSTR(N,M)*PL(N,K)*PL(N,L)
21 SCAT(K,L,M)=PROB
C
C UNIFORMLY DISTRIBUTED ISOTROPIC SOURCE
C UNITS ARE (CM2/GM--STERADIAN)
C
C REWIND 7
C WIDTH=0.
DO 31 M=1,MAT
31 WIDTH=WIDTH+THK(M)
DO 32 K=1,NANG
DO 33 I=1,NX
33 SDISTR(I)=5/WIDTH
32 WRITE(7)(SDISTR(I),I=1,NX)
40 CONTINUE
C
C-----FORMAT STATEMENTS-----
10 FORMAT(18I3)
100 FORMAT(6F10.0)
501 FORMAT(/IX,DX=,E12.5,GM/CM2 ERR=,
1F9.6,2X,ZERO=,F10.7,2X,NX=,13,2X,NANG=,13,2X,NPOS=,13)
502 FORMAT(/IX,ISOTROPIC SCATTERING=)
503 FORMAT(/IX,SCREENED-RUTHERFORD SCATTERING, MATERIAL NO.,12,
15X,ETA =,E12.5)
504 FORMAT(1X,ENERGY OF INCIDENT ELECTRON BEAM = ,F6.3,MEV/
11X,GROUP WIDTH = ,F7.4,MEV)
506 FORMAT(/IX,MATERIAL NO.,12/1X,SLAB THICKNESS = ,E12.5,1X
+,(CM/CM2)/1X,SIG(TOTAL CROSS SECTION) = ,E12.5,1X
+,(CM2/GM)*1X,SIG(SCATTERING CROSS SECTION) = ,E12.5,1X
+,(CM2/GM)/1X,SIG(ABSORPTION CROSS SECTION) = ,E12.5,
+1X,(CM2/GM)/1X,ELAMPR(TOTAL MEAN FREE PATH) = ,E12.5,
+1X,(CM/CM2)/1X,ALBEDO = ,F6.3)
C
C RETURN
C END
C SUBROUTINE LEP(Y,X,N)
C DIMENSION Y(25)
C Y(1)=1.
C IF(N)1,1,2
1 RETURN
2 Y(2)=X
3 IF(N-1)1,1,3
3 DO 4 I=2,N
4 Y(I+1)=G-Y(I-1)+G-(G-Y(I-1))/FLOAT(I)
C RETURN
C END
C SUBROUTINE CROSS
C
C CALCULATION OF SCREENED RUTHERFORD CROSS SECTION LEGENDRE
C COEFFICIENTS(SEE VOL.1, PAGES 209-214)
C
C COMMON SCAT(12,12,2),AMU(12),A(12,2),B(12,2),EFLUX(101)
+CUR(12),S(101,12),SDISTR(100),CFLUX(100,12),EF(100),X(101),
+KRG(12),DX,SIG(2),ALBEDO(2),ERR,ZERO,ETA(2),NX,NANG,NPOS,
+KSRFC,INTRFC,MAT,E,Z(2),AA(2),1PRNT
+DLSTR(13,2),ELAMPR(2),LMAX,PL(13,12),ISO,LEDGE,ISKIP,
+EXCIT(2),STPR(2),DE,ITER(2),NGRPS,NG,ITMAX,IT
C
C DIMENSION SGG(25),SIGSTR(25),C(25),DS(2)

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```

C 9 INTIME/0/
C-----DEFINITION OF PARAMETERS-----
C
C LMAX = ORDER OF LEGENDRE CROSS SECTION EXPANSION
C PLUS ONE (MAXIMUM ALLOWED IS ORDER 36)
C
C AVOG = AVOGADRO'S NUMBER
C
C RO = ELECTRON RADIUS(CM.)
C
C SGG(L) = SCATTERING CROSS SECTION LEGENDRE COEFFICIENTS
C
C SIGSTR(L) = TRANSPORT CORRECT SCATTERING CROSS SECTION
C LEGENDRE COEFFICIENTS
C
C DLSTR(L,M) = TRANSPORT CORRECTED SCATTERING PROBABILITY
C
C C(L) = LEGENDRE COEFFICIENTS FOR MATERIAL M
C
C C(L) = SCREENED RUTHERFORD RECURSION RELATION COEFFICIENTS
C USED IN OBTAINING THE LEGENDRE COEFFICIENTS OF THE
C SCATTERING CROSS SECTION
C
C ELAMPR = TRANSPORT CORRECTED MEAN FREE PATH
C
C ALBEDO = TRANSPORT CORRECTED ALBEDO
C-----
C
C IF(INTIME.NE.0)GO TO 95
C AVOG=6.025E23
C RO=2.8179E-13
C XL2=ALOG(2.)
C FCC=6.2831853*.511*AVOG*RO*RO
C INTIME=1
C 95 CONTINUE
C
C-----CONVERT ENERGY TO MC*2 UNITS
C EMEV=E
C
C USE GROUP MIDPOINT ENERGY
C
C E=(EME*0.5*DE)/.511
C
C DO 100 M=1,MAT
C LMAX=2*NP05+1
C
C-----COMPUTE SCREENING CONSTANT, ETA(M)
C
C ETA(M)=.5*(Z(M)**(1./3.)/121.245)**2/(E*(2.*E))*(1.13+
C 13.76*(Z(M)/137.)*2*(1.*E)**2/(E*(2.*E)))
C
C-----COMPUTE TOTAL SCATTERING CROSS SECTION
C
C 10 SGG(1)=(1.+Z(M))*AVOG*RO*RO*Z(M)/AA(M)*((1.*E)/(E*(2.*E)))**2
C +/ETA(M)*(1.+5*ETA(M)))
C
C COMPUTE STOPPING POWER IN MEV/(GM/CM2)
C
C BETA=(SORT(E*(E+2.)))/(E+1.)
C XLG=ALOG(E*(E+2.)/(2.*(EXCIT(M)/.511)**2))
C FN=1.-BETA*BETA*(1.125*E*(2.*E+1)*XL2)/(E+1.)*2
C STPWR(M)=(FCC*Z(M)/AA(M)/BETA**2)*(XLG+FN)
C
C COMPUTE ELECTRON PATH LENGTH FOR ENERGY GROUP WIDTH DE
C IN MATERIAL M (DS(M) UNITS ARE GM/CM2)
C
C DS(M)=DE/STPWR(M)
C
C-----COMPUTATION OF SCREENED RUTHERFORD RECURSION COEFFICIENTS.C(L)
C E1=1./((1.+5*ETA(M))
C C(1)=0.

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```

C/ 1=ALOG(1.+2./ETA(M))-E1
Z 1/ETA(M)
DMIN=1.E6
DO 1 L=3,LMAX
AL=FLOAT(L-2)
BL=2.*AL+1.
CL=AL+1.
C(L)=(BL*(1.+ETA(M))+C(L-1)-CL*C(L-2)-BL*E1)/AL
DO=ABS(C(L)-Z)
IF(DD.GT.DMIN)GO TO 1
DMIN=DO
LM=L
1 CONTINUE
IF(LM.EQ.LMAX)GO TO 2
DO 3 L=L+1,LMAX
3 C(L)=Z1
2 CONTINUE
C-----COMPUTE UNCORRECTED SCATTERING CROSS SECTION LEGENDRE COEFFICIENTS
DO 30 L=2,LMAX
30 SGG(L)=SGG(1)-C(L)*ETA(M)*(1.+5*ETA(M))*SGG(1)
CLAST=C(LMAX)
C-----COMPUTE TRANSPORT CORRECTED SCATTERING CROSS SECTION LEGENDRE
COEFFICIENTS
DO 31 L=1,LMAX
31 SIGSTR(L)=ETA(M)*(1.+5*ETA(M))*(CLAST-C(L))*SGG(1)
C-----COMPUTE TRANSPORT CORRECTED SCATTERING PROBABILITY LEGENDRE
COEFFICIENTS
DO 50 L=1,LMAX
50 DLSTR(L,M)=SIGSTR(L)/SIGSTR(1)
C-----ABSORPTION CROSS SECTION
ABSORB=1./(DS(M)*SIGSTR(1))
SIGA=ABSORB*SGG(1)
C-----TRANSPORT CORRECTED TOTAL INTERACTION CROSS SECTION
SIG(M)=(SIGSTR(1)+SIGA)
C-----TRANSPORT CORRECTED MEAN FREE PATH
ELMPR(M)=1./SIG(M)
C-----TRANSPORT CORRECTED ALBEDO
ALBEDO(M)=SIGSTR(1)/(SIGA+SIGSTR(1))
PRINT 500, EMEV, Z(M), STPMR(M), DS(M)
PRINT 500, EMEV, Z(M), STPMR(M), DS(M)
C-----PRINT DERIVED QUANTITIES
C
75 LMM=LMAX-1
IF(M.EQ.1)PRINT 502,LMM
76 PRINT 501,M,ETA(M),Z(M),AA(M)
IF(IPRINT.LT.2)GO TO 100
PRINT 503
DO 51 L=1,LMAX
LL=L-1
51 PRINT 504,LL,C(L),SGG(L),SIGSTR(L),DLSTR(L,M)
100 CONTINUE
C-----RESTORE E TO MEV UNITS
E=EMEY
C
C-----FORMATS-----
501 FORMAT(1X,'MATERIAL NUMBER',12.2X,'ETA=',E12.5,
15X,'Z(ATOMIC NO.)=',F7.3,'AA(ATOMIC WGT.)=',F7.3/)
502 FORMAT(1X,'THIS IS AN S-',12,' CALCULATION')
503 FORMAT(10X,'SCREENED RUTHERFORD',5X,'TRUE SCATTERING',4X,
+EXTENDED TRANSPORT,3X,'TRANSPORT CORRECTED',9X,
+RECURRENCE COEFFICIENTS,4X,'CROSS SECTION',6X,
+CORRECTED CROSS,4X,'SCATTERING PROBABILITY',5X,'L',12X,
+CL(L),16X,'(CM2/GM)',8X,'SECTION (CM2/GM)',4X,
+LEGENDRE COEFFICIENTS')
504 FORMAT(1X,15.5X,E16.9,8X,E16.9,4X,E16.9,6X,E16.9)

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510 IF 'NAT(IX,0 ISOTROPIC SCATTERING*)
590 K AT(//IX,0 ENERGY = 0.19.5,0 MEV, Z = 0.17.2,0, STPMR = ( 1.5,
+IX,0 MEV/(GM/CM2), DS = 0.12.5,0 GM/CM2)
C
C RETURN
C END
C SUBROUTINE FLUXES(OUT,JS,NANGP,IS,NXP)
C
C COMMON SCAT(12,12,2),AMU(12),A(12,2),B(12,2),EFLUX(101)
+CUR(12),S(101,12),SOISTR(100),CFLUX(100,12),EF(100),X(101),
+MGT(12),DX,SIG(2),ALBEDO(2),ERR,ZERO,ETA(2),NA,NANG,NPOS,
+MSRC,INTRFC,MAT,E,Z(2),AA(2),IPRNT,
+DLSTR(13,2),ELAMPR(2),LMAX,PL(13,12),ISO,EDGE,ISKIP,
+EXCIT(2),STPMR(2),DE,ITER(2),NGRPS,NG,ITMAX,IT
C
C ZERO OUT NEGATIVE FLUX FIXUP COUNTER FOR THIS ITERATION
C
C NEG=0
C
C IF(JS.NE.1)GO TO 10
C NXP1=NXP+1
C DO 15 I=1,NXP1
C 15 EFLUX(I)=0.
C 10 DO 100 J=1,NPOS
C JR=J+JS+NANGP
C EF(J)=0.
C IF(15.EQ.-1)EFLUX(NXP1)=EFLUX(NXP1)+MGT(J)*EF(J)
C DO 90 I=1,NX
C 90 IF(15.EQ.-1)GO TO 20
C M=1
C IF(1,GE,INTRFC)M=2
C GO TO 21
C 20 M=2
C IF(1,LT,INTRFC)M=1
C 21 IR=15+NXP
C IF(15.EQ.1)EFLUX(IR)=EFLUX(IR)+MGT(J)*EF(J)
C CFLUX(IR,JR)=A(J,M)*EF(J)+B(J,M)*S(IR,JR)
C P=2.*CFLUX(IR,JR)
C EF(J)=P-EF(J)
C IF(EF(J).GE.0.)GO TO 30
C
C NEGATIVE FLUX FIXUP---SET FLUX TO ZERO AND INCREMENT COUNTER
C
C NEG=NEG+1
C EF(J)=0.
C 30 CONTINUE
C IF(15.EQ.-1)EFLUX(IR)=EFLUX(IR)+MGT(J)*EF(J)
C 90 CONTINUE
C IF(15.EQ.1)EFLUX(NXP1)=EFLUX(NXP1)+MGT(J)*EF(J)
C-----DETERMINE TOTAL OUTGOING CURRENT-----
C CUR(JR)=EF(J)*AMU(J)
C OUT=OUT+CUR(JR)*MGT(JR)
C 100 CONTINUE
C
C NEGATIVE FLUX FIXUP PRINTOUT
C
C IF(NEG.GT.0)PRINT 150,IT,NEG
C 150 FORMAT(1X,0 ITERATION NO.,0,15,5X,110,0 NEGATIVE FLUX FIXUPS*)
C RETURN
C END
C SUBROUTINE GRULE(N,X,M)
C-----GENERATE GAUSS QUADRATURE ABSCISSAS AND WEIGHTS---
C DIMENSION X(1),W(1)
C M=(N+1)/2
C ET=N*(N+1)

```



```

DC 114 I=1,NX
  ITR(I)=0.
DO 114 K=1,NANG
  114 SDISTR(I)=SDISTR(I)+S(I,K)*WGT(K)
  PRINT 110,(SDISTR(I),I=1,NX)
  110 FORMAT(1X,'SCATTERING SOURCE DISTRIBUTION',(1X,10E12.5))
  SUM=0.
  DO 115 I=1,NX
    115 SUM=SUM+SDISTR(I)*DX
  PRINT 111,SUM
  111 FORMAT(1X,'INTEGRAL OF SCATTERING SOURCE =',E16.9)
C
C COMPUTE NEW SCATTERING CROSS SECTIONS, LEGENDRE COEFFICIENTS
C
C CALL CROSS
C
C COMPUTE NEW SCATTERING PROBABILITY
C
DO 100 K=1,NANG
DO 100 L=1,NANG
DO 100 M=1,MAT
  PROB=0.
  DO 101 N=1,LMAX
    101 PROB=PROB+.5*FLOAT(2*N-1)*DLSTR(N,M)*PL(N,K)*PL(N,L)
    100 SCAT(K,L,M)=PROB
  C
  C COMPUTE CONSTANTS A AND B
  C
DO 106 J=1,NPOS
DO 106 M=1,MAT
  DENOM=2.*AMU(J)*SIG(M)*DX
  106 A(J,M)=2.*AMU(J)/DENOM
  B(J,M)=DX/DENOM
  RETURN
END

```

Program Listing

- MULTSCT -

(Reference: Vol. I, Section IV.B.3)

```

PROGRAM MULTSCT(INPUT,OUTPUT,TAPE2,TAPE6)

C
C
C PROGRAM MULTSCT IS A CONDENSED COLLISION MONTE CARLO PROGRAM FOR
C SOLVING THE ELECTRON TRANSPORT PROBLEM OF AN ELECTRON BEAM
C INCIDENT ON A MATERIAL SLAB. THE ALGORITHM IS BASED ON THE METHOD
C OF CURGUVEN AND DUMCUMB, WHICH WAS FURTHER DEVELOPED BY LOVE
C AND MYKLEBUST (SEE REFS.15-17, P.265, VOL. I). THE RANGE CORRE-
C SPONDING TO THE ENERGY OF THE INCIDENT ELECTRON IS DIVIDED INTO
C 100 EQUAL TRAJECTORY STEPS. THE ELECTRON ENERGY AT THE MIDPOINT
C OF EACH STEP IS COMPUTED IN SUBROUTINE "ESET" BY RUNGE-KUTTA INTE-
C GRATION OF THE STOPPING POWER FORMULA(EQ.40, P.231, VOL.1)

C
C
C COMMON/MC/NCOLL,CTHO,CTH,STH,PHI,CPHI,SPHI,EGY,CPHIO,SPHIO,DELTE,
C 1EO,N,IFLAG,MAXCOLL,LCHR,TLIM,XO,YO,ZO,X,Y,Z,ECUT,THICK(21),DELZ,
C 2NMAX,ZMIN,ZMAX,INRAN,XIRA,NPRNT,TLAST,NBATCH,ENTAB(100),INCODE(3),
C 3XIRC,MAT.ONSTY(3),STO(4,101),TBO,DRB,RTOTAL,BCHF(3),NRA,MARA,FORCE
C 4,THETA
C
C DIMENSION TIMR(11)

C
C SUBROUTINE "SETRUN" READS IN AND INITIALIZES THE VARIOUS PARAMETERS
C NECESSARY FOR THE MONTE CARLO CALCULATION

C
C CALL SETRUN
C FN=FLOAT(NMAX)
C COLLNO=0.
C MTM=1
C
C CALL SECOND(TIME)
C TIMR(1)=TIME
C
C 5 N=N+1

C
C "SETHS" INITIALIZES THE ELECTRON TRAJECTORY PARAMETERS AND IS
C CALLED AT THE START OF EACH TRAJECTORY.

C
C CALL SETHS
C "SCORE" RECORDS THE COORDINATES, DIRECTION AND ENERGY AT EACH
C ELECTRON COLLISION SITE

C
C CALL SCORE

C
C "PENET" TRANSLATES THE ELECTRON FROM ONE COLLISION SITE TO THE
C NEXT

C
C 20 CALL PENET
C IF(IFLAG.NE.0) GO TO 6

C
C "ENERGY" ASSIGNS THE ELECTRON ENERGY AFTER A COLLISION HAS
C OCCURRED

C
C CALL ENERGY
C IF(IFLAG.NE.0) GO TO 6
C CALL SCORE
C CALL ANGLES
C IF(IFLAG.NE.0)GO TO 6
C GO TO 20

C
C 6 CALL SCORE
C NRAP=NRAP+1
C DO 7 MCC=NRAP,MARA
C RA=РАНF(0)
C RA=РАНF(0)
C 7 CONTINUE

```



```

C      R( IN AND INITIALIZE ALL PARAMETERS NECESSARY FOR THE (
C      CALCULATION.
C
COMMON/MC/MC/COLL,CTHO,CTH,STM,PHI,CPHI,EGY,CPHIO,SPHIO,DELTE,
1EQ,N,IFLAG,MAXCOLL,LCHR,TLM,XO,YO,ZO,X,Y,Z,ECUT,THICK(21),DELZ,
2NMAX,ZMIN,ZMAX,INRAN,XIRA,NPRNT,TLAST,NBATCH,ENTAB(100),INCODE(3),
3XIRC,MAT,DNSTY(3),STO(4,101),TRO,DBR,RTOTAL,BCKF(3),NRA,NKRA,FORCE
4,THETA
DATA CPHIO,SPHIO,ZMIN,ZMAX,N/1.0,0.0,0.0,1000.0/
DATA XO,YO,ZO/3.0,0.0/
DATA DNSTY/1.1,8.96,19.3/
DATA BCKF/1.1,260.417/
READ 1,NMAX,MAT,NPRNT,(INCODE(1),I=1,3)
MAXCOLL=100
C
C      NMAX - TOTAL NO. OF MONTE CARLO HISTORIES
C      MAXCOLL - MAXIMUM NUMBER OF COLLISIONS PER HISTORY TO BE ALLOWED
C      NPRNT - DEBUG PRINT CONTROL PARAMETER
C      NPRNT EQUAL TO ZERO SUPPRESSES DEBUG PRINTOUT
C      INCODE- INPUT KEYS FOR BULK BACKSCATTER FRACTION, CHANGES IN
C      SLAB THICKNESSES, RANDOM NUMBER INITIALIZATION
C      TLM - TIME LIMIT(SEC) FOR CALCULATION
C      CTHO - INITIAL POLAR ANGLE OBLIQUITY COSINE OF BEAM
C      EO - ELECTRON BEAM ENERGY(KEV)
C      ECUT - ELECTRON CUT-OFF ENERGY
C
READ 3,TLM,CTHO,EO,ECUT
ED=.001, J
ECUT=.001*ECUT
IF(INCODE(1).GT.0)READ 3,BCKF(MAT)
IF(INCODE(1).GT.0)PRINT 30,BCKF(MAT)
30 FORMAT(1X,'BULK BACKSCATTER FRACTION =',F7.4)
CALL ESET
DTH=1.5E-5
C
C      FORCE - APPLIED ELECTRIC FIELD (EV/CM)
C
READ 3,FORCE
PRINT 35,FORCE
FORCE=FORCE/DNSTY(MAT)/1.E6
PRINT 36,FORCE
35 FORMAT(1X,'FIELD FORCE (EV/CM)=',E12.5)
36 FORMAT(1X,'FIELD FORCE(MEV PER G/CM2)=',E12.5)
DO 50 I=1,20
50 THICK(I)=I*OTH
20 FORMAT(8F10.0)
THICK(21)=3.15E-4
IF(INCODE(2).GT.0)READ 20,(THICK(I),I=1,21)
ZMAX=THICK(21)
INRAN=0
IF(INCODE(3).GT.0)READ 8,INRAN
EO - INITIAL PARTICLE ENERGY
XO,YO,ZO - INITIAL PARTICLE COORDINATES
PRINT 4,NMAX,MAXCOLL
IF(MAT.EQ.1)PRINT 15
IF(MAT.EQ.2)PRINT 16
IF(MAT.EQ.3)PRINT 17
C      USUALLY "ALUMINUM" IS ASSUMED INSTEAD OF "LUCITE" HERE
C
15 FORMAT(1X,'LUCITE+')
16 FORMAT(1X,'COPPER+')
17 FORMAT(1X,'GOLD+')
9 FORMAT(1X,'EO (INITIAL ENERGY MEV)=',E12.5//1X,'ECUT (LOW ENERGY

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```

1C1" OFF MEV) *E12.5//1X,*XO, YO, ZO, (SOURCE POINT COORDINATES)
3 (.3E12.5//1X,*CTHO (INCIDENT POLAR COSINE) *E12.5// CPHIO
4, S-HIO (INCIDENT AZIMUTH) *E12.5)
8 FORMAT(018)
1 FORMAT(615)
3 FORMAT(8F10.0)
4 FORMAT(//1X,*NMAX (NO. OF HISTORIES) *E16//1X,*MAXCOLL (MAXIMUM
1 ALLOWED NUMBER OF COLLISIONS) *,13)
10 N=0
XIRA=XIRC=INRAN
MXRA=MAXCOLL+10
NBATCH=NMAX/10
CALL RANSET(XIRA)
PRINT 9,EO,ECUT,XO,ZO,CTHO,CPHIO,SPHIO
RETURN
END
SUBROUTINE SETHIS
C
C INITIALIZE ELECTRON HISTORY
COMMON/MC/NCOLL,CTHO,CTH,STH,PHI,CPHI,SPHI,EGY,CPHIO,SPHIO,DELTE,
1EO,N,IFLAG,MAXCOLL,LCHR,TLIM,XO,ZO,X,Y,Z,ECUT,THICK(21),DELZ,
2NMAX,ZMIN,ZMAX,INRAN,XIRA,NPRINT,TLAST,NBATCH,ENTAB(100),INCODE(3),
3XIRC,MAT,DNSTY(3),STO(4,101),TBO,DRB,RTOTAL,BCKF(3),NRA,MARA,FORCE
4,THETA
LCHR=0
NCOLL=0
NRA=0
X=XO
Y=Y0
Z=ZO
CPHI=CPHIO
SPHI=SPHIO
PHI=0.0
CTH=CTHO=1.
THETA=0.
STH=SQRT(1.-CTH**2)
EGY=EO
IFLAG=0
DELTE=0.
RETURN
END
SUBROUTINE PENET
C
C CALCULATE COORDINATES OF NEXT COLLISION POINT AT DISTANCE "DRB"
C AWAY FROM PREVIOUS COLLISION POINT
C
COMMON/MC/NCOLL,CTHO,CTH,STH,PHI,CPHI,SPHI,EGY,CPHIO,SPHIO,DELTE,
1EO,N,IFLAG,MAXCOLL,LCHR,TLIM,XO,ZO,X,Y,Z,ECUT,THICK(21),DELZ,
2NMAX,ZMIN,ZMAX,INRAN,XIRA,NPRINT,TLAST,NBATCH,ENTAB(100),INCODE(3),
3XIRC,MAT,DNSTY(3),STO(4,101),TBO,DRB,RTOTAL,BCKF(3),NRA,MARA,FORCE
4,THETA
S=DRB
X1=X
Y1=Y
Z1=Z
DELZ=.25/EGY*FORCE=DRB*DRB
Z-Z1=S*CTH*DELZ
IF(Z.GT.ZMIN) GO TO 10
Z=ZMIN
S=(ZMIN-Z1)/CTH
IFLAG=1
IF(Z.LT.ZMAX) GO TO 11
Z=ZMAX
S=(ZMAX-Z1)/CTH
IFLAG=1
10
11

```

```

11 X-1+S+STM+CPHI
    +S+STM+SPHI
    NCOLL=NCOLL+1
    IF(NCOLL.EQ.MAXCOLL) IFLAG=1
    RETURN
END
SUBROUTINE ESET
C CALCULATE ELECTRON ENERGIES CORRESPONDING TO 100 EQUAL
C TRAJECTORY STEPS
C
COMMON/MC/NCOLL,CTHO,CTH,STH,PHI,CPHI,SPHI,EGY,CPHIO,SPHIO,DELTE,
1EO,N,IFLAG,MAXCOLL,LCHR,TLM,XO,YO,ZO,X,Y,Z,ECUT,THICK(21),DELZ,
2NMAX,ZMIN,ZMAX,INRAN,XIRA,NPRINT,TLAST,NBATCH,ENTAB(100),INCODE(3),
3XIRC,MAT,DNSTY(3),STO(4,101),TBO,DBR,RTOTAL,BCKF(3),NRA,MXRA,FORCE
4,THETA
DIMENSION ZM(3),AM(3)
DIMENSION ER(20),RBETHE(20,3)
C RBETHE IS A TABLE OF ELECTRON C.S.D.A. RANGES FOR 20 ENERGIES
C CORRESPONDING TO THE COMPUTED ARRAY "ER" BELOW, FOR ALUMINUM,
C COPPER AND GOLD. THIS TABLE IS NOT ALWAYS USED. IN THE PRESENT
C CALCULATION, THE RANGE FOR LUCITE(200 KEV) IS ENTERED VIA
C REPLACEMENT STATEMENT (SEE BELOW . . "RTOTAL =...")
C
DATA RBETHE/3.519E-4,7.074E-4,1.165E-3,1.716E-3,2.356E-3,3.08E-3,
13.883E-3,4.762E-3,5.714E-3,6.735E-3,7.822E-3,8.974E-3,1.019E-2,
21.146E-2,1.279E-2,1.417E-2,1.561E-2,1.710E-2,1.864E-2,2.034E-2,
34.556E-4,8.949E-4,1.453E-3,2.121E-3,2.892E-3,3.760E-3,4.719E-3,
45.766E-3,6.896E-3,8.106E-3,9.392E-3,1.075E-2,1.218E-2,1.368E-2,
51.524E-2,1.686E-2,1.855E-2,2.029E-2,2.209E-2,2.477E-2,2.794E-2,
61.453E-3,2.268E-3,3.226E-3,4.319E-3,5.537E-3,6.873E-3,8.322E-3,
79.876E-3,1.153E-2,1.328E-2,1.513E-2,1.706E-2,1.908E-2,2.117E-2,
82.335E-2,2.580E-2,2.792E-2,3.031E-2,3.274E-2,3.574E-2,
9DATA ZM,AM/5.901,25.,79.,10.785,63.54,197./
FD(X,CY)=CX/X*(ALOG(1.166*X)-CY)
ZA=ZM(MAT)
ER(1)=.01
DO 250 I=2,20
    ER(I)=ER(I-1)+.005
250 ER(I)=ER(I-1)+.005
BF=BCKF(MAT)
TBO=.02209+.10718*BF+.03009*BF*BF+.37555*BF*BF*BF
AA=AM(MAT)
RTOTAL=.27866E-3
DRB=RTOTAL/100.
I=1
H=RB=.5*DRB
EV=ED*1000.
C1=-78500.*ZA/AA
C2=ALOG(.001*(9.76*ZA+58.5)/((ZA**.19)))
EOLD=EV
11 CAY1=FD(EV,C1,C2)*H
    EV=EOLD+.5*CAY1
    CAY2=FD(EV,C1,C2)*H
    EV=EOLD+.5*CAY2
    CAY3=FD(EV,C1,C2)*H
    EV=EOLD+CAY3
    CAY4=FD(EV,C1,C2)*H
    ENTAB(1)=EOLD+1./6.*(CAY1+2.*CAY2+2.*CAY3+CAY4)
    EOLD=EV+ENTAB(1)
    IF(1.EQ.100) GO TO 105
    I=I+1
H=2.*RB
GO TO 11
105 PRINT 52
H=RB

```

```

C 50 I=1,100
C IF 51-H,ENTAB(1)
50 H=1-2-DB
51 FORMAT(1X,E12.5,10X,E12.5)
52 FORMAT(6X,=RANGE=,7X,=MIDPOINT ENERGY(KEV)=)
DO 53 I=1,100
53 ENTAB(I)=.001*ENTAB(1)
IF(ECUT.GT.ENTAB(100))RETURN
ECUT=ENTAB(100)
RETURN
END
SUBROUTINE ANGLES
C COMPUTE POLAR AND AZIMUTHAL ANGLES OF ELECTRON TRAJECTORY AFTER
C COLLISION
C
COMMON/MC/NCOLL,CTHO,CTH,STH,PHI,CPHI,EGY,CPHIO,SPHIO,DELTE,
1EO,N,IFLAG,MAXCOLL,LCHR,TLIM,XO,YO,ZO,X,Y,Z,ECUT,THICK(21),DELZ,
2NMAX,ZMIN,ZMAX,INRAN,XIRA,NPRINT,TLAST,NBATCH,ENTAB(100),INCODE(3),
3XIRC,MAT,DNSTY(3),STO(4,101),TBO,DBR,RTOTAL,BCKF(3),NRA,NXRA,FORCE
4,THETA
DATA TWOPI/6.283185307/
DTH=ABS(2.*DELZ/DBR)
IF(FORCE.GT.0.)THETA=THETA-DTH
IF(THETA.LT.0.)THETA=-THETA
IF(FORCE.LT.0.)THETA=THETA+DTH
CTH=COS(THETA)
STH=SQRT(1.-CTH*CTH)
RD=RANF(0)
T82=(TBO*EQ/EGY)*.2/RD
COM=(1.-T82)/(1.+T82)
SOM=SQRT(1.-COM*COM)
6 RC=RANF(0)
NRA=NRA+1
RHO=TWOPI*RC
CRHO=COS(RHO)
SRHO=SIN(RHO)
STH1=STH
CTH1=CTH
CTH=CTH1*COM+STH1*SOM*CRHO
STH=SQRT(1.-CTH*CTH)
IF(STH1.EQ.0.0.OR.STH.EQ.0.0) GO TO 1
C1=(COM-CTH*CTH1)/(STH*STH1)
S1=SRHO*SOM/STH
CPHI1=CPHI
SPHI1=SPHI
CPHI=CPHI1+C1*SPHI1*S1
SPHI=CPHI1+S1*SPHI1*C1
GO TO 2
1 CPHI=CRHO
SPHI=SRHO
2 CONTINUE
PHI=ATAN2(SPHI,CPHI)
IF(PHI.LT.0)PHI=PHI+TWOPI
THETA=ACOS(CTH)
THETA=ACOS(CTH)
RETURN
END
SUBROUTINE SCORE
C RECORD ELECTRON TRAJECTORY CHARACTERISTICS
C
COMMON/MC/NCOLL,CTHO,CTH,STH,PHI,CPHI,EGY,CPHIO,SPHIO,DELTE,
1EO,N,IFLAG,MAXCOLL,LCHR,TLIM,XO,YO,ZO,X,Y,Z,ECUT,THICK(21),DELZ,
2NMAX,ZMIN,ZMAX,INRAN,XIRA,NPRINT,TLAST,NBATCH,ENTAB(100),INCODE(3),
3XIRC,MAT,DNSTY(3),STO(4,101),TBO,DBR,RTOTAL,BCKF(3),NRA,NXRA,FORCE

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```

NY 4+1
NA I=NHIST+1
IOLD=1
I2TOP=1
Z1=0.
GO TO 101
5 Z=STO(1,L)
NCOLL=IFIX(ABS(STO(3,L))*.1)
EGY=STO(4,L)
MU=STO(2,L)
IF(MU)70,70,80
70 DO 71 I=1,19
71 IF(Z.LT.THICK(I).AND.Z1.GT.THICK(I))EFLNEG(I+1)=EFLNEG(I)+
11000.*EGY
GO TO 85
80 DO 81 I=1,20
81 IF(Z.GT.THICK(I).AND.Z1.LT.THICK(I))EFLPOS(I)=EFLPOS(I)+
11000.*EGY
85 Z1=Z
AMU=ABS(MU)
ARG=2.*AMU-1.
C
C EVALUATE LEGENDRE POLYNOMIALS
C
C CALL LEP(PL,ARG,5)
C
C CLASSIFY EMERGENT ENERGY
C
DO 25 K=1,10
25 IF(EGY.GT.ENERGY(K))GO TO 26
K=10
26 KC=K
EF=EGY/EO
KK=IFIX(20.*EF)+1
FA(I)=FB(I)=1.
DO 111 MA=2,5
FA(MA)=AMU*FA(MA-1)
111 FB(MA)=1000.*EGY*FB(MA-1)
C
C
C IF(Z.GT.ZMIN)GO TO 6
C IF(I2TOP.GT.20)I2TOP=20
C BACKSCATTERED CURRENT
C
DO 11 I=I2TOP,20
BC(I,KC)=BC(I,KC)+1.0
BTOT(I)=BTOT(I)+1.
C
C LEGENDRE COEFFICIENTS OF BACKSCATTERED ANGULAR DISTRIBUTION
C
DO 1101 LP=2,6
BPL(I,KC,LP-1)=BPL(I,KC,LP-1)+PL(LP)
1101 BPLTOT(I,LP-1)=BPLTOT(I,LP-1)+PL(LP)
11 CONTINUE
BFE(KK)=BFE(KK)+1.0
C
C BACKSCATTER ENERGY-ANGLE MOMENTS MATRIX
C
DO 1102 MA=1,5
DO 1102 MB=1,5
1102 BROW(MA,MB)=BROW(MA,MB)+FA(MA)*FB(MB)
EFLNEG(I)=EFLNEG(I)+1000.*EGY
GO TO 102
6 DO 10 IZ=1,21

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```

IF(Z.GT.THICK(I2))GO TO 10
I( 2,IG.1)GO TO 102
I( 2,IG.1)GO TO 102
IF(15.LE.IZTOP)GO TO 102
IZTOP=15
IQ=15-1
C
C TRANSMITTED CURRENT
C
DO 12 IT=IOLD,IQ
TC(IT,KC)=TC(IT,KC)+1.
TTOT(IT)=TTOT(IT)+1.
TFE(KK,IT)=TFE(KK,IT)+1.0
C
C TRANSMISSION ENERGY-ANGLE MOMENTS MATRIX
C
DO 1103 MA=1,5
DO 1103 MB=1,5
1103 TMOM(MA,MB,IT)=TMOM(MA,MB,IT)+FA(MA)*FB(MB)
C
C LEGENDRE COEFFICIENTS OF TRANSMISSION ANGULAR DISTRIBUTION
C
DO 1201 LP=2,6
TPL(IT,KC,LP-1)=TPL(IT,KC,LP-1)+PL(LP)
1201 TPLTOT(IT,LP-1)=TPLTOT(IT,LP-1)+PL(LP)
12 CONTINUE
IOLD=IZTOP
GO TO 102
10 CONTINUE
102 IF(STO(3,L).LT.0.0)GO TO 112
GO TO 101
112 IF(NH.LT.NNN)GO TO 101
KPUT=1
NH=0
1B=1B+1
WRITE(6)1B
C
C ENERGY FLOW CALCULATION FOR 20 ZONES
C
C *EFLNET* IS THE NET ENERGY DEPOSITION
C
EFLNET(1)=1000.*FN*EO-EFLPOS(1)+EFLNEG(2)-EFLNEG(1)
EFLNET(20)=-EFLPOS(20)-EFLNEG(20)+EFLPOS(19)
DO 86 I=2,19
86 EFLNET(I)=EFLNEG(I+1)-(EFLNEG(I)+EFLPOS(I))+EFLPOS(I-1)
DO 103 I=1,20
TTOT(I)=TTOT(I)/FN
BTOT(I)=BTOT(I)/FN
EFLPOS(I)=EFLPOS(I)/FN
EFLNEG(I)=EFLNEG(I)/FN
EFLNET(I)=EFLNET(I)/FN
C
C THE REMAINING SECTION OF CODE CALCULATES THE STATISTICAL
C STANDARD ERROR IN THE COMPUTED OUTPUT QUANTITIES. WHEN ALL
C HISTORIES HAVE BEEN PROCESSED, THE RESULTS ARE PRINTED OUT.
C
DO 1104 MA=1,5
DO 1104 MB=1,5
1104 TMOM(MA,MB,1)=TMOM(MA,MB,1)/FN
DO 103 LP=1,5
BPLTOT(I,LP)=BPLTOT(I,LP)/FN
TPLTOT(I,LP)=TPLTOT(I,LP)/FN
DO 103 KK=1,10
BPL(I,K,LP)=BPL(I,K,LP)/FN
TPL(I,K,LP)=TPL(I,K,LP)/FN
103 CONTINUE
DO 113 I=1,20

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```

DO 113 K=1,10
TC(I,K)=TC(I,K)/FN
113 IF (I.EQ.1) BC(I,K)=BC(I,K)/FN
DO 723 KK=1,20
BFE(KK)=BFE(KK)/FN
DO 723 I=1,20
TFC(KK,I)=TFC(KK,I)/FN
DO 1105 MA=1,5
DO 1105 MB=1,5
1105 RMOM(MA,MB)=RMOM(MA,MB)/FN
IF (I.EQ.10) CALL SECOND(TLAST)
DO 150 NM=1,2
DO 104 K=1,10
DO 105 I=1,20
GO TO (115,125),NM
115 TEMP(I)=TC(I,K)
TFC(I,K)=0.
GO TO 105
125 TEMP(I)=BC(I,K)
BC(I,K)=0.
105 CONTINUE
C
C SUBROUTINE 'STATS' STORES THE OUTPUT QUANTITIES AND COMPUTES
C THEIR STANDARD ERRORS
C
CALL STATS(TEMP,ISIG,20,KPUT,18)
WRITE(6)(TEMP(I),ISIG(I),I=1,20)
IF (I.EQ.10) GO TO 104
DO 156 I=1,20
IF (NM.EQ.1) TFC(I,K)=TEMP(I)
IF (NM.EQ.2) BC(I,K)=TEMP(I)
156 IG(I,K)=ISIG(I)
104 CONTINUE
DO 140 I=1,20
GO TO (141,142),NM
141 TEMP(I)=TTOT(I)
TTOT(I)=0.
GO TO 140
142 TEMP(I)=BTOT(I)
BTOT(I)=0.
140 CONTINUE
CALL STATS(TEMP,ISIG,20,KPUT,18)
WRITE(6)(TEMP(I),ISIG(I),I=1,20)
IF (I.EQ.10) GO TO 150
DO 143 I=1,20
IF (NM.EQ.1) TTOT(I)=TEMP(I)
IF (NM.EQ.2) BTOT(I)=TEMP(I)
143 IGOT(I)=ISIG(I)
IF (I.EQ.10) GO TO 150
PRINT 220
DO 1531 KK=1,2
I1=IL(KK)
I2=IH(KK)
GO TO (114,124),NM
114 PRINT 230
GO TO 153
124 PRINT 231
153 PRINT 236, (THICK(I),I=1,12)
236 PRINT 236, (TNM(I),I=1,12)
230 FORMAT(5X,'NM=',2X,10E12.5/3X,'E(KEV)=')
230 FORMAT(//53X,'TRANSMITTED CURRENT=')
231 FORMAT(//53X,'BACKSCATTERED CURRENT=')
234 FORMAT(53X,'SLAB THICKNESS/2X',(G/CM2),10E12.5)
DO 158 K=1,10
GO TO (1581,1582),NM
1581 PRINT 221,ENERG(K), (TC(I,K),IG(I,K),I=1,12)

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WRITE(2)ENERG(K),(TC(I,K),IG(I,K),I=1,12)
C O 158
1582 PA,NT 221,ENERG(K),(BC(I,K),IG(I,K),I=1,12)
WRITE(2)ENERG(K),(BC(I,K),IG(I,K),I=1,12)
158 CONTINUE
GO TO(1583,1584),MM
1583 PRINT 235,(TOT(I),IGTOT(I),I=1,12)
WRITE(2)((TOT(I),IGTOT(I),I=1,12)
GO TO 1531
1584 PRINT 235,(BTOT(I),IGTOT(I),I=1,12)
WRITE(2)((BTOT(I),IGTOT(I),I=1,12)
235 FORMAT(/2X,'TOTALS',1X,10(F9.5,13))
1531 CONTINUE
150 CONTINUE
DO 175 MM=1,3
DO 180 I=1,20
GO TO (181,182,183),MM
181 TEMP(I)=EFLNEG(I)
EFLNEG(I)=0.
GO TO 180
182 TEMP(I)=EFLPOS(I)
EFLPOS(I)=0.
GO TO 180
183 TEMP(I)=EFLNET(I)
EFLNET(I)=0.
180 CONTINUE
CALL STAYS(TEMP,ISIG,20,KPUT,18)
WRITE(6),TEMP(I),ISIG(I),I=1,20)
IF(18-LT,10)GO TO 175
DO 184 I=1,20
IF(MM.EQ.1)EFLNEG(I)=TEMP(I)
IF(MM.EQ.2)EFLPOS(I)=TEMP(I)
IF(MM.EQ.3)EFLNET(I)=TEMP(I)
184 IGTOT(I)=ISIG(I)
175 CONTINUE
IF(18-LT,10)GO TO 176
WRITE(2)(EFLNET(I),I=1,20)
ZERO=0.
I2=1
PRINT 185
PRINT 186,I2,ZERO,EFLNEG(1),THICK(1),EFLPOS(1),EFLNET(1)
DO 187 I=2,20
187 PRINT 186,I,THICK(I-1),EFLNEG(I),THICK(I),EFLPOS(I),EFLNET(I)
185 FORMAT(/30X,'ENERGY DEPOSITION PROFILE',10X,'ZONE',6X,'Z',10X,
1-EFLNEG,8X,'Z',8X,'EFLPOS',5X,'EFLNET')
186 FORMAT(12X,12,1X,E12.5,1X,F10.7,E12.5,1X,F10.7,1X,F10.7)
176 DO 350 MM=1,2
DO 300 LP=1,5
DO 301 I=1,20
GO TO (302,312),MM
302 TEMP(I)=TPL(I,K,LP)
TPL(I,K,LP)=0.
GO TO 301
312 TEMP(I)=BPL(I,K,LP)
BPL(I,K,LP)=0.
301 CONTINUE
CALL STAYS(TEMP,ISIG,20,KPUT,18)
WRITE(6)(TEMP(I),ISIG(I),I=1,20)
IF(18-LT,10)GO TO 300
DO 303 I=1,20
TPL(I,K,LP)=TEMP(I)
303 IPL(I,K,LP)=ISIG(I)
300 CONTINUE
DO 320 LP=1,5
DO 321 I=1,20

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322      GO TO (322,332),MM
      .(I)=TPLTOT(I,LP)
      TPLTOT(I,LP)=0.
      GO TO 321
322      TEMP(I)=8PLTOT(I,LP)
      8PLTOT(I,LP)=0.
321      CONTINUE
      CALL STATS(TEMP,ISIG,20,KPUT,18)
      WRITE(6)(TEMP(I),ISIG(I),I=1,20)
      IF(18.LT.10)GO TO 320
      DO 323 I=1,20
      TPLTOT(I,LP)=TEMP(I)
323      IG(I,LP)=ISIG(I)
320      CONTINUE
      IF(18.LT.10)GO TO 350
      PRINT 220
      GO TO (314,324),MM
314      PRINT 340
      GO TO 325
324      PRINT 341
325      E1=1000.*ED
      DO 326 K=1,10
      E2=ENERG(K)
      PRINT 342,E1,E2
      E1=E2
      PRINT 343
      DO 327 I=1,20
      GO TO(3271,3272),MM
3271      WRITE(2)TC(I,K),(TPL(I,K,LP),IPL(I,K,LP),LP=1,5)
      IF(TC(I,K).NE.0.0)PRINT 344,THICK(I),TC(I,K),(TPL(I,K,LP),
      1IPL(I,K,LP),LP=1,5)
      GO TO 327
3272      WRITE(2)BC(I,K),(TPL(I,K,LP),IPL(I,K,LP),LP=1,5)
      IF(BC(I,K).NE.0.0)PRINT 344,THICK(I),BC(I,K),(TPL(I,K,LP),
      1IPL(I,K,LP),LP=1,5)
327      CONTINUE
326      CONTINUE
      PRINT 345
      PRINT 343
      DO 328 I=1,20
      GO TO (3281,3282),MM
3281      WRITE(2)TTOT(I),(TPLTOT(I,LP),IG(I,LP),LP=1,5)
      IF(TTOT(I).NE.0.0)PRINT 344,THICK(I),TTOT(I),(TPLTOT(I,LP),
      1IG(I,LP),LP=1,5)
      GO TO 328
3282      WRITE(2)BTOT(I),(TPLTOT(I,LP),IG(I,LP),LP=1,5)
      IF(BTOT(I).NE.0.0)PRINT 344,THICK(I),BTOT(I),(TPLTOT(I,LP),
      1IG(I,LP),LP=1,5)
328      CONTINUE
350      CONTINUE
      DO 725 I=1,20
      DO 726 K=1,20
      TEMP(K)=TFE(K,I)
      TFE(K,I)=0.
726      CALL STATS(TEMP,ISIG,20,KPUT,18)
      WRITE(6)(TEMP(K),ISIG(K),K=1,20)
      IF(18.LT.10)GO TO 725
      DO 727 K=1,20
      TFE(K,I)=TEMP(K)
727      IFE(K,I)=ISIG(K)
725      CONTINUE
      DO 728 K=1,20
      TEMP(K)=BFE(K)
      BFE(K)=0.
728      CALL STATS(TEMP,ISIG,20,KPUT,18)
      WRITE(6)(TEMP(I),ISIG(I),I=1,20)

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17 8.LT.10)GO TO 770
DO 30 K=1,20
  BFE(K)=TEMP(1)
730 IBFE(K)=ISIG(K)
770 DO 775 MA=1,5
  DO 775 MB=1,5
  DO 771 I=1,20
    TEMP(I)=TMOM(MA,MB,I)
771 TMOM(MA,MB,I)=0.
  CALL STAS(TEMP,ISIG,20,KPUT,18)
  WRITE(6)(TEMP(I),ISIG(I),I=1,20)
  IF(18.LT.10)GO TO 775
  DO 772 I=1,20
    TMOM(MA,MB,I)=TEMP(I)
772 ITMOM(MA,MB,I)=ISIG(I)
775 CONTINUE
  DO 785 MA=1,5
  DO 785 MB=1,5
  DO 784 MB=1,5
    TEMP(MB)=BMOM(MA,MB)
784 BMOM(MA,MB)=0.
  CALL STAS(TEMP,ISIG,5,KPUT,18)
  WRITE(6)(TEMP(I),ISIG(I),I=1,5)
  IF(18.LT.10)GO TO 785
  DO 783 MB=1,5
    BMOM(MA,MB)=TEMP(MB)
783 ITBMOM(MA,MB)=ISIG(MB)
785 CONTINUE
  IF(18.LT.10)GO TO 800
  PRINT 761
761 FORMAT(10/42X,'EMERGENT ENERGY DISTRIBUTION - TRANSMISSION*')
  I1=1
  I2=5
  IMAK=20
  IF(12.GT.IMAK)I2=IMAK
  PRINT 751,(THICK(I),I=1,12)
751 FORMAT(//1X,'THICKNESS(G/CM2)',5(E12.5,4X))
  PRINT 752
752 FORMAT(/8X,'E/E0')
  DO 753 K=1,20
753 PRINT 754,EFAC(K),(TFC(K,I),I=1,12)
754 FORMAT(8X,F4.2,2X,5(E12.5,14))
  IF(12.EQ.IMAK)GO TO 755
  I1=11*5
  I2=12*5
  GO TO 756
755 PRINT 757
757 FORMAT(//2X,'EMERGENT ENERGY DISTRIBUTION - BACKSCATTER*')
  PRINT 752
  DO 758 K=1,20
758 PRINT 759,EFAC(K),BFE(K),IBFE(K)
759 FORMAT(8X,F4.2,2X,E12.5,14)
  DO 760 I=1,IMAK
760 WRITE(2)IMAX,(EFAC(K),TFC(K,I),I=1,12),K=1,20)
  WRITE(2)(EFAC(K),BFE(K),IBFE(K),K=1,20)
  PRINT 789
789 FORMAT(1X,'MOMENTS MATRICES - TRANSMISSION*,10X,'ENERGY (DOWN) -
  1 ANGLE(ACROSS)*')
791 FORMAT(//1X,'THICKNESS NO.',13,5X,E12.5,' G/CM2*')
  DO 790 I=1,20
  IF(TTOT(I).EQ.0.0)GO TO 710
  DO 711 MA=1,5
  DO 711 MB=1,5
711 TMOM(MA,MB,I)=TMOM(MA,MB,I)/TTOT(I)
710 PRINT 791,I,THICK(I)
  DO 792 MB=1,5
792 PRINT 793,(TMOM(MA,MB,I),MA=1,5),(ITMOM(MA,MB,I),MA=1,5)

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790 CONTINUE
793 F AT(1X,5E12.5,10X,515)
P .T 794
794 FORMAT(//1X, 'MOMENT MATRIX - - BACKSCATTER', 10X, 'ENERGY (DOWN) - -'
1ANGLE(ACROSS)')
DO 712 MA=1,5
DO 712 MB=1,5
712 BROW(MA,MB)=BROW(MA,MB)/BTOT(20)
DO 795 MB=1,5
795 PRINT 793, (BROW(MA,MB), MA=1,5), (BROW(MA,MB), MA=1,5)
800 CONTINUE
340 FORMAT(1X, 'LEGENDRE COEFFICIENTS - TRANSMISSION')
341 FORMAT(1X, 'LEGENDRE COEFFICIENTS - BACKSCATTER')
342 FORMAT(1X, 'ENERGY GROUP UPPER AND LOWER BOUNDS - ',
1F5.2, 'KEV TO', F5.2, 'KEV')
343 FORMAT(4X, 'THICKNESS', 12X, 'L', 7X, 'Q', 10X, '12X', 2X, '12X',
13X, '12X', 4X, '12X', 5X)
344 FORMAT(1X, E12.5, 10X, F10.7, 3X, 5(F10.7, 13))
345 FORMAT(1X, 'TOTAL OF ALL ENERGIES')
IF(18.EQ.10) PRINT 499, NHIST
101 CONTINUE
220 FORMAT(///)
221 FORMAT(1X, F7.3, 1X, 10(F9.5, 13))
499 FORMAT(//1X, 'TOTAL NUMBER OF HISTORIES PROCESSED = ', I10)
Z=STO(1, LCHR)
DO 600 I=1, 20
IF(1Z.GT. THICK(1)) GO TO 600
IC=1
GO TO 601
600 CONTINUE
IF(1Z.LE.0.0) RETURN
IC=20
601 CHG(IC)=CHG(IC)+1./FNN
IF(1B.LT.10) RETURN
PRINT 602
DO 603 I=1, 20
603 PRINT 604, I, CHG(I)
602 FORMAT(//9X, 'CHARGE DEPOSITION PROFILE', 10X, 'ZONE', 10X, 'CHARGE')
604 FORMAT(11X, 12.9A, F9.7)
PRINT 714
714 FORMAT(///)
RETURN
END
SUBROUTINE LEP(Y,X,N)
C
C COMPUTE LEGENDRE POLYNOMIALS
C
DIMENSION Y(1)
Y(1)=1.
IF(N) 1,1,2
1 RETURN
2 Y(2)=X
IF(N-1) 1,1,3
3 DO 4 I=2,N
G=X*Y(I)
4 Y(I+1)=G-Y(I-1)+G-(G-Y(I-1))/FLOAT(I)
RETURN
END
SUBROUTINE STATS(XVAL, ISIG, N, K, I8)
C
C COMPUTATION OF STATISTICAL STANDARD ERRORS (SEE VOL. I., P238.)
C
DIMENSION XVAL(1), ISIG(1), AVE(20), SIG(20)
DIMENSION ECAGE(4500), ECSIG(4500)
IF(18.NE.1) GO TO 2
DO 1 I=1,N

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```

AVE(I) = XVAL(I)
1  S  I)=99
1  S  I)=0.0
10 DO 100 I=1,N
    INK=I-1
    ECARE(INK)=AVE(I)
    100 EC SIG(INK)=SIG(I)
    N=N+1
    RETURN
2  NSORS=IB
    DO 200 I=1,N
        INK=I-1
        AVE(I)=ECARE(INK)
    200 SIG(I)=EC SIG(INK)
    DO 3 I=1,N
        SIG(I)=(NSORS-2.0)/(NSORS-1.0)*SIG(I)+(XVAL(I)-AVE(I))*2/NSORS
        AVE(I)=(NSORS-1.0)*AVE(I)+XVAL(I)/NSORS
        XVAL(I) = AVE(I)
        ERROR=SQRT(SIG(I)/NSORS)
        ISIG(I)=99
        IF(AVE(I).EQ.0.0) GO TO 3
        ISIG(I)=100.0*ERROR/ABS(AVE(I))+.50001
        IF(ISIG(I).GT.99) ISIG(I)=99
    3  CONTINUE
    GO TO 10
END
SUBROUTINE ENERGY
COMMON/MC/NCOLL,CTHO,CTH,STH,PHI,CPHI,SPHI,EGY,CPHIO,SPHIO,DELTE,
1EQ,N,IFLAG,MAXCOLL,LCHR,TLM,XO,YO,ZO,X,Y,Z,ECUT,THICK(21),DELZ,
2NMAX,ZMIN,ZMAX,INRAN,XIRA,NPRINT,TLAST,NBATCH,ENTAB(100),INCODE(3),
3XIRC,MAT,DNSTY(3),STO(4,101),TBO,DRB,RTOTAL,BCKF(3),NRA,NKRA,FORCE
4,THETA
DATA P12/1.570796327/
PM=PO-1
IF(FORCE.LT.0.)PM=-1.
IF(THETA.GT.P12)PM=-1.
EOLD=EGY
DO 1 I=1,100
1 IF(EOLD.GE.ENTAB(I))GO TO 2
I=100
2 IM=I
IF(I.EQ.1)IM=2
IM1=IM-1
STP=(ENTAB(IM1)-ENTAB(IM))/DRB
DELTE=DELTE-STP*PM*DELZ*CTH+FORCE*CTH*(DRB*PM*PQ*DELZ*CTH)
EGY=ENTAB(NCOLL)+DELTE
RETURN
END

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Program Listing

- MCEL -

(Reference: Vol. I, Section IV.B.2)



```

C
C
C
SUBROUTINE "PROC". IN THIS WAY THE CALCULATION CAN BE CONTINUED
* WITHOUT LOSING THE DATA OBTAINED ITMUS FAR DUE TO A COMPUTE TIME
OY. RUN.

IF(NBATCH.EQ.0)GO TO 60
NTEST=N-NBATCH*(N/NBATCH)
IF(NTEST.NE.0)GO TO 60
MTM=MTM+1
CALL SECOND(TIME)
TIMR(TIM)=TIME
DELT=TIME-TIMR(1)
AVGT=DELT/FLOAT(MTM-1)
TREM=TIM-TIMR-DELT
TLEFT=1.5*AVGT
IPCT=10*(MTM-1)
PRINT 75,IPCT,TREM
75 FORMAT(1X,'THE PROBLEM IS',14,' PERCENT COMPLETE.',1X,
1*TIME TO FINISH IS',F12.3,'SECONDS')
IF(IPCT.EQ.100)GO TO 100
:(TREM.NE.0)GO TO 100
:(TREM.NE.0)GO TO 5
GO TO 100
60 IF(N-NMAX)5,100,100
2 FORMAT(1X,'E16.9)
100 PRINT 64,COLLNG
64 CALL RANGET(IIRC)
64 FORMAT(//1X,'AVERAGE NUMBER OF COLLISIONS PER HISTORY = ',F7.3)
IF(NBATCH.EQ.0)GO TO 200
MTM1=MTM-1
PRINT 65,MTM1,NBATCH
65 FORMAT(//1X,'THERE ARE ',14,' BATCHES OF',15,' HISTORIES')
AVGT=0.
SUM=TIMR(1)
DO 66 M=2,MTM
TIMR(M)=TIMR(M)-SUM
SUM=SUM+TIMR(M)
AVGT=AVGT+TIMR(M)
MM=M-1
66 PRINT 67,MM,TIMR(M)
67 FORMAT(1X,'BATCH NO.',14,' TIME = ',F8.3,' SECONDS')
AVGT=AVGT/FLOAT(MTM1)
PRINT 68,AVGT
68 FORMAT(//1X,'AVERAGE TIME PER BATCH = ',F8.3,' SECONDS')
200 PRINT 190
190 FORMAT(6X,'INRAN',11X,'IRA',12X,'IRC')
PRINT 195,INRAN,XIRA,XIRC
195 FORMAT(3(1X,016))
STOP
END
SUBROUTINE SETRUN
READ IN AND INITIALIZE ALL PARAMETERS NECESSARY FOR THE
CALCULATION.
COMMON/MC/STO(7,601),NCOLL,CTHO,CTH,STH,PHI,CPHI,SPHI,EGY,CPHIO,IC
1,SPHIO,EO,SPAC,AMBDA,N,IFLAG,MAXCOLL,LCHR,STAB(200),ETAB(200),TLIM
2,XO,YO,ZO,X,Y,Z,ECUT,CLAB,THICK(11),NMAX,ZMIN,ZMAX,1E,INRAN,XIRA
3,NPRNT,TLAST,CTHC,NBATCH,ETA,PATH,S,SIGTAB(200),ENTAB(200),XIRC
4,ELOW,MAT,DNSTY(2),ESAVE,INEL
DIMENSION INCODE(3),TK(11,2)
DATA CPHIO,SPHIO,PI,ZMIN,ZMAX,N/1,0,0,0,3,14159265,0,0,1000.,0/
DATA ECUT,SPAC,XO,YO,ZO/0.,1.,3,0./
DATA TK/1.,1.,5.,1.,5.,2.,4.,5.,7.,10.,15.,1000.,
1,3.,5.,7.,1.,5.,2.,2,5,3.,4.,5.,1000./
DATA DNSTY/2,7,19,3/
READ 1,NMAX,MAXCOLL,NPRNT

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RETURN
C
C SUBROUTINE SETHIS
C
C INITIALIZE ELECTRON HISTORY
COMMON/MC/STO(7,601),NCOLL,CTHO,CTH,STH,PHI,CPHI,SPHI,EGY,CPHIO,IC
1,SPHIO,EO,SFAC,AMBDA,N,IFLAG,MAXCOLL,LCHR,STAB(200),ETAB(200),TLIM
2,XO,YO,ZO,X,Y,Z,ECUT,CLAB,THICK(11),NMAX,ZMIN,ZMAX,IE,INRAN,XIRA
3,NPRINT,TLAST,CTHC,NBATCH,ETA,PATH,S,SIGTAB(200),ENTAB(200),XIRC
4,ELON,MAT,DNSTY(2),ESAVE,INEL
LCHR=0
NCOLL=0
PATH=0.
S=0.
X=XO
Y=Y0
Z=ZO
CPHI=CPHIO
SPHI=SPHIO
PHI=0.0
CTH=CTHO=1.
STH=STO(1,-CTH**2)
EGY=EO
IE=1
CALL CROSS
IFLAG=0
RETURN
END
SUBROUTINE PENET
C
C CALCULATE INTER-COLLISION DISTANCE BY SAMPLING EXPONENTIAL
C ATTENUATION LAW AND CALCULATE COORDINATES OF COLLISION POINT
C
COMMON/MC/STO(7,601),NCOLL,CTHO,CTH,STH,PHI,CPHI,SPHI,EGY,CPHIO,IC
1,SPHIO,EO,SFAC,AMBDA,N,IFLAG,MAXCOLL,LCHR,STAB(200),ETAB(200),TLIM
2,XO,YO,ZO,X,Y,Z,ECUT,CLAB,THICK(11),NMAX,ZMIN,ZMAX,IE,INRAN,XIRA
3,NPRINT,TLAST,CTHC,NBATCH,ETA,PATH,S,SIGTAB(200),ENTAB(200),XIRC
4,ELON,MAT,DNSTY(2),ESAVE,INEL
X1=X
Y1=Y
Z1=Z
4 RB=RNRF(0)
IF(RB.GT.0.)GO TO 5
GO TO 4
5 S=-ALOG(RB)*AMBDA
Z=Z1+S*CTH
IF(Z.GT.ZMIN)GO TO 10
Z=ZMIN
S=(ZMIN-Z1)/CTH
IFLAG=1
10 IF(Z.LT.ZMAX)GO TO 11
Z=ZMAX
S=(ZMAX-Z1)/CTH
IFLAG=1
11 X=X1+S*STH*CPHI
Y=Y1+S*STH*SPHI
PATH=PATH+S
NCOLL=NCOLL+1
IF(NCOLL.EQ.MAXCOLL)IFLAG=1
RETURN
END
SUBROUTINE ESET
C
C SET UP A TABLE OF 100 SCREENED-RUTHERFORD CROSS SECTIONS AND
C STOPPING POWERS VS. ENERGY

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C      ON/MC/STO(7,601),NCOLL,CTHO,CTH,STM,PHI,CPHI,SPHI,EGY( 410,IC
1,S=HIO,EO,SFAC,AMBOA,N,IFLAG,MAXCOLL,LCHR,STAB(200),ETAB(200),TLIM
2,XO,YO,ZO,X,Y,Z,ECUT,CLAB,THICK(11),NMAX,ZMIN,ZMAX,IE,INRAN,XIRA
3,NPRINT,TLAST,CTHC,NBATCH,ETA,PATH,S,SIGTAB(200),ENTAB(200),XIRC
4,ELOW,MAT,DNSTY(2),ESAVE,INEL
      DIMENSION ZM(2),EXM(2),AM(2)
      DATA PI,XL2/3.14159265,.693147181/
      DATA AVCG,RO,FCC/6.025E23,2.8179E-13,
1,15360828/
      DATA ZM,AM,EXM/13..79..27..197..163.E-6,797.E-6/
      ZA=ZM(MAT)
      AA=AM(MAT)
      EXCIT=EXM(MAT)
      ENTAB(1)=EO
      DE=.01*EO
      DO 1 II=2,100
1 ENTAB(II)=ENTAB(II-1)-DE
      DO 3 II=1,100
      E=ENTAB(II)/.511
      E1=(1.+E)*(1.+E)
      E2=E*(2.+E)
      BETA=SQRT(E2/E1)
      XLG=ALOG(E*E2/(2.-(EXCIT/.511)**2))
      FM=1.-BETA*BETA*(.125*E-E-(2.+E+1.)*XL2)/E1
      STAB(II)=(FCC-ZA/AA/BETA**2)*(XLG+FM)
      ETAB(II)=0.5*(ZA*(1./3.)/121.245)**2/E2*(1.13+3.76*(ZA/137.))**2*
1E1/E2
      SIGTAB(II)=2.*PI*(1.+ZA)*AVCG*RO*RO*ZA/AA*E1/(E2*E2)/(ETAB(II)
1*(1.+S*ETAB(II)))
      3 CONTINUE
      PRINT 75
75 FORMAT(1X,'CROSS SECTION AND STOPPING POWER TABLES',//1X,'*ENERGY(M
1EV) STOPPING POWER
      ETA
      SIGMA*//)
      DO 76 II=1,100
76 PRINT 77,ENTAB(II),STAB(II),ETAB(II),SIGTAB(II)
77 FORMAT(1X,F10.6,3E16.9)
      DO 50 II=1,100
      IF(STAB(II).GT.0.0)GO TO 50
      IM=II-1
      50 CONTINUE
      GO TO 51
      IM=100
      51 ECUT=ENTAB(IM)
      RETURN
      END
      SUBROUTINE ANGLES
      COMPUTE POLAR AND AZIMUTHAL ANGLES OF ELECTRON TRAJECTORY AFTER
      COLLISION
      COMMON/MC/STO(7,601),NCOLL,CTHO,CTH,STM,PHI,CPHI,SPHI,EGY,CPHIO,IC
1,SPHIO,EO,SFAC,AMBOA,N,IFLAG,MAXCOLL,LCHR,STAB(200),ETAB(200),TLIM
2,XO,YO,ZO,X,Y,Z,ECUT,CLAB,THICK(11),NMAX,ZMIN,ZMAX,IE,INRAN,XIRA
3,NPRINT,TLAST,CTHC,NBATCH,ETA,PATH,S,SIGTAB(200),ENTAB(200),XIRC
4,ELOW,MAT,DNSTY(2),ESAVE,INEL
      DATA TWOPI/6.283185307/
      RD=RNAN(0)
      IF(IC.EQ.1)GO TO 5
      CAY=EGY*ELOW/(EGY-ELOW)
      W=ELOW*CAY/(CAY-RD*ELOW)
      SOW=SQRT(W/EGY)
      COS=SQRT(1.-W/EGY)
      EGY=EGY-W*ESAVE
      IF(EGY.LT.ECUT)IFLAG=1
      GO TO 6

```

```

5 CPM=(2.*RD*(1.-ETA)-ETA)/(2.*RD+ETA)
6 SORT(1.-COM**2)
7 R=JANF(0)
RHO=TWOPI*RC
CRHO=COS(RHO)
SRHO=SIN(RHO)
STH1=STH
CTH1=CTH
CTH=CTH1*COM+STH1*SOM+CRHO
STH=STH1*COM+STH1*SOM+CRHO
IF(STH1.EQ.0.0.OR.STH.EQ.0.0) GO TO 1
C1=(COM-CTH*CTH1)/(STH*STH1)
S1=SRHO*SOM/STH
CPH11=CPH1
SPH11=SPH1
CPH1=CPH11*C1-SPH11*S1
SPH1=CPH11*S1+SPH11*C1
GO TO 2
1 CPH1=CRHO
SPH1=SRHO
2 CONTINUE
PHI=ATAN2(SPH1,CPH1)
IF(PHI.LT.0.0)PHI=PHI+TWOPI
RETURN
END
SUBROUTINE SCORE
RECORD ELECTRON TRAJECTORY CHARACTERISTICS AND STORE IN BUFFER
ARRAY "STO"
COMMON/MC/STO(7,601),NCOLL,CTHO,CTH,STH,PHI,CPH1,SPH1,EGY,CPH10,IC
1,SPH10,ED,SPAC,AMBD,N,IFLAG,MACCOLL,LCHR,STAB(200),ETAB(200),TLIM
2,XO,YO,ZO,X,Y,Z,ECUT,CLAB,THICK(11),NMAX,ZMIN,ZMAX,IE,INRAN,XIRA
3,NPRINT,TLAST,CTHC,NBATCH,ETA,PATH,S,SIGTAB(200),ENTAB(200),XIRC
4,ELOW,MAT,DNSTY(2),ESAVE,INEL
LCHR=LCHR+1
STO(1,LCHR)=Z
STO(2,LCHR)=CTHO
STO(3,LCHR)=CTH
STO(4,LCHR)=STH
STO(5,LCHR)=PHI
STO(6,LCHR)=EGY
STO(7,LCHR)=FLOAT(NCOLL)
IF(IFLAG.NE.0)STO(7,LCHR)=FLOAT(NCOLL)
RETURN
END
SUBROUTINE PROC
THIS SUBROUTINE PROCESSES THE ELECTRON TRAJECTORY DATA AND
COMPUTES THE TRANSMISSION AND BACKSCATTER FRACTION AND EMERGENT
FLUX VALUES FOR 10 SLAB THICKNESSES
THE HISTORIES ARE PROCESSED IN BATCHES OF 601 AT A TIME. THE ARRAY
"STO" IS USED AS THE BUFFER. IT IS FILLED IN SUBROUTINE "SCORE".
COMMON/MC/STO(7,601),NCOLL,CTHO,CTH,STH,PHI,CPH1,SPH1,EGY,CPH10,IC
1,SPH10,ED,SPAC,AMBD,N,IFLAG,MACCOLL,LCHR,STAB(200),ETAB(200),TLIM
2,XO,YO,ZO,X,Y,Z,ECUT,CLAB,THICK(11),NMAX,ZMIN,ZMAX,IE,INRAN,XIRA
3,NPRINT,TLAST,CTHC,NBATCH,ETA,PATH,S,SIGTAB(200),ENTAB(200),XIRC
4,ELOW,MAT,DNSTY(2),ESAVE,INEL
DIMENSION ENERGY(20),ANGLE(5),TC(10,20),BC(10,20),TNM(11),
2TF(10,20),BF(10,20),TEMP(10),ISIG(10),SUM(10),ENERK(20)
DATA TC,TF,BC,BF/800*0./
DATA ANGLE/.2,.4,.6,.8,1./
IF(ITIME.GT.0)GO TO 1
ITIME=1

```

```

DO 24 I=1,11
24 TC(I)=1.E+7*THICK(I)/DNSTY(MAT)
18...M=NHIST=0
REWIND 3
RTDEG=180./3.14159265
NNN=MMAX/10
DE=EO/20.
ENERGY(1)=EO-DE
ENERK(1)=1000.*ENERGY(1)
DO 2 K=2,20
ENERGY(K)=ENERGY(K-1)-DE
2 ENERK(K)=1000.*ENERGY(K)
1 WRITE(3)LCNR,1(STO(L1,L2),L1=1,7),L2=1,LCNR)
DO 101 L=1,LCNR
IF(STO(2,L)).EQ.0.0.AND.STO(6,L).EQ.0.0)STOP 5
IF(STO(7,L).NE.0.0)GO TO 5
NH=NH+1
NHIST=NHIST+1
TOLD=1
IZTOP=1
GO TO 101
5 Z=STO(1,L)
C
C
C
C CLASSIFY EMERGENT ENERGY
DO 25 K=1,20
25 IF(STO(6,L).GT.ENERGY(K))GO TO 26
K=20
26 KC=K
C
C
C
C IF(Z.GT.ZMIN)GO TO 6
IF(IZTOP.GT.10)IZTOP=10
C
C BACKSCATTER FRACTION(BC) AND FLUX(BF)
DO 11 I=IZTOP,10
BC(I,KC)=BC(I,KC)+1.0
11 BF(I,KC)=BF(I,KC)+1./ABS(STO(3,L))
GO TO 102
6 DO 10 IZ=1,11
IF(Z.GT.THICK(IZ))GO TO 10
IF(IZ.EQ.1)GO TO 102
15=12
IF(15.LE.IZTOP)GO TO 102
IZTOP=IS
IQ=IS-1
C
C TRANSMISSION FRACTION(TC) AND FLUX(TF)
DO 12 IT=IOLD,10
TC(IT,KC)=TC(IT,KC)+1.
12 TF(IT,KC)=TF(IT,KC)+1./STO(3,L)
IOLD=IZTOP
GO TO 102
10 CONTINUE
102 IF(STO(7,L).LT.0.0)GO TO 112
GO TO 101
112 IF(NH.LT.NNN)GO TO 101
KPUT=1
NH=0
18=18+1
REWIND 2
FN=FLOAT(NNN)
DO 103 I=1,10
DO 103 K=1,20

```

```

TC(I,K)=TC(I,K)/PN
BC(I,K)=BC(I,K)/PN
TF(I,K)=TF(I,K)/PN
103 BF(I,K)=BF(I,K)/PN
    IF(IB.EQ.10)CALL SECOND(TLAST)
    DO 150 NM=1,4
    DO 93 I=1,10
99 SUM(I)=0.
    IF(IB.LT.10)GO TO 154
    PRINT 220
    GO TO (114,124,134,144),NM
114 PRINT 230
124 PRINT 231
134 PRINT 232
144 PRINT 233
153 PRINT 234,(THICK(I),I=1,10)
    PRINT 236,(TNM(I),I=1,10)
236 FORMAT(4X,NM=3X,10E12.5)
230 FORMAT(//45X,'TRANSMITTED CURRENT=')
231 FORMAT(//45X,'BACKSCATTERED CURRENT=')
232 FORMAT(//45X,'TRANSMITTED FLUX=')
233 FORMAT(//45X,'BACKSCATTERED FLUX=')
234 FORMAT(45X,'SLAB THICKNESS (G/CM2)=9X,10E12.5/3X,'E(KEV)=')
154 DO 104 K=1,20
    DO 105 I=1,10
    GO TO (115,125,135,145),NM
115 TEMP(I)=TC(I,K)
    TC(I,K)=0.
    GO TO 105
125 TEMP(I)=BC(I,K)
    BC(I,K)=0.
    GO TO 105
135 TEMP(I)=TF(I,K)
    TF(I,K)=0.
    GO TO 105
145 TEMP(I)=BF(I,K)
    BF(I,K)=0.
105 CONTINUE
    CALL STATS(TEMP,ISIG,10,KPUT,18)
    WRITE(2)ENERGY(K),(TEMP(I),ISIG(I),I=1,10)
    IF(IB.LT.10)GO TO 104
156 SUM(I)=SUM(I)+TEMP(I)
    PRINT 221,ENERG(K),(TEMP(I),ISIG(I),I=1,10)
104 CONTINUE
    IF(IB.EQ.10)PRINT 235,(SUM(I),I=1,10)
235 FORMAT(/2X,'TOTALS ',10(E12.5))
150 CONTINUE
    IF(IB.EQ.10)PRINT 499,NHIST
101 CONTINUE
220 FORMAT(//)
221 FORMAT(1X,F7.3,1X,10(F9.5,13))
499 FORMAT(1X,'TOTAL NUMBER OF HISTORIES PROCESSED = ',I10)
    RETURN
    END
    SUBROUTINE STATS(XVAL,ISIG,N,K,18)
C
C "STATS" STORES THE OUTPUT QUANTITIES AND COMPUTES THE STANDARD
C STATISTICAL ERROR FOR EACH
C
    DIMENSION XVAL(1),ISIG(1),AVE(10),SIG(10)
    DIMENSION ECAGE(1000),ECSIG(1000)
    IF(IB.NE.1) GO TO 2

```

```

DO 1 I=1,N
  X(I) = XVAL(I)
  SIG(I)=0.0
  DO 100 I=1,N
    IKK=K+I-1
    ECAGE(IKK)=AVE(I)
    EC SIG(IKK)=SIG(I)
  K=K+N
  RETURN
2 XNSORS=IB
DO 200 I=1,N
  IKK=K+I-1
  AVE(I)=ECAGE(IKK)
  SIG(I)=EC SIG(IKK)
200 SIG(I)=EC SIG(IKK)
DO 3 I=1,N
  SIG(I)=(XNSORS-2.0)/(XNSORS-1.0)*SIG(I)+(XVAL(I)-AVE(I))*2/XNSORS
  AVE(I) = ((XNSORS-1.0)*AVE(I)+XVAL(I))/XNSORS
  XVAL(I) = AVE(I)
  ERROR=SQRT(SIG(I)/XNSORS)
  ISIG(I)=99
  IF(AVE(I).EQ.0.0) GO TO 3
  ISIG(I)=100.0*ERROR/ABS(AVE(I))+.50001
  IF(ISIG(I).GT.99) ISIG(I)=99
3 CONTINUE
GO TO 10
END
SUBROUTINE ENERGY
C
C *ENERGY* PERFORMS A TABLE LOOK-UP ON THE ELECTRON ENERGY
C AND CHOOSES THE CORRESPONDING CROSS SECTION VALUE.
C
COMMON/MC/STO(7,601),NCOLL,CTHO,CTH,STH,PHI,CPHI,SPHI,EGY,CPHIQ,IC
1,SPHIQ,EO,SFAC,AMBDA,N,IFLAG,MACCOLL,LCHR,STAB(200),ETAB(200),TLIM
2,XO,YO,ZO,X,Y,Z,ECUT,CLAB,THICK(1),NMAX,ZMIN,ZMAX,IE,INRAN,XIRA
3,NPRINT,TLAST,CTHC,NBATCH,ETA,PATH,S,SIGTAB(200),ENTAB(200),XIRC
4,ELOW,MAT,DNSTY(2),ESAVE,INEL
ETO=1000.*EO
EI=1000.*EGY
DD=100./ETO
IE=1+IFIX(DD*(ETO-ET))
DE=STAB(IE)*S
EGY=EGY-DE
IF(EGY.GT.ECUT)GO TO 4
IFLAG=1
EGY=ECUT
ENTRY CROSS
4 ETA=ETAB(IE)
IF(EGY.GT.ECUT)GO TO 5
IC=1
RETURN
5 IF(INEL.GT.0)GO TO 6
IC=1
AMBDA=1./SIGTAB(IE)
RETURN
C
C INELASTIC SCATTER
C
6 WAVG=EGY*ALOG(EGY/ELOW)
AMBIN=WAVG/STAB(IE)
SIGIN=1./AMBIN
SIGTOT=SIGIN+SIGTAB(IE)
AMBDA=1./SIGTOT
RA=RA+NF(0)
RATIO=SIGTAB(IE)*AMBDA
IC=2

```

C  
C  
C  
C

ESAVE=DE  
1 ( A.LT.RATIO) IC=1  
RL JRN  
END

Program Listing

- BIGSLAB -

(Reference: Vol. I, Section IV.B.4)

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PROGRAM BIGSLAB(INPUT,OUTPUT,TAPE2,TAPE3)

PROGRAM "BIGSLAB" GENERATES TRANSMISSION AND ALBEDO DISTRIBUTIONS
OF ELECTRONS SCATTERED OUT OF VERY THICK HOMOGENEOUS MEDIA(GOLD
OR ALUMINUM). THE TRANSMISSION DATA GENERATED HERE ARE USED AS
INPUT SOURCE FUNCTIONS FOR THE SECOND MONTE CARLO PROGRAM "INTFC"
WHICH TREATS THE MATERIAL INTERFACE REGION. THE ALBEDO DATA
GENERATED HERE ARE ALSO USED IN "INTFC" TO PROVIDE A BACKSCATTER
DISTRIBUTION FROM THICK SLABS WHEN AN ELECTRON ESCAPES FROM THE
INTERFACE REGION. IN THIS WAY A COSTLY MONTE CARLO CALCULATION IN
THE THICK REGIONS SURROUNDING THE MATERIAL INTERFACE REGION MAY
BE AVOIDED. A LIST OF THE QUANTITIES CALCULATED BY "BIGSLAB" IS
GIVEN IN VOL. 1, PAGES 253-257.

COMMON/MC/STO(7,51),NCOLL,CTHO,CTH,PHI,CPI,EGY,CPHI0,
1SPHI0,ED,SFAC,AMBDA,N,IFLAG,MAXCOLL,LCHR,STAB(200),ETAB(200),TLIM,
2XO,YO,ZO,X,Y,Z,ECUT,CLAB,THICK(11),NMAX,ZMIN,ZMAX,IE,INRAN,XIRA
3,NPNT,TLAST,CTHC,NBATCH,ETA,PATH,S,SIGTAB(200),ENTAB(200),XINC
DIMENSION TIMR(11)

SUBROUTINE "SETRUN" READS IN AND INITIALIZES THE VARIOUS PARAMETERS
NECESSARY FOR THE MONTE CARLO CALCULATION

CALL SETRUN
FN=FLOAT(NMAX)
COLLN=0.
NIM=1
CALL SECOND(TIME)
TIMR(1)=TIME
5 N=N+1

"SETH" INITIALIZES THE ELECTRON TRAJECTORY PARAMETERS AND IS
CALLED AT THE START OF EACH TRAJECTORY

CALL SETHS

"SCORE" RECORDS THE COORDINATES, DIRECTION AND ENERGY AT EACH
ELECTRON COLLISION SITE.

CALL SCORE

"PENET" TRANSLATES THE ELECTRON FROM ONE COLLISION SITE TO THE
NEXT

20 CALL PENET

THE VARIABLE "IFLAG >0" SIGNIFIES THE END OF THE CASE HISTORY.
THIS MAY OCCUR VIA TRANSMISSION, BACKSCATTER, ENERGY BELOW CUT OFF
OR COLLISION NUMBER EXCEEDING THE MAXIMUM ALLOWED NUMBER.

IF(IFLAG.NE.0) GO TO 6
CALL ENERGY
IF(IFLAG.NE.0) GO TO 6
CALL SCORE

"ANGLES" COMPUTES THE POLAR AND AZIMUTHAL ANGLES OF THE ELECTRON
TRAJECTORY AFTER A COLLISION HAS OCCURRED

CALL ANGLES
GO TO 20
6 CALL SCORE
CALL PROC
IF(IPRNT.EQ.2)PRINT 2,((STO(K,L),K=1,7),L=1,LCHR)

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```

C C COLLNO=COLLNO+FLOAT(NCOLL)/FN
C C
C C      CALCULATION IS ORGANIZED SO THAT THE TOTAL NUMBER(NMAX) F
C C      HISTORIES IS DIVIDED INTO 10 EQUAL PARTS. THIS SECTION OF CODE
C C      COMPUTES THE AVERAGE TIME REQUIRED TO PROCESS EACH GROUP OF
C C      (NMAX/10) HISTORIES AND DETERMINES WHETHER SUFFICIENT TIME HAS
C C      BEEN ALLOWED TO FINISH THE CALCULATION. IF THERE IS NOT ENOUGH
C C      TIME REMAINING, THE CALCULATION IS TERMINATED NORMALLY WITH THE
C C      INTERMEDIATE RESULTS WRITTEN ON A PERMANENT FILE(TAPE3) BY
C C      SUBROUTINE "PROC". IN THIS WAY THE CALCULATION CAN BE CONTINUED
C C      WITHOUT LOSING THE DATA OBTAINED THUS FAR DUE TO A COMPUTER TIME
C C      OVERRUN.
C C
C C      IF(NBATCH.EQ.0)GO TO 60
C C      NTEST=N-NBATCH*(N/NBATCH)
C C      IF(NTEST.LE.0)GO TO 60
C C      MTM=MTM+1
C C      CALL SECOND(TIME)
C C      IF(MTM.EQ.1)TIME=TLAST
C C      TIMR(MTM)=TIME
C C      DELT=TIME-TIMR(1)
C C      AVGT=DELT/FLOAT(MTM-1)
C C      TREMAIN=TLIM-DELT
C C      TLEFT=1.5*AVGT
C C      IPCT=10*(MTM-1)
C C      PRINT 75,IPCT,TREMAIN
C C      FORMAT(1X,THE PROBLEM IS,14, PERCENT COMPLETE.,1X,
C C      1*TIME TO FINISH IS,12.3,SECONDS*)
C C      IF(IPCT.EQ.100)GO TO 100
C C      IF(TREMAIN.LE.TLEFT)GO TO 5
C C      GO TO 100
C C
C C      60 IF(N-NMAX)5,100,100
C C      2 FORMAT(1X,7E16.9)
C C      100 PRINT 64,COLLNO
C C      CALL RANGE(XIRC)
C C      64 FORMAT(//1X,AVERAGE NUMBER OF COLLISIONS PER HISTORY = ,F7.3)
C C      IF(NBATCH.EQ.0)GO TO 200
C C      MTM=MTM-1
C C      PRINT 65,MTM,NBATCH
C C      65 FORMAT(//1X,THERE ARE ,14, BATCHES OF,15, HISTORIES//)
C C      SUM=TIMR(1)
C C      DO 66 M=2,MTM
C C      TIMR(M)=TIMR(M)-SUM
C C      SUM=SUM+TIMR(M)
C C      AVGT=AVGT+TIMR(M)
C C      MM=M-1
C C      66 PRINT 67,MM,TIMR(M)
C C      67 FORMAT(1X,BATCH NO.,14, TIME = ,F9.3, SECONDS*)
C C      AVGT=AVGT/FLOAT(MTM)
C C      PRINT 68,AVGT
C C      68 FORMAT(//1X,AVERAGE TIME PER BATCH = ,F9.3, SECONDS*)
C C      200 PRINT 190
C C      190 FORMAT(/6X,INRAN,11X,IRA,12X,IRC*)
C C      PRINT 195,INRAN,XIRA,XIRC
C C      195 FORMAT(3(1X,O16))
C C      STOP
C C      END
C C      SUBROUTINE SETRUN
C C
C C      READ IN AND INITIALIZE ALL PARAMETERS NECESSARY FOR THE
C C      CALCULATION
C C
C C      COMMON/MC/STD(7,51),NCOLL,CTHO,CTH,STH,PHI,CPHI,SPHI,EGY,CPHIO,
C C      1SPHIO,EO,SPAC,AMBDA,N,IFLAG,NMAXCOLL,LCHR,STAB(200),ETAB(200),TLIM,
C C      2XD,YO,ZO,X,Y,Z,ECUT,CLAB,THICK(11),NMAX,ZMIN,ZMAX,1E,INRAN,XIRA

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```

3,NPRINT,TLAST,CTHC,NBATCH,ETA,PATH,S,SIGTAB(200),ENTAB(200),YIRC
D1(SIGN INCODE(3))
DATA CPHIO,SPHIO,PI,ZMIN,ZMAX,N/1.0,0.0,0.3,14159265,0.0,1000.,0.0/
DATA ECUT,SFAC,XO,YO,ZO/0.,800.,3*0./
DATA THICK/2.5,5,7,5,10,12,5,15,17,5,20,25,50,10000./
READ 1,NMAX,MAXCOLL,NPRINT,INPT,NSLABS
C
C NMAX - TOTAL NO. OF MONTE CARLO HISTORIES
C STORED ON TAPE OR DISK UNTIL PROCESSING TIME
C MAXCOLL - MAXIMUM NUMBER OF COLLISIONS PER HISTORY TO BE ALLOWED
C NPRINT - DEBUG PRINT CONTROL PARAMETER
C NPRINT EQUAL TO ZERO SUPPRESSES DEBUG PRINTOUT
C NPRINT NOT EQUAL TO ZERO ACTIVATES DEBUG PRINTOUT
C INPT(0/1) - (DEFAULT/USER PROVIDED) VALUES FOR INITIAL RANDOM
C NUMBER "INRAN". STOPPING POWER FUDGE FACTOR "SFAC".
C AND INITIAL PARTICLE COORDINATES "XO,YO,ZO".
C
C
READ 3,TLIM,CTHO,EO,ECUT
IF(NSLABS.EQ.0)GO TO 50
READ 3,THICK(1),I=1,11)
50 DO 51 I=1,11
51 THICK(I)=THICK(I)*1.E-5
IF(INPT.GT.0)GO TO 40
INRAN=0
GO TO 45
40 READ 1,((INCODE(1),I=1,3)
IF(INCODE(1).EQ.1)READ 8,INRAN
IF(INCODE(2).EQ.1)READ 3,SFAC
IF(INCODE(3).EQ.1)READ 3,XO,YO,ZO
C
C EO - INITIAL PARTICLE ENERGY
C XO,YO,ZO - INITIAL PARTICLE COORDINATES
C SFAC - STOPPING POWER FUDGE FACTOR
C
45 PRINT 4,NMAX,MAXCOLL
PRINT 9,EO,ECUT,XO,YO,ZO,CTHO,CPHIO,SPHIO
PRINT 11,SPAC
11 FORMAT(1X,'STOPPING POWER FUDGE FACTOR (SFAC) =',F10.5)
9 FORMAT(1X,'EO (INITIAL ENERGY MEV)',E12.5//1X,'ECUT (LOW ENERGY
ICUT OFF MEV) ',E12.5//1X,'XO, YO, ZO, (SOURCE POINT COORDINATES)
3 ',E12.5//1X,'CTHO (INCIDENT POLAR COSINE) ',E12.5//1X,'CPHIO
4,SPHIO (INCIDENT AZIMUTH) ',E12.5)
6 FORMAT(018)
1 FORMAT(6110)
3 FORMAT(BF10.0)
4 FORMAT(1//1X,'NMAX (NO. OF HISTORIES) ',E16//1X,'MAXCOLL (MAXIMUM
1 ALLOWED NUMBER OF COLLISIONS) ',I3)
10 N=0
XIRA=XIRC=INRAN
NBATCH=NMAX/10
CALL RANSET(XIRA)
CALL ESET
RETURN
END
SUBROUTINE SETHIS
C
C INITIALIZE ELECTRON HISTORY
C
COMMON/MC/STO(17,51),NCOLL,CTHO,CTH,STH,PHI,CPHI,SPHI,EGY,CPHIO,
1SPHIO,EO,SFAC,AMBD,A,N,IFLAG,MAXCOLL,LCHR,STAB(200),ETAB(200),TLIM,
2XO,YO,ZO,X,Y,Z,ECUT,CLAB,THICK(11),NMAX,ZMIN,ZMAX,IE,INRAN,XIRA
3,NPRINT,TLAST,CTHC,NBATCH,ETA,PATH,S,SIGTAB(200),ENTAB(200),XIRC
LCHR=0
NCOLL=0
PATH=0.
S=0.

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      BETASQRT(E2/E1)
      X(1)=LOG((E-E2)/(2.0*(EXCIT/.511)**2))
      FM...BETA=BETA*(1.125+E-12.0+E+1.0)*XL2/E1
      STAB(11)=(FCC*ZA/AA/BETA**2)*(XLG+FM)*SFAC
      ETAB(11)=0.5*(ZA**2/(1./3.)/121.245)**2/E2*(1.13+3.76*(ZA/137.0)**2
      1E1/E2)
      SIGTAB(11)=ZA*(1.+ZA)*AVOG*RO*RO*ZA/AA*E1/(E2+E2)/(ETAB(11)
      1*11+.5*ETAB(11)))
      ETAB(11)=.035276
      3 IF(11.GT.91)ETAB(11)=.103378
      IF(NPRNT.EQ.0)RETURN
      PRINT 75
      75 FORMAT(1X,'CROSS SECTION AND STOPPING POWER TABLES',//,1X,'ENERGY(M
      1EV) STopping POWER ETA
      SIGMA//)
      DO 76 11=1,191
      76 PRINT 77,ETAB(11),STAB(11),ETAB(11),SIGTAB(11)
      77 FORMAT(1X,F10.6,3E16.9)
      RETURN
      END
      SUBROUTINE ANGLES
      C
      C COMPUTE POLAR AND AZIMUTHAL ANGLES OF ELECTRON TRAJECTORY AFTER
      C COLLISION
      C
      COMMON/NC/STO(7,51),NCOLL,CTHO,CTH,STM,PHI,CPHI,SPHI,EGY,CPHI0,
      1SPHI0,EO,SFAC,AMBDA,N,IFLAG,MAXCOLL,LCHR,STAB(200),ETAB(200),TLIM,
      2XO,YO,ZO,X,Y,Z,ECUT,CLAB,THICK(11),NMAX,ZMIN,ZMAX,IE,INRAN,XIRA
      3,NPRNT,TLAST,CTHC,NBATCH,ETA,PATN,S,SIGTAB(200),ENTAB(200),XIRC
      DATA TPOI/6.283185307/
      RD=2.*RANF(0)
      COM=(RD*(1.+ETA)-ETA)/(RD+ETA)
      SCM=SQRT(1.-COM**2)
      RC=RANF(0)
      RHO=TPOI*RC
      CRMO=COS(RHO)
      SRMO=SIN(RHO)
      STH=CTH
      CTH=CTH
      CTH=CTH1*COM+STM1*SCM+CRMO
      STH=SQRT(1.-CTH**2)
      IF(STM1.EQ.0.0.OR.STH.EQ.0.0) GO TO 1
      C1=(COM-CTH*CTH1)/(STM*STM1)
      S1=SRMO*SCM/STH
      CPHI1=CPHI
      SPHI1=SPHI
      CPHI=CPHI1+C1-SPHI1*S1
      SPHI=CPHI1+S1+SPHI1*C1
      GO TO 2
      1 CPHI=CRMO
      SPHI=SRMO
      2 CONTINUE
      PHI=ATAN2(SPHI,CPHI)
      IF(PHI.LT.0.0)PHI=PHI+TPOI
      RETURN
      END
      SUBROUTINE SCORE
      C
      C RECORD ELECTRON TRAJECTORY CHARACTERISTICS IN BUFFER ARRAY "STO"
      C
      COMMON/NC/STO(7,51),NCOLL,CTHO,CTH,STM,PHI,CPHI,SPHI,EGY,CPHI0,
      1SPHI0,EO,SFAC,AMBDA,N,IFLAG,MAXCOLL,LCHR,STAB(200),ETAB(200),TLIM,
      2XO,YO,ZO,X,Y,Z,ECUT,CLAB,THICK(11),NMAX,ZMIN,ZMAX,IE,INRAN,XIRA
      3,NPRNT,TLAST,CTHC,NBATCH,ETA,PATN,S,SIGTAB(200),ENTAB(200),XIRC
      LCHR=LCHR+1
      STO(1,LCHR)=Z

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```

300 STO(2,LCNR)=CTHO
301 SIF(3,LCNR)=CTH
302 SIF(4,LCNR)=STM
303 STO(5,LCNR)=PHI
304 STO(6,LCNR)=EGY
305 STO(7,LCNR)=FLOAT(NCOLL)
306 IF(FLAG.NE.0)STO(7,LCNR)=--FLOAT(NCOLL)
307 RETURN
308 END
309 SUBROUTINE PROC
310
311 PROCESS TRAJECTORY DATA. FOR LIST OF COMPUTED QUANTITIES, SEE
312 VOL. I, PAGES 253-257. ALL COMPUTED QUANTITIES ARE PRINTED OUT
313 WITH THEIR STATISTICAL STANDARD ERRORS. THE TRANSMISSION AND AL-
314 BEDO SPHERICAL HARMONICS COEFFICIENTS ARE WRITTEN OUT TO A PERMA-
315 NENT FILE(TAPE2).
316
317 COMMON/MC/STO(7,51),NCOLL,CTHO,CTH,STM,PHI,CPHI,SPHI,EGY,CPHID,
318 1SPHID,EO,SEAC,AMBOA,N,IFLAG,MAXCOLL,LCNR,STAB(200),ETAB(200),TLIM,
319 2XO,YO,ZO,X,Y,Z,ECUT,CLAB,THICK(11),NMAX,ZMIN,ZMAX,IE,INRAN,XIRA
320 3,NPRINT,TLAST,CTHC,NBATCH,ETA,PATH,5,SIGTAB(200),ENTAB(200),XIRC
321 4,DIMENSION TRANC(10,5,20),BACAC(10,5,20),TRANF(10,5,20),BACKF(10,5,
322 5,20),ENERGY(20),ANGLE(5),TC(10,20),BC(10,20),CMEAN(20),NKC(20),
323 6,2TF(10,20),BF(10,20),TEMP(10),ISIG(10),SUM(10),SCMEAN(20),ECRSE(10)
324 7,DIMENSION ALBEDO(10,10,12),CTFINE(10),PHFINE(12),SIGX(20),SIGZ(10,10,12)
325 8,1SIGZ(20),CURTX(20),CURTZ(20),EXPCOF(10,10),PL(10),ISG(10,10,12)
326 9,1CURTAC(10),CURTZC(10)
327 10,DIMENSION APLUS(9,9,10),AMINUS(9,9,10),YCOF(9,9),CP(9),
328 11,1SPI(9),PLM(9,9),1SGAP(9,9,10),1SGAM(9,9,10),1SGEX(10,10)
329 12,DIMENSION TPLUS(9,9,10),TMINUS(9,9,10),ITP(9,9,10),ITM(9,9,10)
330 13,EQUIVALENCE(APLUS(1),TPLUS(1)),EQUIVALENCE(AMINUS(1),TMINUS(1))
331 14,EQUIVALENCE(1SGAP(1),ITP(1)),EQUIVALENCE(1SGAM(1),ITM(1))
332 15,EQUIVALENCE(1SIGZ(1),BF(1,1)),EQUIVALENCE(1SIGX(1),BF(1,2))
333 16,EQUIVALENCE(CURTZC(1),BF(1,3)),EQUIVALENCE(CURTAC(1),BF(1,4))
334 17,EQUIVALENCE(ALBEDO(1),TRANC(1))
335 18,EQUIVALENCE(ALBEDO(100),TRANF(1))
336 19,EQUIVALENCE(1SG(1),BACAC(1))
337 20,EQUIVALENCE(1SG(100),BACKF(1))
338 21,EQUIVALENCE(1SIGZ(1),TF(1,1)),EQUIVALENCE(1SIGX(1),TF(1,3))
339 22,EQUIVALENCE(CURTZ(1),TF(1,5)),EQUIVALENCE(CURTX(1),TF(1,7))
340 23,EQUIVALENCE(CTFINE(1),TF(1,9)),EQUIVALENCE(PHFINE(1),TF(1,10))
341 24,EQUIVALENCE(ECRSE(1),TF(1,12))
342 25,DATA CMEAN,SCMEAN,NKC/40.0,20.0/
343 26,DATA CMC,SMC,NKCR/20.0,10.0/
344 27,DATA ZMIN/0./
345 28,DATA ANGLE/2.4,6.8,1./
346 29,DATA YCOF,PLM/162.0/
347 30,DATA TRANC,BACAC,TRANF,BACKF/4000.0./
348 31,DATA TPLUS,TMINUS,EXPCOF,ITP,ITM/1720.0,1620.0/
349 32,DATA ITIME/0/
350 33,IF(1TIME.GT.0)GO TO 1
351 34,DO 30 LI=1,9
352 35,FL=(FLOAT(LI)*2.+1.)/6.283185307
353 36,DO 31 MI=1,L1
354 37,LM=L1-MI+1
355 38,LM=L1+MI
356 39,F1=1.
357 40,DO 32 I=LM1,LM
358 41,F1=F1*FLOAT(I)
359 42,F1=F1*FLOAT(I)
360 43,31 YCOF(L1,MI)=SORT(FL/F1)
361 44,30 CONTINUE
362 45,ITIME=1
363 46,IB-NH-NHIST=0
364 47,REWINO 3
365 48,RTDEG=160./3.14159265

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```

NNN=MAX/10
FN=LOAT(NNN)
DO 1 /20
ENERGY(1)=ED-DE
DO 2 K=2,20
2 ENERGY(K)=ENERGY(K-1)-DE
1 WRITE(3)LCNR,((STO(L1,L2),L1=1,7),L2=1,LCNR)
DO 101 L=1,LCNR
IF(STO(2,L).EQ.0.0.AND.STO(6,L).EQ.0.0)STOP 5
IF(STO(7,L).NE.0.0)GO TO 5
NH=NH+1
NHIST=NHIST+1
IOLD=1
IZTOP=1
GO TO 101
5 Z=STO(1,L)
C
C CLASSIFY EMERGENT ANGLE
C
DO 15 J=1,5
15 IF(ABS(STO(3,L)).LT.ANGLE(J))GO TO 16
J=5
16 JC=J
C
C CLASSIFY EMERGENT ENERGY
C
DO 25 K=1,20
25 IF(STO(6,L).GT.ENERGY(K))GO TO 26
K=20
26 KC=K
C
C
IF(IZ.GT.ZMIN)GO TO 6
IF(IZTOP.GT.10)IZTOP=10
DO 11 I=IZTOP,10
CTC=ABS(STO(3,L))
BACKC(1,JC,KC)=BACKC(1,JC,KC)+1.0
BACKF(1,JC,KC)=BACKF(1,JC,KC)+1./ABS(STO(3,L))
11 CPH=COS(STO(5,L))
CMEAN(KC)=CMEAN(KC)+CTC
SCMEAN(KC)=SCMEAN(KC)+CPH*STO(4,L)
NKC(KC)=NKC(KC)+1
KCR=(KC+1)/2
CMC(KCR)=CMC(KCR)+CTC
SMC(KCR)=SMC(KCR)+CPH*STO(4,L)
NKCR(KCR)=NKCR(KCR)+1
GO TO 102
6 DO 10 IZ=1,11
IF(IZ.GT.THICK(IZ))GO TO 10
IF(IZ.EQ.1)GO TO 102
IS=IZ
IF(15.-LE.IZTOP)GO TO 102
IZTOP=IS
10=IS-1
DO 12 IT=IOLD,IQ
TRANC(IT,JC,KC)=TRANC(IT,JC,KC)+1.
12 TRANF(IT,JC,KC)=TRANF(IT,JC,KC)+1./STO(3,L)
IOLD=IZTOP
IF(IQ.NE.10)GO TO 102
CTHET=STO(3,L)
STHET=STO(4,L)
PHI=STO(5,L)
CALL LEP(PL,CTHET,9)
KCR=(KC+1)/2
DO 1430 LL=1,10
1430 EXPCOF(LL,KCR)=EXPCOF(LL,KCR)+PL(LL)/FN

```

```

DO 1338 M=1,9
  CP( )=COS(M*PHI)
  SP(M)=SIN(M*PHI)
  CALL ASSOC(PL,CTHET,STNET,PLM)
DO 1341 LL=1,9
  DO 1339 M=1,LL
    FM=YCOF(LL,M)*PLM(LL,M)/FN
    TPLUS(LL,M,KCR)=TPLUS(LL,M,KCR)+FM*CP(M)
    TMINUS(LL,M,KCR)=TMINUS(LL,M,KCR)+FM*SP(M)
  1341 CONTINUE
  GO TO 102
10 CONTINUE
102 IF(STO(7,L).LT.0.0)GO TO 112
  GO TO 101
112 IF(NH.LT.NNN)GO TO 101
  KPUT=1
  NH=0
  IB=IB+1
  REWIND 2
DO 103 I=1,10
  DO 103 K=1,20
    TC(I,K)=BC(I,K)=TF(I,K)=BF(I,K)=0.
  DO 103 J=1,5
    TC(I,K)=TC(I,K)+TRANC(I,J,K)/FN
    BC(I,K)=BC(I,K)+BACKC(I,J,K)/FN
    TF(I,K)=TF(I,K)+TRANF(I,J,K)/FN
    BF(I,K)=BF(I,K)+BACKF(I,J,K)/FN
  103 TRANC(I,J,K)=BACKC(I,J,K)=TRANF(I,J,K)=BACKF(I,J,K)=0.0
  IF(IB.EQ.10)CALL SECOND(LAST)
  DO 150 MM=1,4
    DO 93 I=1,10
      93 SUM(I)=0.
      IF(IB.LT.10)GO TO 154
      PRINT 220
      GO TO (114,124,134,144),MM
114 PRINT 230
124 PRINT 231
134 PRINT 232
144 PRINT 233
153 PRINT 234,(THICK(I),I=1,10)
230 FORMAT(//45X,'TRANSMITTED CURRENT=/')
231 FORMAT(//45X,'BACKSCATTERED CURRENT=/')
232 FORMAT(//45X,'TRANSMITTED FLUX=/')
233 FORMAT(//45X,'BACKSCATTERED FLUX=/')
154 DO 104 K=1,20
  DO 105 I=1,10
    GO TO (115,125,135,145),MM
115 TEMP(I)=TC(I,K)
  GO TO 105
125 TEMP(I)=BC(I,K)
  GO TO 105
135 TEMP(I)=TF(I,K)
  GO TO 105
145 TEMP(I)=BF(I,K)
105 CONTINUE
  CALL STATS(TEMP,ISIG,10,KPUT,IB)
  WRITE(2)ENERGY(K),(TEMP(I),ISIG(I),I=1,10)
  IF(IB.LT.10)GO TO 104
  DO 156 I=1,10
    156 SUM(I)=SUM(I)+TEMP(I)
  PRINT 221,ENERGY(K),(TEMP(I),ISIG(I),I=1,10)
104 CONTINUE

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IF(18.EQ.10)PRINT 235,(SUM(I),I=1,10)
235 FOR I=1,2X,*TOTALS *,10(E12.5)
150 CO NUC
DO 1700 I=1,9
DO 1700 J=1,9
DO 1701 K=1,10
TEMP(K)=TPUS(I,J,K)
1701 TPUS(I,J,K)=0.
CALL STATS(TEMP,ISIG,10,KPUT,18)
IF(18.LT.10)GO TO 1700
DO 1702 K=1,10
TPUS(I,J,K)=TEMP(K)
1702 ITP(I,J,K)=ISIG(K)
1700 CONTINUE
DO 1710 I=1,9
DO 1710 J=1,9
DO 1711 K=1,10
TEMP(K)=TMINUS(I,J,K)
1711 TMINUS(I,J,K)=0.
CALL STATS(TEMP,ISIG,10,KPUT,18)
IF(18.LT.10)GO TO 1710
DO 1712 K=1,10
TMINUS(I,J,K)=TEMP(K)
1712 ITM(I,J,K)=ISIG(K)
1710 CONTINUE
DO 1720 I=1,10
AL=1
FAL=SQRT((2.*AL-1.)/12.56637061)
DO 1721 J=1,10
TEMP(J)=EXPCOF(I,J)
1721 EXPCOF(I,J)=0.
CALL STATS(TEMP,ISIG,10,KPUT,18)
IF(18.LT.10)GO TO 1720
DO 1722 J=1,10
EXPCOF(I,J)=FAL*TEMP(J)
1722 ISGEX(I,J)=ISIG(J)
1720 CONTINUE
IF(18.EQ.10)PRINT 499,NHIST
101 CONTINUE
IF(NHIST.LT.NMAX)RETURN
220 FORMAT(///)
221 FORMAT(1X,F7.3,1X,10(F9.5,13))
499 FORMAT(1X,*TOTAL NUMBER OF HISTORIES PROCESSED = *,110)
PRINT 179
DO 180 K=1,20
IF(NKC(K).EQ.0)GO TO 180
CMEAN(K)=CMEAN(K)/FLOAT(NKC(K))
SCMEAN(K)=SCMEAN(K)/FLOAT(NKC(K))
180 PRINT 181,ENERGY(K),CMEAN(K),SCMEAN(K)
179 FORMAT(///10X,*AVERAGE COS(THETA)=,5X,*AVERAGE SIN(THETA)COS(PHI)*
1/3X,*E(MEV)=)
181 FORMAT(1X,F7.3,2X,E12.5,14X,E12.5)
PRINT 179
DO 185 K=1,10
FDE=FLOAT(2*K)*DE
ECRSE(K)=EO-FDE
IF(NKCR(K).EQ.0)GO TO 185
CMC(K)=CMC(K)/FLOAT(NKCR(K))
SMC(K)=SMC(K)/FLOAT(NKCR(K))
185 PRINT 181,ECRSE(K),CMC(K),SMC(K)
PRINT 1823
1823 FORMAT(///45X,*LEGENDRE COEFFICIENTS OF TRANSMISSION*/
14X,*E(MEV) L=,4X,*0=,11X,*1=,11X,*2=,11X,*3=,11X,*4=,
211X,*5=,11X,*6=,11X,*7=,11X,*8=,11X,*9=)
DO 1826 K=1,10
1826 PRINT 526,ECRSE(K),(EXPCOF(LL,K),LL=1,10)

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      PK 651
      DO 525 K=1,10
1525 PRINT 524,ECRSE(K),(ISGEX(LL,K),LL=1,10)
      DO 1345 KE=1,10
      PRINT 1347,ECRSE(KE)
1347 FORMAT(/IX,*ENERGY = *.F10.5,*MEV*/IX,*SPHERICAL HARMONICS COEFF
      ICIENTS FOR TRANSMISSION, TPLUS*/)
      DO 1343 L=1,9
1343 PRINT 349,L,(TPLUS(L,M,KE),M=1,9)
      PRINT 651
      DO 1652 L=1,9
1652 PRINT 653,L,(ITP(L,M,KE),M=1,9)
      PRINT 1348
1348 FORMAT(IX,*SPHERICAL HARMONICS COEFFICIENTS FOR TRANSMISSION, TMIN
      IUS*/)
      DO 1342 L=1,9
1342 PRINT 349,L,(TMINUS(L,M,KE),M=1,9)
      PRINT 651
      DO 1654 L=1,9
1654 PRINT 653,L,(ITM(L,M,KE),M=1,9)
      WRITE(2)(EXPCOF(L,KE),L=1,10)
      DO 1251 L=1,9
1251 PRINT 1252,L,(TPLUS(L,M,KE),M=1,9)
      WRITE(2)(TPLUS(L,M,KE),M=1,9)
      DO 1251 L=1,9
1251 PRINT 1252,L,(TMINUS(L,M,KE),M=1,9)
      WRITE(2)(TMINUS(L,M,KE),M=1,9)
1351 CONTINUE
1345 CONTINUE
      REWIND 3
      IB=NH=0
      DO 300 I=1,10
      DO 337 L=1,9
      DO 337 M=1,9
1337 APLUS(L,M,I)=AMINUS(L,M,I)=0.
      SIGZC(I)=SIGAC(I)=CURTZX(I)=CURTXC(I)=0.
      CTFINE(I)=0.1*FLOAT(I)
      DO 300 J=1,10
      DO 300 K=1,12
300 ALBEDO(I,J,K)=0.
      DO 289 K=1,12
289 PHFINE(K)=30.*FLOAT(K)
      DO 302 K=1,20
302 SIGX(K)=SIGZ(K)=CURTX(K)=CURTZ(K)=0.
      DO 312 K=1,10
      DO 312 L=1,10
      ISGEX(I,K)=0.
312 EXPCOF(I,K)=0.
298 READ(3)LCR,((STO(L1,L),L1=1,7),L=1,LCR)
299 NH=NH+1
      DO 301 L=2,LCR
      IF(STO(L1,L).GT.ZMIN)GO TO 301
      CTHET=ABS(STO(3,L))
      STHET=STO(4,L)
      PHI=STO(5,L)
      CLASSIFY EMERGENT POLAR ANGLE
      DO 315 J=1,10
315 IF(CTHET.LT.CTFINE(J))GO TO 316
      J=J+1
316 JC=J
      CLASSIFY EMERGENT ENERGY
      DO 325 I=1,10
325 IF(STO(6,L).GT.ECRSE(I))GO TO 326
      I=I+1
326 IC=I
      CLASSIFY AZIMUTH
      PP=RTDEG*PHI
      NN=2*E*V-1,12

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335 IF (NH.LT.NNN)GO TO 336
336 K=1
C ALBEDO(IC,JC,KC)=ALBEDO(IC,JC,KC)+1./FN
C COMPUTE VARIANCE,STD. DEV. AND SKEWNESS OF DIRECTION COSINES
C ON FINE ENERGY GRID
C CLASSIFY ENERGY ON FINE GRID
DO 425 K=1,20
425 IF (STO(6,L).GT.ENERGY(K))GO TO 426
K=20
426 KE=K
DIF=CTHET-CMEAN(KE)
SIGZ(KE)=SIGZ(KE)+DIF*DIF
CURTZ(KE)=CURTZ(KE)+DIF*DIF*DIF
DIF=STHET-COS(PHI)-SCMEAN(KE)
SIGX(KE)=SIGX(KE)+DIF*DIF
CURTX(KE)=CURTX(KE)+DIF*DIF*DIF
CLASSIFY ENERGY ON COARSE GRID
KCRSE=(KE+1)/2
DIF=CTHET-CMC(KCRSE)
SIGZC(KCRSE)=SIGZC(KCRSE)+DIF*DIF
CURTZC(KCRSE)=CURTZC(KCRSE)+DIF*DIF*DIF
DIF=STHET-COS(PHI)-SMC(KCRSE)
SIGXC(KCRSE)=SIGXC(KCRSE)+DIF*DIF
CURTXC(KCRSE)=CURTXC(KCRSE)+DIF*DIF*DIF
GET LEGENDRE COEFFICIENTS OF POLAR ANGULAR DISTRIBUTION
CALL LEPI(PL,CTHET,9)
DO 430 LL=1,10
430 EXPCOF(LL,KCRSE)=EXPCOF(LL,KCRSE)+PL(LL)/FN
CP(M)=COS(M*PHI)
SP(M)=SIN(M*PHI)
338 CALL ASSOC(PL,CTHET,STHET,PLM)
DO 341 LL=1,9
431 FM=YCOF(LL,M)*PLM(LL,M)
FM=FM/FN
APLUS(LL,M,KCRSE)=APLUS(LL,M,KCRSE)+FM*CP(M)
339 AMINUS(LL,M,KCRSE)=AMINUS(LL,M,KCRSE)+FM*SP(M)
341 CONTINUE
301 CONTINUE
IF (NH.LT.NNN)GO TO 298
NH=0
18=18+1
KPUT=1
DO 600 K=1,12
DO 600 J=1,10
DO 601 I=1,10
TEMP(I)=ALBEDO(I,J,K)
601 ALBEDO(I,J,K)=0.
CALL STATS(TEMP,ISIG,10,KPUT,18)
IF (18.LT.10)GO TO 600
DO 602 I=1,10
ALBEDO(I,J,K)=TEMP(I)
602 ISG(I,J,K)=ISIG(I)
600 CONTINUE
DO 700 I=1,9
DO 701 K=1,10
TEMP(K)=APLUS(I,J,K)
701 APLUS(I,J,K)=0.
CALL STATS(TEMP,ISIG,10,KPUT,18)
IF (18.LT.10)GO TO 700
DO 702 K=1,10
APLUS(I,J,K)=TEMP(K)
702 ISGAP(I,J,K)=ISIG(K)

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```

700 CONTINUE
DO 0 I=1,9
DO 10 J=1,9
DO 711 K=1,10
TEMP(K)=AMINUS(I,J,K)
711 AMINUS(I,J,K)=0.
CALL STATST(TEMP,ISIG,10,KPUT,18)
IF(18.LT.10)GO TO 710
DO 712 K=1,10
AMINUS(I,J,K)=TEMP(K)
712 ISGAM(I,J,K)=ISIG(K)
710 CONTINUE
DO 720 I=1,10
DO 721 J=1,10
TEMP(J)=EXPCOF(I,J)
721 EXPCOF(I,J)=0.
CALL STATST(TEMP,ISIG,10,KPUT,18)
IF(18.LT.10)GO TO 720
DO 722 J=1,10
EXPCOF(I,J)=TEMP(J)
722 ISGEX(I,J)=ISIG(J)
720 CONTINUE
GO TO 298
C
NORMALIZE AND PRINT
500 DO 510 I=1,10
DO 501 J=1,10
SUM(J)=0.
DO 511 K=1,12
511 SUM(J)=SUM(J)+ALBEDO(I,J,K)
501 CONTINUE
ELDW=0.
EHIGH=EO
IF(1.GT.1)ENIGH=ECRSE(I-1)
PRINT 502,EO,ENIGH,ECRSE(I),(CTFINE(J),J=1,10)
502 FORMAT(//20X,'ALBEDOS --- INCIDENT ENERGY =',F7.3,'MEV',
15X,'EMERGENT ENERGY BOUNDS =',F7.3,' TO ',F7.3,' MEV',
245X,'EMERGENT POLAR ANGLE COSINE=9X,10E12.5/1X,PHI(DEG)=')
DO 503 K=1,12
503 PRINT 504,PHFINE(K),(ALBEDO(I,J,K),ISG(I,J,K),J=1,10)
504 FORMAT(1X,F7.3,1X,10(F9.5,13))
510 CONTINUE
C
FINAL COMPUTATION OF VARIANCES, SKEWNESS AND LEGENDRE COEFFS
PRINT 519
DO 520 KE=1,20
STDX=STDZ=GAMX=GAMZ=0.
IF(NKCR(KE).EQ.0)GO TO 520
FNK=FLOAT(NKCR(KE))
SIGZ(KE)=SIGZ(KE)/FNK
CURTZ(KE)=CURTZ(KE)/FNK
SIGX(KE)=SIGX(KE)/FNK
CURTX(KE)=CURTX(KE)/FNK
STDZ=SQRT(SIGZ(KE))
STDX=SQRT(SIGX(KE))
GAMZ=CURTZ(KE)/(STDZ**3)
GAMX=CURTX(KE)/(STDX**3)
1STDZ,CURTZ(KE),GAMZ
520 PRINT 521,ENERGY(KE),SIGZ(KE),STDZ,CURTZ(KE),GAMZ,SIGX(KE),
1STDX,CURTX(KE),GAMX
519 FORMAT(//,20X,'ANGLE ALBEDO STATISTICS=',3X,'E(MEV)=',4X,'VARZ=
19X,'STDZ=',8X,'KURTOSZ=',8X,'SKWZ=',8X,'VARX=',8X,'KURTDX=
26X,'SKWAX=')
521 FORMAT(1X,F7.3,1X,10E12.5)
DO 620 K=1,10
STDX=STDZ=GAMX=GAMZ=0.
IF(NKCR(K).EQ.0)GO TO 620

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FNK=FLOAT(NKCR(K))
SI(K)=SIGZC(K)/FNK
CURC(K)=CURTZC(K)/FNK
SIGXC(K)=SIGXC(K)/FNK
CURTAC(K)=CURTAC(K)/FNK
STDX=SQRT(SIGZC(K))
STDX=SQRT(SIGZC(K))
GAMZ=CURTZC(K)/(STDX**3)
GAMZ=CURTZC(K)/(STDX**3)
1STDX,CURTAC(K),GAMZ
620 PRINT 521,ECRSE(K),SIGZC(K),STDX,CURTAC(K),GAMZ,SIGXC(K),
C PRINT LEGENDRE COEFFICIENTS
523 FORMAT(//45X,*,LEGENDRE COEFFICIENTS OF ALBEDO/4X,*,E(MEV) L=*,
14X,*,0.11X,*,1.11X,*,2.11X,*,3.11X,*,4.11X,*,5.11X,*,6.11X,*,7.
211X,*,8.11X,*,9*)
DO 625 LL=1,10
AL=LL
FAL=SQRT((2.*AL-1.)/12.56637061)
DO 625 K=1,10
EXPCOF(LL,K)=EXPCOF(LL,K)*FAL
625 CONTINUE
PRINT 523
DO 626 K=1,10
626 PRINT 526,ECRSE(K), (EXPCOF(LL,K),LL=1,10)
526 FORMAT(3X,F7.3,1X,10E12.5)
PRINT 651
DO 525 K=1,10
525 PRINT 524,ECRSE(K), (ISGEX(LL,K),LL=1,10)
524 FORMAT(3X,F7.3,1X,10I4)
DO 345 KE=1,10
3461 PRINT 347,ECRSE(KE)
347 FORMAT(//1X,*,ENERGY = *,F10.5,*,MEV*/1X,*,SPHERICAL HARMONICS COEFF
1CIENTS - APLUS*/
DO 343 L=1,9
343 PRINT 349,L,(APLUS(L,M,KE),M=1,9)
349 FORMAT(1X,L=*,15,1X,9E12.5)
PRINT 651
651 FORMAT(1X,*,PERCENT ERRORS IN ABOVE ITEMS*)
DO 652 L=1,9
652 PRINT 653,L,(ISGAP(L,M,KE),M=1,9)
348 PRINT 348
348 FORMAT(1X,*,SPHERICAL HARMONICS COEFFICIENTS - AMINUS*/
DO 342 L=1,9
342 PRINT 349,L,(AMINUS(L,M,KE),M=1,9)
PRINT 651
DO 654 L=1,9
654 PRINT 653,L,(ISGAM(L,M,KE),M=1,9)
653 FORMAT(1X,L=*,15,1X,9I4)
WRITE(2)(EXPCOF(L,KE),L=1,10)
DO 351 L=1,9
WRITE(2)(APLUS(L,M,KE),M=1,9)
WRITE(2)(AMINUS(L,M,KE),M=1,9)
351 CONTINUE
345 CONTINUE
RETURN
END
SUBROUTINE LEP(Y,X,N)
C COMPUTE LEGENDRE POLYNOMIALS
C
C DIMENSION Y(1)
Y(1)=1.
IF(N)1,1,2
1 RETURN
2 Y(2)=X
IF(N-1)1,1,3

```

```

3 DOX I=2,N
4 G=X(I)
5 Y(I+1)=G-Y(I-1)+G-(G-Y(I-1))/FLOAT(I)
RETURN
END
SUBROUTINE STATS(XVAL,ISIG,N,M,IB)
C
C COMPUTATION OF STATISTICAL STANDARD ERRORS (SEE VOL 1, PAGE 238)
C
DIMENSION XVAL(1),ISIG(1),AVE(10),SIG(10)
DIMENSION ECAGE(3000),ECSIG(3000)
IF(IB.NE.1) GO TO 2
DO 1 I=1,N
AVE(I) = XVAL(I)
ISIG(I)=99
1 SIG(I)=0.0
10 DO 100 I=1,N
IKK=K+I-1
ECAGE(IKK)=AVE(I)
ECSIG(IKK)=SIG(I)
K=K+N
RETURN
2 XNSORS=IB
DO 200 I=1,N
IKK=K+I-1
AVE(I)=ECAGE(IKK)
SIG(I)=ECSIG(IKK)
200 SIG(I)=ECSIG(IKK)
DO 3 I=1,N
SIG(I)=(XNSORS-2.0)/(XNSORS-1.0)*SIG(I)+(XVAL(I)-AVE(I))*2/XNSORS
AVE(I) = ((XNSORS-1.0)*AVE(I)+XVAL(I))/XNSORS
XVAL(I) = AVE(I)
ERRORS=SQRT(SIG(I)/XNSORS)
ISIG(I)=99
IF(AVE(I).EQ.0.0) GO TO 3
ISIG(I)=100.0*ERROR/ABS(AVE(I))*50001
IF(ISIG(I).GT.99) ISIG(I)=99
3 CONTINUE
GO TO 10
END
SUBROUTINE ENERGY
C
C "ENERGY" PERFORMS A TABLE LOOK-UP ON THE ELECTRON ENERGY AND
C PROVIDES THE CORRESPONDING CROSS SECTION VALUE.
C
COMMON/MC/STO(7,51),NCOLL,CTMO,CTH,STM,PHI,CPHI,SPHI,EGY,CPHIO,
1SPHIO,EO,SPAC,AMBDA,N,IFLAG,MAXCOLL,LCHR,STAB(200),ETAB(200),TLIM,
2AO,YO,ZC,X,Y,Z,ECUT,CLAB,THICK(11),NMAX,ZMIN,ZMAX,IE,INRAN,XIRA
3,NPRINT,TLAST,CTHC,NBATCH,ETA,PATH,S,SIGTAB(200),ENTAB(200),KIRC
IF(EGY-.1)1,2,2
2 IE=1+IFIX(100.*(1.-EGY))
GO TO 3
1 IF(EGY.LT.0.01)EGY=.01
IE=91+IFIX(1000.*(1.-EGY))
3 DE=STAB(IE)*S
EGY=EGY-DE
IF(EGY.GT.ECUT)GO TO 4
IFLAG=1
EGY=ECUT
ENTRY CROSS
4 ETA=ETAB(IE)
AMBDA=1./SIGTAB(IE)
RETURN
END
SUBROUTINE ASSOC(PL,CX, SX,P)
C
C "ASSOC" COMPUTES THE ASSOCIATED LEGENDRE FUNCTIONS"

```

```

C
      SIGN PL(10),P(9,9)
      IF(JX.GT.0.0)GO TO 1
      DO 2 L=1,9
      DO 2 M=1,9
      2 P(L,M)=0.0
      RETURN
      1 P(1,1)=SX
      P(2,1)=3.*CX*SX
      P(2,2)=2.*CX/SX*P(2,1)-6.*PL(3)
      M=1
      LOW=M+1
      DO 10 L=LOW,8
      10 P(L+1,M)=((2*L+1)*CX*P(L,M)-(L+M)*P(L-1,M))/FLOAT(L-M+1)
      P(3,2)=2.*CX/SX*P(3,1)-12.*PL(4)
      DO 100 M=2,8
      LOW=M+1
      IF(LOW.EQ.9)GO TO 51
      DO 50 L=LOW,8
      50 P(L+1,M)=((2*L+1)*CX*P(L,M)-(L+M)*P(L-1,M))/FLOAT(L-M+1)
      51 P(LOW,M+1)=(2*M)*P(LOW,M)*CX/SX-((LOW+M)*P(LOW,M+1))*P(LOW,M-1)
      IF(M.EQ.8)GO TO 100
      P(LOW+1,M+1)=(2*M)*P(LOW+1,M)*CX/SX-((LOW+M+1)*P(LOW+M+1))*P(LOW+1,M-1)
      100 CONTINUE
      RETURN
      END

```

Program Listing

- INTFC -

(Reference: Vol. I, Section IV.B.4)

```

OVERLAY(COMM,0.0)
PROGRAM INTFC(INPUT,OUTPUT,TAPE2,TAPE3)

THIS PROGRAM MAKES USE OF THE ELECTRON TRANSMISSION AND ALBEDO
DATA FROM "BIGSLAB" AND PERFORMS A DETAILED MONTE CARLO CALCULA-
TION IN THE REGION OF A TWO MATERIAL INTERFACE. IF THE ELECTRON
ESCAPES FROM THE INTERFACE REGION AND ENTERS THE THICK SLAB POR-
TION OF EITHER MATERIAL, THE ALBEDO DISTRIBUTION FROM "BIGSLAB" IS
SAMPLED SO THAT THE BACKSCATTERED ELECTRON IS POINTED BACK TOWARD
THE INTERFACE REGION WITH THE APPROPRIATE ENERGY AND DIRECTION.
THE TRANSMISSION DISTRIBUTION FROM "BIGSLAB" IS USED AS THE
INITIAL SOURCE FUNCTION INTO THE INTERFACE REGION. EXCEPT FOR
THESE FEATURES, THIS PROGRAM AND "BIGSLAB" ARE STRUCTURED
SIMILARLY. THE OUTPUT QUANTITIES ARE DEFINED IN VOL. I, PAGES
257-260.

COMMON/MC,STOI(7,201),NCOLL,CTHO,CTH,STH,PHI,CPHI,SPHI,EGY,CPHIO,
1SPHID,ED,SPAC,AMBD,A,N,IFLAG,MAXCOLL,LCHR,STAB(200),ETAB(200),TLIM,
2XO,YO,ZO,X,Y,Z,ECUT,CLAB,THICK(11),NMAX,ZMIN,ZMAX,IE,INRAN,XIRA
3,NPRNT,TLAST,CTHCH,NBATCH,ETA,PATH,S,SIGTAB(200),ENTAB(200),XIRC,
4PLI(10,101),WF,WMIN
DATA CPHIO,SPHIO,ZMIN,ZMAX,N/1.0,0.0,1000.0/
DATA ECUT,SPAC,XO,YO,ZO/0.800,2*0.5-E-4/
DATA THICK/50.51...52.100000./
DATA WMIN/1.E-5/

THE FIRST OVERLAY IS THE MONTE CARLO CALCULATION

CALL OVERLAY(4HCOMM,1,0)

THE SECOND OVERLAY IS THE PROCESSING PROGRAM FOR THE TRAJECTORY
CHARACTERISTICS. THE OUTPUT QUANTITIES ARE COMPUTED HERE.

CALL OVERLAY(4HCOMM,2,0)
CALL EXIT
END
OVERLAY(COMM,1.0)
PROGRAM MCTR
CALL MCTR
END
SUBROUTINE MCTR
COMMON/MC,STOI(7,201),NCOLL,CTHO,CTH,STH,PHI,CPHI,SPHI,EGY,CPHIO,
1SPHID,ED,SPAC,AMBD,A,N,IFLAG,MAXCOLL,LCHR,STAB(200),ETAB(200),TLIM,
2XO,YO,ZO,X,Y,Z,ECUT,CLAB,THICK(11),NMAX,ZMIN,ZMAX,IE,INRAN,XIRA
3,NPRNT,TLAST,CTHCH,NBATCH,ETA,PATH,S,SIGTAB(200),ENTAB(200),XIRC,
4PLI(10,101),WF,WMIN
DIMENSION TIMR(11)
CALL SETRUN
CALL SETRN
FN=FLOAT(NMAX)
COLNO=0.
MTM=1
CALL SECOND(TIME)
TIMR(1)=TIME
5 N=N+1
CALL SETHIS
CALL SCORE
CALL PENET
IF(IFLAG.EQ.1) GO TO 6
CALL ENERGY
IF(IFLAG.EQ.1) GO TO 6
20 SAMPLE THE ALBEDO DISTRIBUTION FROM "BIGSLAB"

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IF (FLAG.EQ.7) CALL BACKS(EGY,CTH,STH,PHI,CPhi,SPHI,NF)
IF (LT.WMIN) GO TO 6
IF (LAG.NE.7) CALL ANGLES
FLAG=0
GO TO 20
6 COLLNO=COLLNO+FLOAT(NCOLL)/FN
IF (NBATCH.EQ.0) GO TO 60
NTEST=N-NBATCH*(N/NBATCH)
IF (NTEST.NE.0) GO TO 60
MTM=MTM+1
CALL SECOND(TIME)
TIMR(MTM)=TIME
DELT=TIME-TIMR(1)
AVGT=DELT/FLOAT(MTM-1)
TREM=TIME-TIM-DELT
TLEFT=1.5*AVGT
IPCT=10*(MTM-1)
PRINT 75,IPCT,TREM
75 FORMAT(1X,'THE PROBLEM IS',14,' PERCENT COMPLETE.',1X,
1'TIME TO FINISH IS',F12.3,'SECONDS')
IF (IPCT.EQ.100) GO TO 100
IF (TREM.NE.0) LEFT=1.5*AVGT
GO TO 100
60 IF (N-NMAX) 5,100,100
2 FORMAT(1X,F16.9)
100 PRINT 64,COLLNO
IF (LCHR.GT.0) AND (LCHR.LE.201) WRITE(2)((STO(K,L),K=1,7),L=1,201)
IF (LCHR.EQ.0) WRITE(2)((STO(K,L),K=1,7),L=1,201)
CALL RANGET(XIRC)
64 FORMAT(1X,'AVERAGE NUMBER OF COLLISIONS PER HISTORY = ',F7.3)
IF (NBATCH.EQ.0) GO TO 200
MTM=MTM-1
PRINT 65,MTM,NBATCH
65 FORMAT(1X,'THERE ARE ',14,' BATCHES OF',15,' HISTORIES')
AVGT=0.
SUM=TIMR(1)
DO 66 M=2,MTM
TIMR(M)=TIMR(M)-SUM
SUM=SUM+TIMR(M)
AVGT=AVGT+TIMR(M)
MMI=M-1
66 PRINT 67,MM,TIMR(M)
67 FORMAT(1X,'BATCH NO.',14,' TIME = ',F6.3,' SECONDS')
AVGT=AVGT/FLOAT(MTM)
PRINT 68,AVGT
68 FORMAT(1X,'AVERAGE TIME PER BATCH = ',F6.3,' SECONDS')
200 PRINT 190
190 FORMAT(1X,'INRAN',11X,'IRA',12X,'IRC')
PRINT 195,INRAN,XIRA,XIRC
195 FORMAT(3(1X,016))
RETURN
END
SUBROUTINE SETRUN
COMMON/MC/STO(7,201),NCOLL,CTHO,CTH,STH,PHI,CPhi,SPHI,EGY,CPhiD,
1SPHID,EG,SPAC,AMBD,N,IFLAG,MAXCOLL,LCHR,STAB(200),ETAB(200),TLIM,
2XO,YO,ZO,X,Y,Z,ECUT,CLAB,THICK(11),NMAX,ZMIN,ZMAX,IE,INRAN,XIRA,
3,NPRNT,TLAST,CTHC,NBATCH,ETA,PATH,5,SIGTAB(200),ENTAB(200),XIRC,
4PLI(10,101),WF,WMIN
DIMENSION INCDEL(3)
READ 1,NMAX,MAXCOLL,NPRNT,INPT,NSLABS
NMAX - TOTAL NO. OF MONTE CARLO HISTORIES
STORED ON TAPE OR DISK UNTIL PROCESSING TIME
MAXCOLL - MAXIMUM NUMBER OF COLLISIONS PER HISTORY TO BE ALLOWED
NPRNT - DEBUG PRINT CONTROL PARAMETER
NPRNT EQUAL TO ZERO SUPPRESSES DEBUG PRINTOUT
NPRNT NOT EQUAL TO ZERO ACTIVATES DEBUG PRINTOUT

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READ 3, TLIM, CTHD, ED, ECUT
IF (LABS.EQ.0) GO TO 50
RE 3, (THICK(1), I=1, 11)
DO 51 I=1, 4
51 THICK(I)=THICK(1)+I.E-5
IF (INPT.GT.0) GO TO 40
INRAN=0
GO TO 45
40 READ 1, (NCODE(1), I=1, 3)
IF (NCODE(1).EQ.1) READ 8, INRAN
IF (NCODE(2).EQ.1) READ 3, SFAC
IF (NCODE(3).EQ.1) READ 3, XO, YO, ZO
ED - INITIAL PARTICLE ENERGY
XO, YO, ZO - INITIAL PARTICLE COORDINATES
SFAC - STOPPING POWER FUDGE FACTOR
45 PRINT 4, NMAX, MAXCOLL
PRINT 9, ED, ECUT, XO, YO, ZO, CTHD, CPHI, SPHIO
PRINT 11, SFAC
11 FORMAT(1X, 'STOPPING POWER FUDGE FACTOR (SFAC) = ', F10.5)
9 FORMAT(1X, 'ED (INITIAL ENERGY MEV) = ', E12.5//1X, 'ECUT (LOW ENERGY
ICUT OFF MEV) = ', E12.5//1X, 'XO, YO, ZO, (SOURCE POINT COORDINATES)
3 = ', E12.5//1X, 'CTHD (INCIDENT POLAR COSINE) = ', E12.5//1X, 'CPHIO
4, SPHIO (INCIDENT AZIMUTH) = ', E12.5)
8 FORMAT(D18)
1 FORMAT(G110)
3 FORMAT(8F10.0)
4 FORMAT(1X, 'NMAX (NO. OF HISTORIES) = ', I6//1X, 'MAXCOLL (MAXIMUM
1 ALLOWED NUMBER OF COLLISIONS) = ', I3)
10 N=0
LCNR=0
REWIND 2
XIRA=XIRC=INRAN
NBATCH=NMAX/10
CALL RANSET(XIRA)
CALL ESET
DO 20 K=1, 7
DO 20 L=1, 201
20 STO(K, L)=0.
DO 30 I=1, 10
DO 30 J=1, 101
CZ=.01*(I-1)
30 PLI(L, I)=PLINT(L, CZ)
RETURN
END
SUBROUTINE SETHIS
COMMON/NC/STO(7, 201), NCOLL, CTHD, CTH, STH, PHI, CPHI, SPHI, EGY, CPHIO,
1SPHIO, ED, SFAC, AMBDA, N, IFLAG, MAXCOLL, LCNR, STAB(200), ETAB(200), TLIM,
2XO, YO, ZO, X, Y, Z, ECUT, CLAB, THICK(11), NMAX, ZMIN, ZMAX, IE, INRAN, XIRA
3, NPRINT, TLAST, CTHC, NBATCH, ETA, PATH, S, SIGTAB(200), ENTAB(200), XIRC,
4PLI(10, 101), WF, WMIN
DIMENSION ESCR(9), PSRC(8), NG(8)
DATA ESCR/.4, .35, .3, .25, .2, .15, .1, .05, .01/
DATA PSRC/.023713, .13685, .18255, .18768, .13856, .11498, .075147, .1405
12/
DATA JS/1/
IF (N.GT.1) GO TO 1
NN=0
DO 2 JU=1, 8
NG(JU)=NMAX+PSRC(JU)
2 NN=NN+NG(JU)
NLEFT=NMAX-NN
NG(8)=NG(8)+NLEFT
DO 3 JU=2, 8
3 NG(JU)=NG(JU)+NG(JU-1)
PRINT 17, (NG(JU), JU=1, 8)
17 FORMAT(1X, 'CUMULATIVE TOTAL OF HISTORIES FOR THE 8 ENERGY GROUPS=')

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1 1X R(10)
1 PA 0.
NCLL=0
S=0.
X=X0
Y=Y0
Z=Z0
WF=.32736
CPHI=CPHIO
SPHI=SPHIO
PHI=0.0
IF (N.GT.NG(JS))JS=JS+1
RA=RA*(0)
ED=RA*(SRC(JS)+ESRC(JS+1))*(1.-RA)
USRC=(JS+1)/2

SAMPLE THE SOURCE ENERGY ANGLE DISTRIBUTION FROM A "BIGSLAB" RUN

CALL SAMPMU(,SRC)
CTH=CTH0
STH=SORT(1.-CTH**2)
EGY=ED
IF (EGY-.1)7,8,8
8 IE=1+IFIX(100.*(1.-EGY))
GO TO 9
7 IF (EGY.LT..001)EGY=.001
IE=91+IFIX(1000.*(1.-EGY))
9 CALL CROSS
IFLAG=0
RETURN
END

SUBROUTINE PENET
COMMON/MC/STG(7,201),NCOLL,CTH0,CTH,STH,PHI,CPHI,SPHI,EGY,CPHIO,
1SPHID,ED,SPAC,AMBDA,N,IFLAG,MAXCOLL,LCHR,STAB(200),ETAB(200),TLIM,
2X0,Y0,Z0,X,Y,Z,CUT,CLAB,THICK(11),NMAX,ZMIN,ZMAX,IE,NRAN,XIRA,
3NPINT,TLAST,CTHIC,CTHICBATCH,ETA,PATH,S,SIGTAB(200),ENTAB(200),XIRC,
4PLI(10,101),WF,WMIN
X1=X
X1=X
Z1=Z
Z1=Z
4 RB=RA*(0)
IF (RB.GT.0.)GO TO 5
GO TO 4
5 S=-ALOG(RB)*AMBDA
Z=Z1+S*CTH
NCOLL=NCOLL+1
IF (Z.GT.THICK(2).AND.Z1.LE.THICK(2))CALL SCORE
IF (Z.LE.THICK(2).AND.Z1.GT.THICK(2))CALL SCORE
IF (Z.GE.THICK(1).AND.Z.LE.THICK(3))GO TO 10
IFLAG=7
6 IF (CTH)6,6,7
7 IF (Z.LT.THICK(1))Z=THICK(1)
GO TO 8
7 IF (Z.GT.THICK(3))Z=THICK(3)
8 IF (CTH.NE.0.0)GO TO 89
S=1.E20
GO TO 10
88 S=(Z-Z1)/CTH
10 X=X1+S*STH*CPHI
Y=Y1+S*STH*SPHI
PATH=PATH+S
IF (NCOLL.EQ.MAXCOLL) IFLAG=1
RETURN
END

SUBROUTINE ESET
COMMON/MC/STG(7,201),NCOLL,CTH0,CTH,STH,PHI,CPHI,SPHI,EGY,CPHIO,

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```

158 1,ED,SPAC,AMBDA,N,IFLAG,MAXCOLL,LCHR,STAB(200),ETAB(200),TLIM,
2X0,3,ED,X,Y,Z,ECUT,CLAB,THICK(11),NMAX,ZMIN,ZMAX,IE,INPRAN,RA
3,NPRT,TLAST,CTHC,NBATCH,ETA,PATH,S,SIGTAB(200),ENTAB(200),XIRC,
4PLI(10,101),WF,WMIN
DATA PI,XL2/3.14159265,.593147181/
DATA ZA,AA,AVOG,RO,DNSTY,EXCIT,FCC/13,.26,98,6.025E23,2.8179E-13,
12.7,163,E-6,.15360628/
ENTAB(1)=1.0
DE=.01
DO 1 11=2,91
1 ENTAB(11)=ENTAB(11-1)-DE
DE=.001
DO 2 11=92,190
2 ENTAB(11)=ENTAB(11-1)-DE
DO 3 11=1,190
E=ENTAB(11)/.511
E1=(1.+E)*(1.+E)
E2=E*(2.+E)
BETA=SQRT(E2/E1)
XLG=ALOG(E2/(2.*(EXCIT/.511)**2))
FM=1.-BETA*BETA*(.125+E*(2.+E+.1)*XL2)/E1
STAB(11)=(FCC-ZA/AA/BETA**2)*(XLG-FM)*SFAC
ETAB(11)=0.5*(ZA**((1./3.)/121.245)**2/E2*(1.13+3.76*(ZA/137.))**2*
1E1/E2)
SIGTAB(11)=ZA*(1.+ZA)*AVOG*RO*RO*ZA/AA*E1/(E2*E2)/(ETAB(11)
1*(1.+S*ETAB(11)))
ETAB(11)=.035276
3 IF(11.GT.91)ETAB(11)=.103378
IF(NPRT.EQ.0)RETURN
PRINT 75
75 FORMAT(1X,'CROSS SECTION AND STOPPING POWER TABLES',//,1X,'ENERGY(M
1EV) STOPPING POWER ETA SIGMA')
DO 76 11=1,190
76 PRINT 77,ENTAB(11),STAB(11),ETAB(11),SIGTAB(11)
77 FORMAT(1X,F10.6,3E16.9)
RETURN
END
SUBROUTINE ANGLES
COMMON/NC/STO(7,201),NCOLL,CTHO,CTH,STH,PHI,CPHI,SPHI,EGY,CPHIQ,
1SPHIQ,EO,SPAC,AMBDA,N,IFLAG,MAXCOLL,LCHR,STAB(200),ETAB(200),TLIM,
2X0,Y0,Z0,X,Y,Z,ECUT,CLAB,THICK(11),NMAX,ZMIN,ZMAX,IE,INPRAN,XIRA
3,NPRT,TLAST,CTHC,NBATCH,ETA,PATH,S,SIGTAB(200),ENTAB(200),XIRC,
4PLI(10,101),WF,WMIN
DATA TWOPI/6.283185307/
RD=2.*RANF(0)
COM=(RD*(1.+ETA)-ETA)/(RD+ETA)
SOM=SQRT(1.-COM**2)
RC=RANF(0)
RHO=TWOPI*RC
CRHO=COS(RHO)
SRHO=SIN(RHO)
STH=SIN(RHO)
CTH=CTH
CTH=CTH*COM+STH*SOM*CRHO
STH=SQRT(1.-CTH**2)
IF(STH1.EQ.0.0.OR.STH.EQ.0.0) GO TO 1
C1=(COM-CTH*CTH1)/(STH*STH1)
S1=SRHO*SOM/STH
CPHI1=CPHI
SPHI1=SPHI
CPHI=CPHI1+C1*SPHI1*S1
SPHI=CPHI1+S1*SPHI1*C1
GO TO 2
1 CPHI=CRHO
SPHI=SRHO
2 CONTINUE

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```

PH=ATAN2(SPHI,CPHI)
I=11.LT.0.0)PHI=PHI+TWOP1
RE=JRN
END
SUBROUTINE LEP(Y,X,N)
  DIMENSION Y(1)
  Y(1)=1.
  IF(N1).1.2
  1 RETURN
  2 Y(2)=X
  IF(N-1).1.3
  3 DO 4 I=2,N
    G=X*Y(I)
  4 Y(I+1)=G-Y(I-1)+G-(G-Y(I-1))/FLOAT(I)
  RETURN
END
SUBROUTINE ENERGY
COMMON/MC/STO17,201),NCOLL,CTHO,CTH,STH,PHI,CPHI,SPHI,EGY,CPHIO,
1SPHIO,EO,SPAC,AMBDA,N,IFLAG,MAXCOLL,LCHR,STAB(200),ETAB(200),TLIM,
2XO,YO,ZO,X,Y,Z,ECUT,CLAB,THICK(11),NMAX,ZMIN,ZMAX,IE,INRAN,XIRA
3,NPRINT,TLAST,CTHC,NBATCH,ETA,PATH,S,SIGTAB(200),ENTAB(200),XIRC,
4PLI(10,101),WF,WMIN
  IF(EGV-1).1.2.2
  2 IE=1+FIX(100.*(1.-EGY))
  GO TO 3
  1 IF(EGY.LT.0.001)EGY=.001
  IE=91+FIX(1000.*(1.-EGY))
  3 DE=STAB(IE)*S
  EGY=EGY-DE
  IF(EGY.GT.ECUT)GO TO 4
  IFLAG=1
  EGY=ECUT
  ENTRY CROSS
  4 ETA=ETAB(IE)
  AMBDA=1./SIGTAB(IE)
  RETURN
END
SUBROUTINE ASSOQ(PL,CX,SX,P)
  COMPUTE ASSOCIATED LEGENDRE FUNCTIONS
  DIMENSION PL(10),P(9,9)
  IF(SX.GT.0.0)GO TO 1
  DO 2 L=1,9
    DO 2 M=1,9
      P(L,M)=0.0
  RETURN
  1 P(1,1)=SX
  P(2,1)=3.*CX*SX
  P(2,2)=2.*CX/SX+P(2,1)-8.*PL(3)
  M=1
  LOW=M+1
  DO 10 L=LOW,8
    P(L+1,M)=((2*L+1)*CX*P(L,M)-(L*N)*P(L-1,M))/FLOAT(L-M+1)
    P(3,2)=2.*CX/SX+P(3,1)-12.*PL(4)
    DO 100 M=2,8
      LOW=M+1
      IF(LOW.EQ.9)GO TO 51
      DO 50 L=LOW,8
        P(L+1,M)=((2*L+1)*CX*P(L,M)-(L*N)*P(L-1,M))/FLOAT(L-M+1)
        51 P(LOW,M+1)=(2*M)*P(LOW,M)*CX/SX-((LOW+M)*P(LOW,M+1))+P(LOW,M-1)
        IF(M.EQ.8)GO TO 100
        P(LOW+1,M+1)=(2*M)*P(LOW+1,M)*CX/SX-((LOW+M+1)*P(LOW+1,M+2))+
        P(LOW+1,M-1)
      100 CONTINUE
    RETURN

```

```

END
SUP  UTINE SAMPNU(JSRC)
C
C THIS ROUTINE SAMPLES THE TRANSMISSION DISTRIBUTION DATA FROM A
C "BIGSLAB" RUN
C
COMMON/NC/STOI(7,201),NCOLL,CTHO,CTH,STH,PHI,CPHI,SPHI,EGV,CPHIO,
15PHIO,EQ,EFAC,AMBDA,N,IFLAG,MAXCOLL,LCRR,STAB(200),ETAB(200),ILIM,
2XO,YO,ZO,X,Y,Z,ECUT,CLAB,THICK(11),NMAX,ZMIN,ZMAX,IE,INRAN,XIRA,
3,NPRNT,TLAST,CTHC,NBATCH,ETA,PATH,S,SIGTAB(200),ENTAB(200),XIRC,
4PLI(10,101),WF,WMIN
DIMENSION LMAX(4),PCUM(101),TLEG(10,4)
DATA TLEG/.14827,.22613,.22719,.18634,.13199,.083137,.047188,.0240
163,.01107,.0052421,.34190,.47020,.3737,.19678,.054955,-.012527,
2-.029918,-.03217,-.033122,-.030991,.23414,.29725,.19183,.055359,
3-.01838,-.024905,-.0090189,-.002056,.0031651,.0089282,
4.19916,.2388,.12831,.0085602,-.033786,-.013766,.0091091,
5.008585,.00045677,.00098932/
DATA JOLD/O/
DATA LMAX/8,7,7,7/
IF(JSRC.EQ.JOLD)GO TO 10
JOLD=JSRC
LM=LMAX(JSRC)
DO 1 K=1,101
PCUM(K)=0.
DO 1 L=1,LM
AL=L
FAC=SQRT(2.*AL-1.)
PCUM(K)=PCUM(K)+FAC*TLEG(L,JSRC)*PLI(L,K)
1 CONTINUE
DO 2 K=1,101
IF(PCUM(K).LT.O.)PCUM(K)=0.
2 PCUM(K)=PCUM(K)/PCUM(101)
PRINT 3,JSRC,(PCUM(K),K=1,101)
3 FORMAT(1X,'SOURCE GROUP NO.',12/
11X,'CUMULATIVE PROBABILITIES OF EMERGENT SOURCE ANGLE(0.-1...01)')
2(1X,10E12.5))
10 RA=RAHF(O)
DO 11 K=1,101
IF(RA-PCUM(K))12,12,11
11 CONTINUE
K=101
12 CTHO=.01*(K-1)-.01*(PCUM(K)-RA)/(PCUM(K)-PCUM(K-1))
RETURN
END
FUNCTION PLINT(L,X)
C
C COMPUTE INTEGRALS OF LEGENDRE POLYNOMIALS TO ORDER 10.
C
Y=X*X
Z=Y*Y
GO TO (1,2,3,4,5,6,7,8,9,10),L
1 PLINT=X
RETURN
2 PLINT=0.5*Y
RETURN
3 PLINT=0.5*X*(Y-1.)
RETURN
4 PLINT=0.25*Y*(2.5*Y-3.)
RETURN

```

```

5 PLINT=0.125*X*(7.*Z-10.*Y+3.)
6 PL(N=0.0625*Y*(21.*Z-35.*Y+15.))
7 RETURN
8 PLINT=0.0625*X*(33.*Y+Z-63.*Z+35.*Y-5.)
9 RETURN
10 PLINT=0.0625*(429./8.*Z+Z-231./2.*Z+Y+315./4.*Z-35./2.*Y)
11 RETURN
12 PLINT=X/128.*(715.*Z+Z-1716.*Y+Z+1386.*Z-420.*Y+35.)
13 RETURN
14 PLINT=Y/128.*(2431./2.*Z+Z-6435./2.*Z+Y+3003.*Z-1155.*Y+315./2.)
15 RETURN
16 END
17 SUBROUTINE SCORE
18 COMMON/MC/STO(7,201),NCOLL,CTHD,CTH,STH,PHI,CPHI,SPHI,EGY,CPHIG,
19 1SPHIG,ED,EFAC,AMSDA,N,IFLAG,MAXCOLL,LCHR,STAB(200),ETAB(200),TLIM,
20 2XO,YO,ZO,X,Y,Z,ECUT,CLAB,THICK(11),NMAX,ZMIN,ZMAX,IE,INRAN,XIRA
21 3,NPRINT,TLAST,CTHC,NBATCH,ETA,PATH,S,SIGTAB(200),ENTAB(200),XIRC,
22 4PLI(10,101),WF,WMIN
23 70 LCHR=LCHR+1
24 STO(1,LCHR)=1.0
25 IF(NCOLL.EQ.0)STO(1,LCHR)=100.
26 STO(2,LCHR)=CTHD
27 STO(3,LCHR)=CTH
28 STO(4,LCHR)=STH
29 STO(5,LCHR)=PHI
30 STO(6,LCHR)=EGY
31 STO(7,LCHR)=WF
32 IF(LCHR.EQ.201)GO TO 95
33 WRITE(2)((STO(MM,LL),MM=1,7),LL=1,LCHR)
34 IF(NPRINT.EQ.2)PRINT 2,((STO(MM,LL),MM=1,7),LL=1,LCHR)
35 2 FORMAT(1X,7E16.9)
36 IF(LCHR.EQ.0)WRITE(2)((STO(K,L),K=1,7),L=1,201)
37 DO 1 MM=1,7
38 DO 1 LL=1,LCHR
39 1 STO(MM,LL)=0.
40 LCHR=0
41 95 IF(NCOLL.EQ.0)RETURN
42 100 CONTINUE
43 RETURN
44 END
45 SUBROUTINE BACKS(EGY,CTH,STH,PHI,CPHI,SPHI,WF)
46 SAMPLE ALBEDO DISTRIBUTIONS FROM "BIGSLAB".
47 DIMENSION AZERO(30,10,10),AP(9),PTEST(10),
48 1ETOP(5),AM(9),FL(9),EDUT(11,5),PCUM(101),FAC(10),
49 2LMAX(10,6,5),TOTALB(5,6),EPROB(5,6,11),YCOF(9,9),PLM(9,9),
50 3CPI(9,11),SPI(9,11),PPH(11),PL(10),PPHCUM(11),PLI(10,101),
51 DIMENSION APLUS(30,10,45),AMINUS(30,10,45)
52 REAL MU(6),RATIO(10)
53 DATA IEQ,ITIME/0,0/
54 DATA MU/1.,.9,.7,.5,.3,.1/
55 DATA ETOP/4.,3.,2.,1.,.05/
56 DATA SOFPI,PI/3.544907702,3.141592654/
57 DATA TMOP1/6.283185307/
58 IF(ITIME.EQ.1)GO TO 100
59 IWO=1
60 1 IEO=IEO+1
61 REWIND 3
62 ITIME=1
63 IF(IEQ.GT.1)IWO=IEO-1
64 DO 10 IWO=1,6
65 IO=6*(IWO-1)+IWO
66 DO 9 KE=1,10
67 READ(3)(AZERO(IO,KE,L),L=1,10)

```

```

      LM=L-1.9
      DO 10 L=1,9
        READ(3)(AP(M),M=1,L)
        READ(3)(AM(M),M=1,L)
        DO 7 M=1,L
          LM=LM+1
          APLUS(10,KE,LM)=AP(M)
          7 AMINUS(10,KE,LM)=AM(M)
        8 CONTINUE
        9 CONTINUE
        10 CONTINUE
        IF(IEO.LT.6)GO TO 1
        PPH(1)=0.
        DO 16 L=1,10
          DO 34 J=1,101
            CZ=.01*(J-1)
            34 PLI(L,J)=PLINT(L,CZ)
          AL=L
          16 FAC(L)=SORT(2.*AL-1.)
          DFHI=PI/10.
          DO 29 KP=2,11
            PPH(KP)=PPH(KP-1)+DPHI
          29 PPH(KP)=PPH(KP-1)+DPHI
          DO 30 L=1,9
            FL(L)=(2.*FLOAT(L)+1.)/6.283185307
            DO 31 M=1,L
              LM=L-M+1
              LM=L+M
              FI=1.
              DO 32 I=LM1,LM
                FI=FI+FLOAT(I)
              32 FI=FI+FLOAT(I)
              31 YCOF(L,M)=SQRT(FL(L)/FI)
              DO 33 KP=1,11
                CP(L,KP)=COS(L*PPH(KP))
                SP(L,KP)=SIN(L*PPH(KP))
              33 SP(L,KP)=SIN(L*PPH(KP))
            30 CONTINUE
            EPROB IS THE CUMULATIVE PROBABILITY OF EXIT WITH ENERGY E
            C
            C
            GIVEN EO,MUO
            DO 5 IEO=1,5
              EOUT(1,IEO)=ETOP(IEO)
              DE=ETOP(IEO)/10.
              DO 4 KE=2,11
                4 EOUT(KE,IEO)=EOUT(KE-1,IEO)-DE
              DO 3 IMO=1,6
                TOTALB(IEO,IMO)=0.
                IO=6*(IEO-1)+IMO
                DO 2 KE=1,10
                  2 TOTALB(IEO,IMO)=TOTALB(IEO,IMO)+SQFPI*AZERO(10,KE,1)
                  EPROB(IEO,IMO,1)=0.
                  DO 3 KE=2,11
                    EPROB(IEO,IMO,KE)=SQFPI*AZERO(10,KE-1,1)/TOTALB(IEO,IMO)
                  3 CONTINUE
                5 CONTINUE
              DO 11 IEO=1,5
                DO 11 IMO=1,6
                  DO 11 KE=2,11
                    11 EPROB(IEO,IMO,KE)=EPROB(IEO,IMO,KE)+EPROB(IEO,IMO,KE-1)
                  PRINT 60,IEO,IMO
                  IO=6*(IEO-1)+IMO
                  DO 40 KE=1,10
                    IF(AZERO(10,KE,1).EQ.0.0)GO TO 40
                    PC=0.
                    DO 41 L=1,10
                      AL=L
                      FC=SQRT(2.*AL-1.)/SQFPI+2.*PI

```

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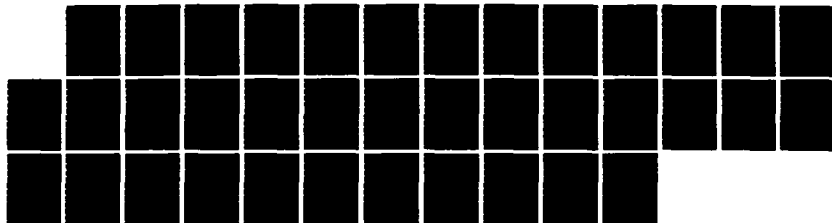
MATHEMATICAL MODELLING OF WAVEGUIDING TECHNIQUES AND  
ELECTRON TRANSPORT VOLUME 2(U) ARCON CORP WALTHAM MA  
S WOOLF ET AL. JAN 84 RADC-TR-83-313-VOL-2  
F19628-78-C-0188

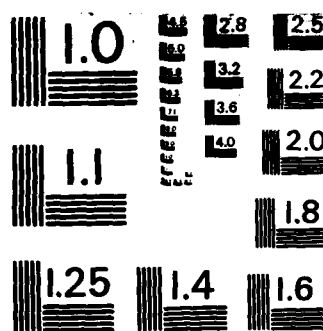
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NL





MICROCOPY RESOLUTION TEST CHART  
NATIONAL BUREAU OF STANDARDS-1963-A

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PQ 7-FC-AZERO(10,KE,L)*PLI(L,101)
41 P I(L)=PC
ATEST-SOPFI-AZERO(10,KE,1)
DIFF=1.E6
DO 42 L=1,10
RATIO(L)=(PTEST(L)/AZERO(10,KE,1))*2/(4.*PI)
DD=RATIO(L)-1.
IF(DD)/42,47,48
47 LOW=L
GO TO 49
48 LOW=L
AD=1.-RATIO(L-1)
IF(AD.LT.DD)LOW=L-1
GO TO 49
42 CONTINUE
LOW=10
49 IF(LOW.EQ.10)GO TO 491
IF(LOW.NE.(L-1))GO TO 491
IF(RATIO(LOW+1).EQ.RATIO(LOW))LOW=LOW+1
491 CONTINUE
LMAX(KE,IMO,IED)=LOW
40 CONTINUE
50 CONTINUE
DO 58 IMO=1,5
DO 58 IEO=1,6
PRINT 57,IED,IMO
58 FORMAT(1X,1015)
57 FORMAT(1X,1,IED=.,IS,1X,IMO=.,IS)
60 FORMAT(1X,1,IEO=.,IS,2X,IMO=.,IS)
C CLASSIFY INCIDENT ENERGY AND ANGLE
C FIND CLOSEST VALUES IN TABLE
100 DTEST=100.
DO 101 II=1,5
DE=ABS(EGY-ETOP(II))
IF(DE.GT.DTEST)GO TO 101
IED=II
DTEST=DE
101 CONTINUE
DTEST=100.
CT=ABS(CTH)
DO 102 II=1,6
DD=ABS(CT-MUO(II))
IF(DD.GT.DTEST)GO TO 102
IMO=II
DTEST=DD
102 CONTINUE
C WEIGHT FACTOR
WF=TOTALB(IEO,IMO)*WF
C COMPUTE NEW ENERGY
590 RA=RAHF(0)
DO 103 KE=2,11
IF(RA-EPROB(IEO,IMO,KE))/104,104,103
103 CONTINUE
KE=11
104 DELTA=(EOUT(KE,IED)-EOUT(KE-1,IED))/(EPROB(IEO,IMO,KE)-
1 ETST-EOUT(KE,IED)-EPROB(IEO,IMO,KE-1))
IF(ETST-GE.EGY)GO TO 590
EGY=ETST
C COMPUTE EXIT POLAR COSINE MU FROM LEGENDRE COEFFICIENTS
KEM=KE-1
LMAX=LMAX(KEM,IMO,IED)
DO 105 I=1,101
105 PCUM(I)=0.
DO 107 I=2,101

```

```

DO 106 LL=1,LWX
106 PC(I)=PCUM(I)*FAC(LL)*PLI(LL,I)*AZERO(IO,KEM,LL)
107 CONTINUE
DO 108 I=1,101
IF(PCUM(I).LT.0.)PCUM(I)=0.
PCUM(I)=PCUM(I)/PCUM(101)
108 IF(PCUM(I).GT.1.0)PCUM(I)=1.0
RA=RA*F(0)
DO 109 I=1,101
IF(RA-PCUM(I))110,110,109
109 CONTINUE
110 SI=1.
IF(CTH.GT.0.)SI=-1.
CTH=SI*(1.01*(I-1)-.01*(PCUM(I)-RA)/(PCUM(I)-PCUM(I-1)))
DETERMINE EXIT AZIMUTH
STH=SQRT(1.-CTH*CTH)
AC=ABS(CTH)
CALL LEP(PL,AC,9)
CALL ASSOC(PL,AC,STH,PLM)
254 SUM1=0.
DO 201 L=1,LWX
201 SUM1=SUM1+FAC(L)/SOFPI*AZERO(IO,KEM,L)*PL(L)
LWX1=LWX-1
SUM2=0.
DO 202 KP=1,11
A1=PPH(KP)*SUM1
SUM3=0.
DO 203 L=1,LWX1
SUM2=0.
DO 204 M=1,L
LM=(L*(L+1))/2-(L-M)
EN=M
204 SUM2=SUM2+YCOF(L,M)/EM*PLM(L,M)*
1(APLUS(IO,KEM,LM)*SP(M,KP)-AMINUS(IO,KEM,LM)*(CP(M,KP)-1.))
SUM3=SUM3+SUM2
203 CONTINUE
PPHCUM(KP)=A1+SUM3
202 CONTINUE
DO 255 KP=2,11
IF(PPHCUM(KP).LT.PPHCUM(KP-1))GO TO 256
255 CONTINUE
GO TO 253
256 LMX=LWX-1
IF(LMX.EQ.1)GO TO 257
GO TO 254
257 DO 258 KP=2,11
258 PPHCUM(KP)=PPHCUM(KP-1)+.1
253 DO 205 KP=1,11
205 PPHCUM(KP)=PPHCUM(KP)/PPHCUM(11)
RA=RA*F(0)
DO 206 KP=1,11
IF(RA-PPHCUM(KP))208,208,206
206 CONTINUE
KP=11
208 PPH=PPH(KP)-DPHI*(PPHCUM(KP)-RA)/(PPHCUM(KP)-PPHCUM(KP-1))
RA=RA*F(0)
IF(RA.GE.0.5)PHP=THOP1-PPH
PHI=PHI+PPH
IF(PHI.GT.TWOPI)PHI=PHI-TWOPI
CPHI=COS(PHI)
SPHI=SIN(PHI)
RETURN
END
OVERLAY(COMM,2.0)
PROGRAM PROX

```

```

CALL PROC
END
SUB JTINE PROC
COMMON/NC/STOI(7,201),NCOLL,CTHO,CTH,STH,PHI,CPHI,SPHI,EGY,C,IO,
1SPHIO,EO,SPAC,AMBDA,N,IPLAG,MAXCOLL,LCHRR,STAB(200),ETAB(200),TLM,
2XO,YO,ZO,X,Y,Z,ECUT,CLAB,THICK(11),NMAX,ZMIN,ZMAX,IE,INRAN,XIRA,
3,NPRINT,TLAST,CTHC,NBATCH,ETA,PATH,S,SIGTAB(200),ENTAB(200),XIRC,
4PLI(10,101),WF,WMIN
DIMENSION TRANC(10,10),TRANF(10,10),BACKC(10,10),BACKF(10,10),
1APLUS(9,9,10),AMINUS(9,9,10),TPLUS(9,9,10),TMINUS(9,9,10),
2ALEG(10,10),TLEG(10,10),ENERGY(10),YCOF(9,9),ANGLE(10),PL(10),
3PLN(9,9),CP(9),SP(9),SUM(10),TEMP(10),ISIG(10),ITP(9,9,10),
4ITM(9,9,10),ISGEX(10,10),IPR(10)
DO 1 L=1,9
FL=(2.-FLOAT(L)+1.)/6.283185307
DO 2 M=1,L
LM1=L-M+1
LM=L-M
FI=1.
DO 3 I=LMI,LM
3 FI=FI-FLOAT(I)
2 YCOF(L,M)=SQRT(FL/FI)
1 CONTINUE
REWIND 2
NN=NMAX/10
FN=FLOAT(NNN)
EO=0.-4
DE=EO/10.
ENERGY(1)=EO-DE
ANGLE(1)=.1
DO 4 K=2,10
ANGLE(K)=ANGLE(K-1)+.1
4 ENERGY(K)=ENERGY(K-1)-DE
IPLANE=1
100 REWIND 2
IB=NH=NHIST=0
IPLANE=IPLANE+1
PRINT 777,THICK(IPLANE)
777 FORMAT('1=,40X,0DETECTOR PLANE AT=,E12.5)
DO 40 I=1,10
IPR(I)=0
DO 41 J=1,10
TRANC(I,J)=TRANF(I,J)=BACKC(I,J)=BACKF(I,J)=0.
41 ALEG(I,J)=TLEG(I,J)=0.
DO 40 J=1,9
DO 40 K=1,9
40 APLUS(J,K,I)=AMINUS(J,K,I)=TPLUS(J,K,I)=TMINUS(J,K,I)=0.
5000 READ(2)((STO(K,L),K=1,7),L=1,201)
DO 90 L=1,201
IF(STO(L,L).EQ.0.)GO TO 11
IF(STO(L,L).LT.10.)GO TO 10
NH=NH+1
11 NHIST=NHIST+1
GO TO 50
10 W=STOI(7,L)
CLASSIFY EMERGENT ANGLE
CA=ABS(STOI(3,L))
DO 80 J=1,10
80 IF(CA.LT.ANGLE(J))GO TO 81
J=10
81 JC=J
C CLASSIFY EMERGENT ENERGY
DO 82 K=1,10
82 IF(STO(6,L).GT.ENERGY(K))GO TO 83
K=10
83 KC=K

```

```

C
1PR(KC)=1
C/ POSITIVE OR NEGATIVE?
C/ -STD(3,1)
STH=I=STD(4,1)
PHI=STD(5,1)
CALL LEP(PL,CTHET,9)
DO 70 M=1,9
CP(M)=COS(M*PHI)
CALL ASSOC(PL,CTHET,STHET,PLM)
IF(CTHET)84,85,85
84 BACKC(JC,KC)=BACKC(JC,KC)*W/FN
BACKF(JC,KC)=BACKF(JC,KC)-W/CTHET/FN
DO 86 LL=1,10
86 ALEG(LL,KC)=ALEG(LL,KC)+PL(LL)*W/FN
DO 87 LL=1,9
DO 88 M=1,LL
FM=YCOF(LL,M)*PLM(LL,M)/FN
APLUS(LL,M,KC)=APLUS(LL,M,KC)+FM*CP(M)*W
AMINUS(LL,M,KC)=AMINUS(LL,M,KC)+FM*SP(M)*W
87 CONTINUE
GO TO 90
85 TRANC(JC,KC)=TRANC(JC,KC)*W/FN
TRANF(JC,KC)=TRANF(JC,KC)+W/FN/CTHET
DO 89 LL=1,10
89 TLEG(LL,KC)=TLEG(LL,KC)+PL(LL)*W/FN
DO 91 M=1,LL
FM=YCOF(LL,M)*PLM(LL,M)/FN
TPLUS(LL,M,KC)=TPLUS(LL,M,KC)+FM*CP(M)*W
TMINUS(LL,M,KC)=TMINUS(LL,M,KC)+FM*SP(M)*W
91 CONTINUE
GO TO 90
50 IF(NHIST.LE.NMN)GO TO 90
NHIST=1
KPUT=1
18=18+1
DO 150 NM=1,4
DO 164 I=1,10
164 SUM(I)=0.
IF(18.LT.10)GO TO 154
PRINT 220
GO TO (114,124,134,144),NM
114 PRINT 230
GO TO 153
124 PRINT 231
GO TO 153
134 PRINT 232
GO TO 153
144 PRINT 233
153 PRINT 234,(ANGLE(I),I=1,10)
230 FORMAT(///45X,*,POSITIVE CURRENT*/)
231 FORMAT(///45X,*,NEGATIVE CURRENT*/)
232 FORMAT(///45X,*,POSITIVE FLUX*/)
233 FORMAT(///45X,*,NEGATIVE FLUX*/)
234 FORMAT(45X,*,EMERGENT POLAR COSINE*/9X,10E12.5/3X,*,E(MEV)*)
154 DO 104 N=1,10
DO 105 I=1,10
GO TO (115,125,135,145),NM
115 TEMP(I)=TRANC(I,K)
TRANC(I,K)=0.
GO TO 105
125 TEMP(I)=BACKC(I,K)
BACKC(I,K)=0.
GO TO 105
135 TEMP(I)=TRANF(I,K)

```





```

1 (PR(K).EQ.0)GO TO 626
PR..T 526,ENERGY(M),(ALEG(LL,K),LL=1,10)
628 CONTINUE
526 FORMAT(3X,F7.3,1X,10E12.5)
PRINT 651
DO 525 K=1,10
IF(IPR(K).EQ.0)GO TO 525
PRINT 524,ENERGY(K),(ISGE(LL,K),LL=1,10)
525 CONTINUE
524 FORMAT(3X,F7.3,1X,10I4)
DO 345 K=1,10
IF(IPR(K).EQ.0)GO TO 345
3461 PRINT 347,ENERGY(K)
347 FORMAT(/1X,ENERGY = ,F10.5,SPHERICAL HARMONICS COEFF
1CIENTS - APLUS/)
DO 343 LL=1,9
343 PRINT 349,LL,(APLUS(LL,M,KE),M=1,LL)
349 FORMAT(1X,L=,15,1X,9E12.5)
PRINT 651
651 FORMAT(1X,PERCENT ERRORS IN ABOVE ITEMS=)
DO 652 LL=1,9
652 PRINT 653,LL,(ITP(LL,M,KE),M=1,LL)
PRINT 348
348 FORMAT(1X,SPHERICAL HARMONICS COEFFICIENTS - AMINUS=)
DO 342 LL=1,9
342 PRINT 349,LL,(AMINUS(LL,M,KE),M=1,LL)
PRINT 651
DO 654 LL=1,9
654 PRINT 653,LL,(ITM(LL,M,KE),M=1,LL)
345 CONTINUE
653 FORMAT(1X,L=,15,1X,9I4)
GO TO 200
90 CONTINUE
GO TO 5000
200 RETURN
END
SUBROUTINE LEP(Y,X,N)
DIMENSION Y(1)
Y(1)=1.
IF(N)1,1.2
1 RETURN
2 Y(2)=X
IF(N-1)1,1.3
3 DO 4 I=2,N
G=X*Y(I)
4 Y(I+1)=G-Y(I-1)+G-(G-Y(I-1))/FLOAT(I)
RETURN
END
SUBROUTINE STATS(AVAL,ISIG,N,K,IB)
DIMENSION XVAL(1),ISIG(1),AVE(10),SIG(10)
DIMENSION ECAVE(4200),ECSIG(4200)
IF(18.NE.1) GO TO 2
DO 1 I=1,N
AVE(I)=XVAL(I)
ISIG(I)=99
1 SIG(I)=0.0
10 DO 100 I=1,N
IKK=K+I-1
ECAVE(IKK)=AVE(I)
100 ECSIG(IKK)=SIG(I)
K=K+N
RETURN
2 XNSORS=IB
DO 200 I=1,N
IKK=K+I-1
AVE(I)=ECAVE(IKK)

```

```

200 SIG(I)=ECSIG(IKK)
DO 1 I=1,N
  SIG(I)=(XNSORS-2.0)/(XNSORS-1.0)*SIG(I)+(XVAL(I)-AVE(I))*2.0/XNSORS
  AVE(I) = ((XNSORS-1.0)*AVE(I)+XVAL(I))/XNSORS
  XVAL(I) = AVE(I)
  ERROR=SQRT(SIG(I)/XNSORS)
  ISIG(I)=99
  IF(AVE(I).EQ.0.0) GO TO 3
  ISIG(I)=100.0*ERROR/ABS(AVE(I))+.50001
  IF(ISIG(I).GT.99) ISIG(I)=99
3 CONTINUE
GO TO 10
END
SUBROUTINE ASSOC(PL,CX, SX,P)
  DIMENSION PL(10),P(9,9)
  IF(SX.GT.0.0)GO TO 1
  DO 2 L=1,9
    DO 2 M=1,9
      2 P(L,M)=0.0
  RETURN
  1 P(1,1)=SX
  P(2,1)=3.*CX*SX
  P(2,2)=2.*CX/SX*P(2,1)-6.*PL(3)
  M=1
  LOW=M+1
  DO 10 L=LOW,8
    10 P(L+1,M)=((2*L+1)*CX*P(L,M)-(L+M)*P(L-1,M))/FLOAT(L-M+1)
    P(3,2)=2.*CX/SX*P(3,1)-12.*PL(4)
    DO 100 M=2,8
      LOW=M+1
      IF(LOW.EQ.9)GO TO 51
      DO 50 L=LOW,8
        50 P(L+1,M)=((2*L+1)*CX*P(L,M)-(L+M)*P(L-1,M))/FLOAT(L-M+1)
        51 P(LOW,M+1)=(2*M)*P(LOW,M)*CX/SX-((LOW+M)*P(LOW,M+1)+P(LOW,M-1)
          IF(M.EQ.8)GO TO 100
          P(LOW+1,M+1)=(2*M)*P(LOW+1,M)*CX/SX-((LOW+M+1)*P(LOW+M+1)+P(LOW+M+2))
          1P(LOW+1,M-1)
        100 CONTINUE
      RETURN
    END
  END

```

Program Listing

- MCSPLIT -

(Reference: Vol. I, Section IV.B.4)

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PROGRAM MCSPLIT(INPUT,OUTPUT,TAPE2)

THE PURPOSE OF THIS PROGRAM IS TO SERVE AS A TEST OF THE
APPLICATION OF THE DISTRIBUTIONS COMPUTED BY "BIGSLAB" TO THE
INTERFACE CODE "INTFC". THE INTENTION IS TO USE THIS CODE
"MCSPILT" AS A BENCHMARK PROGRAM. IT MAKES USE OF THE STATISTICAL
BIASING TECHNIQUE KNOWN AS HISTORY SPLITTING(SEE REF.3, VOL. 1,
PAGE 264). IT IS A TECHNIQUE COMMONLY USED IN DEEP PENETRATION
MONTE CARLO CALCULATIONS. "MCSPILT" IS A STRAIGHTFORWARD SINGLE
SCATTERING ANALOG MONTE CARLO PROGRAM FOR CALCULATING ELECTRON
TRANSPORT IN A VERY THICK MEDIUM (GOLD-ALUMINUM) WITH THE MATERIAL
INTERFACE DEEPLY IMBEDDED. THE ELECTRON BEAM IS ASSUMED NORMALLY
INCIDENT ON THE LEFT SURFACE OF THE SLAB. WHEN AN ELECTRON
CROSSES A BOUNDARY(IMAGINARY) KNOWN AS A "SPLITTING PLANE" IT
SPLITS INTO N IDENTICAL PARTICLES EACH WITH STATISTICAL WEIGHT
GIVEN BY 1/N OF THE ORIGINAL VALUE. THE MONTE CARLO PROCEDURE IS
THEN CARRIED OUT INDEPENDENTLY ON EACH OF THESE FRACTIONAL PARTI-
CLES. A LIST OF THE QUANTITIES CALCULATED BY THIS PROGRAM IS GIVEN
IN VOL. 1, PAGES 259-260. EXCEPT FOR THE HISTORY SPLITTING
FEATURE, THIS PROGRAM IS STRUCTURED IN A MANNER SIMILAR TO THAT
OF "BIGSLAB".

COMMON/MC/STO(7,201),NCOLL,CTHO,CTH,STH,PHI,CPHI,EGY,CPHIO,
15PHIO,EO,SFAC,AMBDA,N,IFLAG,MAXCOLL,LCHR,STAB(200),ETAB(200),TLM,
2XD,YD,ZD,X,Y,Z,ECUT,CLAB,THICK(11),NMAX,ZMIN,ZMAX,IE,INRAN,XIRA
3,NPRINT,TLAST,CTHC,NBATCH,ETA,PATH,S,SIGTAB(200),ENTAB(200),XIRC,
421,WF,NCOUNT(2),ICROSS(2),PCROSS(2),NTRUE
DIMENSION TJMR(11)
CALL SETRUN
NOTDONE=0
PREFLOAT(NMAX)
COLLNO=0.
MTM=1
CALL SECOND(TIME)
TMR(1)=TIME
ICROSS(1)=ICROSS(2)=0
5 N=N+1
IF(ICROSS(1).EQ.0.AND.ICROSS(2).EQ.0)CALL SETHIS
IF(ICROSS(1).EQ.1.AND.ICROSS(2).EQ.0)CALL SPLIT(1)
IF(ICROSS(2).EQ.1)CALL SPLIT(2)
IF(IFLAG.EQ.1)GO TO 5
20 CALL PENET
CALL ENERGY
CALL SCORE
IF(IFLAG.GT.0) GO TO 6
IF(NCOUNT(1).GT.0)GO TO 400
IF(Z.GE.PCROSS(1).AND.ICROSS(1).EQ.0)CALL SPLIT(1)
400 IF(NCOUNT(2).GT.0)GO TO 401
IF(Z1.GT.PCROSS(2).AND.Z.LT.PCROSS(2).AND.ICROSS(2).EQ.0)
1CALL SPLIT(2)
401 CONTINUE
IF(IFLAG.GT.0) GO TO 6
IF(NCOUNT(1).EQ.0.AND.ICROSS(1).EQ.1)GO TO 402
IF(NCOUNT(2).EQ.0.AND.ICROSS(2).EQ.1)GO TO 402
402 IF(IFLAG.LT.0)GO TO 5
CALL ANGLES
GO TO 20
6 COLLNO=COLLNO+FLOAT(NCOLL)/FN
IF(NBATCH.EQ.0)GO TO 60
IF(ICROSS(1).EQ.0)GO TO 47
IF(ICROSS(2).EQ.0.AND.NCOUNT(1).EQ.10)GO TO 47
IF(ICROSS(1).EQ.1.AND.NCOUNT(1).EQ.10.AND.NCOUNT(2).EQ.10)GO TO 47
NOTDONE=1

```

```

G      7 60
47 C    INUE
      NOTDONE=0
      NTEST=NTRUE-NBATCH*(NTRUE/NBATCH)
      IF (NTEST.NE.0) GO TO 60
      MTM=MTM+1
      CALL SECOND(TIME)
      TIMR(TIM)=TIME
      DELT=TIME-TIMR(1)
      AVGT=DELT/FLOAT(MTM-1)
      TREMAIN=TLIM-DELT
      TLEFT=1.5*AVGT
      IPCT=10*(MTM-1)
      PRINT 75,IPCT,TREMAIN
75 FORMAT(1X,'THE PROBLEM IS',14,' PERCENT COMPLETE',1X,
1,TIME TO FINISH IS',F12.3,'SECONDS')
      IF (IPCT.EQ.100) GO TO 100
      IF (TREMAIN.GE.TLEFT) GO TO 5
      GO TO 100
60 IF (NTRUE-NMAX) IS .55.55
55 IF (NOTDONE.E2.1) GO TO 5
2 FORMAT(1X,7E16.9)
100 PRINT 64,COLLND
      IF (LCMR.GT.0.AND.LCMR.LE.201) WRITE(2)((STO(K,L),K=1,7),L=1,201)
      IF (LCMR.GT.0.AND.LCMR.LE.201.AND.NPRINT.EQ.2) PRINT 2,
1((STO(K,L),K=1,7),L=1,LCMR)
      IF (LCMR.EQ.0) WRITE(2)((STO(K,L),K=1,7),L=1,201)
      CALL RANGET(XIRC)
64 FORMAT(//1X,'AVERAGE NUMBER OF COLLISIONS PER HISTORY =',F7.3)
      IF (NBATCH.EQ.0) GO TO 200
      MTM=MTM-1
      PRINT 65,MTM,NBATCH
65 FORMAT(//1X,'THERE ARE',14,' BATCHES OF',15,' HISTORIES')
      AVGT=0.
      SUM=TIMR(1)
      DO 66 M=2,MTM
      TIMR(M)=TIMR(M)-SUM
      SUM=SUM+TIMR(M)
      AVGT=AVGT+1/MR(M)
      MM=M-1
66 PRINT 67,MM,TIMR(M)
87 FORMAT(1X,'BATCH NO.',14,' TIME =',F6.3,' SECONDS')
      AVGT=AVGT/FLOAT(MTM1)
      PRINT 68,AVGT
68 FORMAT(//1X,'AVERAGE TIME PER BATCH =',F6.3,' SECONDS')
200 PRINT 190
190 FORMAT(1,6X,'INRAN',11X,'IRA',12X,'IRC')
      PRINT 195,INRAN,XIRA,XIRC
195 FORMAT(31X,O16)
      CALL SECOND(T1)
      CALL PROC
      CALL SECOND(T2)
      T3=T2-T1
      PRINT 196,T3
196 FORMAT(1X,'TIME REQUIRED FOR PROCESSING HISTORIES =',F10.3,'SEC')
      STOP
      END
      SUBROUTINE SETRUN
      COMMON/MC/STO(17,201),NCOLL,CTHQ,CTH,STM,PHI,CPHI,SPHI,EGY,CPHIO,
1SPHIO,EC,SFAC,AMBDA,N,IFLAG,MAXCOLL,LCMR,STAB(200),ETAB(200),TLIM,
2HO,YO,ZO,X,Y,Z,ECUT,CLAB,THICK(11),AMAX,ZMIN,ZMAX,IE,INRAN,XIRA,
3NPRINT,TLAST,CTHC,NBATCH,ETA,PATH,S,SIGTAB(200),ENAB(200),XIRC,
4Z1,WF,NCOUNT(2),ICROSS(2),PCROSS(2),NTRUE
      DIMENSION INCODE(3)
      DATA CPHIO,SPHIO,PI,ZMIN,ZMAX,N/1.0,0.0,3.14159265,0.0,1000.0,0/
      DATA ECUT,SFAC,XO,YO,ZO/O.,.800.,.3*O./

```

```

DATA THICK/11-3.
C 1. NMAX, MAXCOLL, NPRINT, INPT, NSLABS
C 2. TOTAL NO. OF MONTE CARLO HISTORIES
C 3. STORED ON TAPE OR DISK UNTIL PROCESSING TIME
C 4. MAXCOLL - MAXIMUM NUMBER OF COLLISIONS PER HISTORY TO BE ALLOWED
C 5. NPRINT - DEBUG PRINT CONTROL PARAMETER
C 6. NPRINT EQUAL TO ZERO SUPPRESSES DEBUG PRINTOUT
C 7. NPRINT NOT EQUAL TO ZERO ACTIVATES DEBUG PRINTOUT

READ 3, TLIM, CTHO, EO, ECUT
IF (NSLABS.EQ.0) GO TO 50
READ 3, (THICK(I), I=1,7)
DO 51 I=1,7
51 THICK(I)=THICK(I)*1.E-5
PCROSS(1)=THICK(2)
4 IF (INPT.GT.0) GO TO 40
INRAN=0
GO TO 45
40 READ 1, (INCODE(I), I=1,3)
IF (INCODE(1).EQ.1) READ 8, INRAN
IF (INCODE(2).EQ.1) READ 3, SFAC
IF (INCODE(3).EQ.1) READ 3, XO, YO, ZO
EO - INITIAL PARTICLE ENERGY
XO, YO, ZO - INITIAL PARTICLE COORDINATES
SFAC - STOPPING POWER FUDGE FACTOR
EFAC - ETA FUDGE FACTOR
45 PRINT 4, NMAX, MAXCOLL
PRINT 9, EO, ECUT, XO, YO, ZO, CTHO, CPHO, SPHO
PRINT 11, SFAC
PRINT 33, (THICK(I), I=1,11)
33 FORMAT(1X, 'SLAB BOUNDARIES',/(1X, 10E12.5))
11 FORMAT(1X, 'STOPPING POWER FUDGE FACTOR (SFAC) =', F10.5)
9 FORMAT(1X, 'EO (INITIAL ENERGY MEV)', E12.5//1X, 'ECUT (LOW ENERGY
1CUT OFF MEV) ', E12.5//1X, 'XO, YO, ZO (SOURCE POINT COORDINATES)
3 ' -3E12.5//1X, 'CTHO (INCIDENT POLAR COSINE) ', E12.5//1X, 'CPHO
4, 'SPHO (INCIDENT AZIMUTH) ', 2E12.5)
8 FORMAT(018)
1 FORMAT(610)
3 FORMAT(8F10.0)
4 FORMAT(1X, 'NMAX (NO. OF HISTORIES) ', I6//1X, 'MAXCOLL (MAXIMUM
1 ALLOWED NUMBER OF COLLISIONS) ', I3)
10 N=0
NTRUE=0
LCHR=0
XIRA=XIRC=INRAN
NBATCH=NMAX/10
CALL RANSET(XIRA)
CALL ESET
REWIND 2
DO 20 K=1,7
DO 20 L=1,201
20 STO(K,L)=0.0
RETURN
END
SUBROUTINE SETHIS
COMMON/MC/STO(7,201), NCOLL, CTHO, CTH, STH, PHI, CPHI, SPHI, EGY, CPHIO,
1SPHO, EO, SFAC, AMEDA, N, IFLAG, MAXCOLL, LCHR, STAB(200), ETAB(200), TLIM,
2XO, YO, ZO, X, Y, Z, ECUT, CLAB, THICK(11), NMAX, ZMIN, ZMAX, IE, INRAN, XIRA
3,NPRINT, TLAST, CTHC, NBATCH, ETA, PATH, S, SIGTAB(200), ENTAB(200), XIRC,
4Z1, WF, NCOUNT(2), ICROSS(2), PCROSS(2), NTRUE
NCOUNT(1)=NCOUNT(2)+1
WF=1.
ICROSS(1)=ICROSS(2)+0
NCOLL=0
PATH=0.
S=0.

```

```

X=Y
Y=X
Z=Z0
CPHI=CPHIO
SPHI=SPHIO
PHI=0.0
CTH=CTHO
STH=STH0(1.-CTH**2)
EGV=EO
IE=1
NTRUE=NTRUE+1
CALL CROSS
IFLAG=0
CALL SCORE
RETURN
END
SUBROUTINE PENET
COMMON/MC/STOI(7,201),NCOLL,CTHO,CTH,STH,PHI,CPHI,SPHI,EGV,CPHIO,
1SPHIO,EO,SFAC,AMBDA,N,IFLAG,MAXCOLL,LCHR,STAB(200),ETAB(200),TLIM,
2X0,Y0,Z0,X,Y,Z,ECUT,CLAB,THICK(11),NMAX,ZMIN,ZMAX,IE,INRAN,XIRA,
3,NPRINT,TLAST,CTHC,NBATCH,ETA,PATH,S,SIGTAB(200),ENTAB(200),XIRC,
4Z1,WF,NCOUNT(2),ICROSS(2),PCROSS(2),NTRUE
X1=X
Y1=Y
Z1=Z
4 RB=RAMF(0)
IF(RB.GT.0.)GO TO 5
GO TO 4
5 S=-ALOG(RB)*AMBDA
Z=Z1+S*CTH
IF(Z.GT.ZMIN) GO TO 10
Z=ZMIN
S=(ZMIN-Z1)/CTH
IFLAG=1
10 X=X1+S*STH*CPHI
Y=Y1+S*STH*SPHI
PATH=PATH+S
NCOLL=NCOLL+1
IF(NCOLL.EQ.MAXCOLL) IFLAG=1
IF(Z.GT.ZMAX)IFLAG=1
RETURN
END
SUBROUTINE ESET
COMMON/MC/STOI(7,201),NCOLL,CTHO,CTH,STH,PHI,CPHI,SPHI,EGV,CPHIO,
1SPHIO,EO,SFAC,AMBDA,N,IFLAG,MAXCOLL,LCHR,STAB(200),ETAB(200),TLIM,
2X0,Y0,Z0,X,Y,Z,ECUT,CLAB,THICK(11),NMAX,ZMIN,ZMAX,IE,INRAN,XIRA,
3,NPRINT,TLAST,CTHC,NBATCH,ETA,PATH,S,SIGTAB(200),ENTAB(200),XIRC,
4Z1,WF,NCOUNT(2),ICROSS(2),PCROSS(2),NTRUE
DATA PI,XL2/3.14159265,693147181/
DATA ZA,AA,AVOG,RO,DNSTY,EACIT,FCC/13.,26.98,6.025E23,2.8179E-13,
12.7,163.E-6,.15360628/
ENTAB(1)=1.0
DE=.01
DO 1 II=2,91
1 ENTAB(II)=ENTAB(II-1)-DE
DE=.001
DO 2 II=92,181
2 ENTAB(II)=ENTAB(II-1)-DE
DO 3 III=1,161
E=ENTAB(II)/.511
E1=(1.+E)*(1.+E)
E2=E*(2.+E)
BETA=SQRT(E2/E1)
XLG=ALOG(E*E2/(2.*(EXCIT/.511)**2))
FM=1.-BETA*BETA*(.125+E*(2.*E+1.)*XL2)/E1
STAB(1)=(FCC*ZA/AA/BETA**2)*(XLG+FM)*SFAC

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ETAB(11)=0.5*(ZA**((1./3.)/(121.245)**2/E2*(1.13+3.76*(ZA/137.))**2.
1E1))
SI AB(11)=ZA*(1.+ZA)*AVOG*RO*RO*ZA/AA*E1/(E2+E2)/(ETAB(11),
1*(1.+5*ETAB(11)))
ETAB(11)=.035276
3 IF(11.GT.91)ETAB(11)=.103378
IF(NPRT.EQ.0)RETURN
PRINT 75
75 FORMAT(1X,'CROSS SECTION AND STOPPING POWER TABLES',//,1X,'ENERGY(M
1EV) STOPPING POWER ETA SIGMA*//)
DO 76 11=1,181
76 PRINT 77,ETAB(11),STAB(11),ETAB(11),SIGTAB(11)
77 FORMAT(1X,F10.6,3E16.9)
RETURN
END
SUBROUTINE ANGLES
COMMON/MC/STO(7,201),NCOLL,CTHO,CTH,STM,PHI,CPHI,SPHI,EGY,CPHIO,
1SPHIO,ED,SPAC,AMSDA,N,IFLAG,MAXCOLL,LCHR,STAB(200),ETAB(200),TLIM,
2XO,YO,ZO,X,Y,Z,ECUT,CLAB,THICK(11),NMAX,ZMIN,ZMAX,IE,INRAN,XIRA
3,NPRT,TLAST,CTHC,NBATCH,ETA,PATN,S,SIGTAB(200),ENTAB(200),XIRC,
4Z1,WF,NCOUNT(2),ICROSS(2),PCROSS(2),NTRUE
DATA TMOPI/6.283185307/
RD=2.*RANF(0)
COM=RD*(1.+ETA)-ETA)/(RD+ETA)
SOM=SQRT(1.-COM**2)
RC=RANF(0)
RHO=TMOPI*RC
CRHO=COS(RHO)
SRHO=SIN(RHO)
STH=STM
CTH=CTH
CTH=CTH*COM+STM*STM+SOM*CRHO
STM=SQRT(1.-CTH**2)
IF(STM.EQ.0.0.OR.STM.EQ.0.0) GO TO 1
C1=(COM-CTH*CTH)/(STM*STM)
S1=SRHO*SOM/STM
CPHI=CPHI
SPHI=SPHI
CPHI=CPHI+C1-SPHI*S1
SPHI=CPHI*S1+SPHI*C1
GO TO 2
1 CPHI=CRHO
SPHI=SRHO
2 CONTINUE
PHI=ATAN2(SPHI,CPHI)
IF(PHI.LT.0.0)PHI=PHI+TMOPI
RETURN
END
SUBROUTINE SCORE
COMMON/MC/STO(7,201),NCOLL,CTHO,CTH,STM,PHI,CPHI,SPHI,EGY,CPHIO,
1SPHIO,ED,SPAC,AMSDA,N,IFLAG,MAXCOLL,LCHR,STAB(200),ETAB(200),TLIM,
2XO,YO,ZO,X,Y,Z,ECUT,CLAB,THICK(11),NMAX,ZMIN,ZMAX,IE,INRAN,XIRA
3,NPRT,TLAST,CTHC,NBATCH,ETA,PATN,S,SIGTAB(200),ENTAB(200),XIRC,
4Z1,WF,NCOUNT(2),ICROSS(2),PCROSS(2),NTRUE
DO 100 1=3,5
IF(NCOLL.EQ.0)GO TO 70
96 IF(CTH)97,97,99
99 IF(21.LT.THICK(1).AND.Z.GT.THICK(1))GO TO 70
GO TO 100
97 IF(2.LE.THICK(1).AND.Z1.GT.THICK(1))GO TO 70
GO TO 100
70 LCHR=LCHR+1
STO(1,LCHR)=FLOAT(1)
IF(NCOLL.EQ.0)STO(1,LCHR)=100.
STO(2,LCHR)=CTHO
STO(3,LCHR)=CTH

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      STO(4,LCHR)=STM
      5( 3,LCHR)=PHI
      ST(6,LCHR)=EGY
      STO(7,LCHR)=WF
      IF(LCHR.LT.201)GO TO 95
      WRITE(2)((STO(MM,LL),MM=1,7),LL=1,LCHR)
      IF(INPRT.EQ.2)PRINT 2,((STO(MM,LL),MM=1,7),LL=1,LCHR)
2   FORMAT(1X,7E16.9)
      IF(LCHR.EQ.0)WRITE(2)((STO(K,L),K=1,7),L=1,201)
      DO 1 MM=1,7
      DO 1 LL=1,LCHR
1   STO(MM,LL)=0.
      LCHR=0
95  IF(NCOLL.EQ.0)RETURN
100 CONTINUE
      RETURN
      END
      SUBROUTINE LEP(Y,X,N)
      DIMENSION Y(1)
      Y(1)=1.
      IF(N)1,1,2
1   RETURN
2   Y(2)=X
      IF(N-1)1,1,3
3   DO 4 I=2,N
      G=X*Y(I)
4   Y(I+1)=G-Y(I-1)+G-(G-Y(I-1))/FLOAT(I)
      RETURN
      END
      SUBROUTINE STATS(XVAL,ISIG,N,K,IB)
      DIMENSION XVAL(1),ISIG(1),AVE(10),SIG(10)
      DIMENSION ECAVE(4200),ECSIG(4200)
      IF(18.NE.1) GO TO 2
      DO 1 I=1,N
      AVE(I)=XVAL(I)
      ISIG(I)=99
1   SIG(I)=0.0
10  DO 100 I=1,N
      INK=I-1
      ECAVE(INK)=AVE(I)
100  ECSIG(INK)=SIG(I)
      K=K+N
      RETURN
2   XNSORS=18
      DO 200 I=1,N
      INK=I-1
      AVE(I)=ECAVE(INK)
      SIG(I)=ECSIG(INK)
200  SIG(I)=ECSIG(INK)
      DO 3 I=1,N
      SIG(I)=(XNSORS-2.0)/(XNSORS-1.0)*SIG(I)+(XVAL(I)-AVE(I))*2/XNSORS
      AVE(I)=((XNSORS-1.0)*AVE(I)+XVAL(I))/XNSORS
      XVAL(I)=AVE(I)
      ERROR=SQRT(SIG(I)/XNSORS)
      ISIG(I)=99
      IF(AVE(I).EQ.0.0) GO TO 3
      ISIG(I)=100.0*ERROR/ABS(AVE(I))+.50001
      IF(1SIG(I).GT.99) ISIG(I)=99
3   CONTINUE
      GO TO 10
      END
      SUBROUTINE ENERGY
      COMMON/MC/STO(7,201),NCOLL,CTHO,CTH,STH,PHI,CPHI,SPHI,EGY,CPHIO,
      1SPHIQ,EO,SFAC,AMBOA,N,IFLAG,MAXCOLL,LCHR,STAB(200),ETAB(200),TLIM,
      2XD,YO,ZO,X,Y,Z,ECUT,CLAB,THICK(11),MMAX,ZMIN,ZMAX,IE,INRAN,XIPA,
      3,NPRT,TLAST,CTMC,NBATCH,ETA,PATH,S,SIGTAB(200),ENTAB(200),XIRC,
      4Z1,WF,NCOUNT(2),ICROSS(2),PCROSS(2),NTRUE

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1 IF EGY=1)1,2,2
2 IF >FIX(100.*(1.-EGY))
3 GO 0,3
1 IF EGY.LT.0.01)EGY=.01
1E-91+FIX(1000.*(1.-EGY))
3 DE=STAB(IE)*S
EGY=EGY-DE
IF EGY.GT.ECUT)GO TO 4
IF LAG=1
ENTRY CROSS
EGY=ECUT
ETA=ETAB(IE)
ANBDA=1./SIGTAB(IE)
RETURN
END
SUBROUTINE ASSOC(PL,CX,SX,P)
DIMENSION PL(10),P(9,9)
IF(SX.GT.0.0)GO TO 1
DO 2 L=1,9
DO 2 M=1,9
2 P(L,M)=0.0
RETURN
1 P(1,1)=SX
P(2,1)=3.*CX*SX
P(2,2)=2.*CX/SX*P(2,1)-6.*PL(3)
M=1
LOW=M+1
DO 10 L=LOW,8
10 P(L+1,M)=((2*L+1)*CX*P(L,M)-(L*M)*P(L-1,M))/FLOAT(L-M+1)
P(3,2)=2.*CX/SX*P(3,1)-12.*PL(4)
DO 100 M=2,8
LOW=M+1
IF(LOW.EQ.9)GO TO 51
DO 50 L=LOW,8
50 P(L+1,M)=((2*L+1)*CX*P(L,M)-(L*M)*P(L-1,M))/FLOAT(L-M+1)
51 P(LOW,M+1)=((2*M)*P(LOW,M)+CX/SX-(LOW+M)*P(LOW,M-1))/
IF(M.EQ.8)GO TO 100
P(LOW+1,M+1)=(2*M)*P(LOW+1,M)+CX/SX-((LOW+M+1)*P(LOW,M+2))+
1P(LOW+1,M-1)
100 CONTINUE
RETURN
END
SUBROUTINE SPLIT(K)
END
C "SPLIT" PERFORMS THE HISTORY SPLITTING. IT IS CALLED WHEN A
C SPLITTING PLANE IS CROSSED.
C
COMMON/NC/STO(17,201),NCOLL,CTHO,CTH,STH,PHI,CPI,SPHI,EGY,CPIIO,
1SPHI0,EQ,SFAC,AMBA,N,IFLAG,MAXCOLL,LCHR,STAB(200),ETAB(200),TLIM,
2X0,Y0,Z0,X,Y,Z,ECUT,CLAB,THICK(11),NMAX,ZMIN,ZMAX,IE,INRAN,XIRA,
3,NPRNT,TLAST,CTHC,NBATCH,ETA,PATH,S,SIGTAB(200),ENTAB(200),XIRC,
4Z1,MF,NCOUNT(2),ICROSS(2),PCROSS(2),NTRUE
DIMENSION XSV(2),YSV(2),ZSV(2),ESV(2),CTS(2),STS(2),CPS(2),SPS(2),
1PSV(2),IESV(2),NCSV(2),WFO(2)
DATA WFO/1,...,1/
IF(NCOUNT(K).LT.10)GO TO 1
IF(K.EQ.1)NCOUNT(1)=NCOUNT(2)-1
NCOUNT(K)=-1
IF LAG=1
ICROSS(K)=0
RETURN
1 IF(NCOUNT(K).GE.0)GO TO 10
XSV(K)=X
YSV(K)=Y
ZSV(K)=Z
ESV(K)=EGY

```

```

CTH(K)=CTH
S(4)=STH
CP(K)=CPHI
SPS(K)=SPHI
PSV(K)=PATH
IESV(K)=IE
NCSV(K)=NCOLL
IFLAG=-1
NCOUNT(K)=NCOUNT(K)+1
ICROSS(K)=1
RETURN
10 X=XSV(K)
Y=YSV(K)
Z=ZSV(K)
EGY=ESV(K)
CTH=CTS(K)
STH=STS(K)
CPHI=CPS(K)
SPHI=SPS(K)
PATH=PSV(K)
WF=WFO(K)/10.
N=N-1
IE=IESV(K)
S=0.
CALL CROSS
IFLAG=0
NCOLL=NCSV(K)
NCOUNT(K)=NCOUNT(K)+1
RETURN
END
SUBROUTINE PROC
COMMON/MC/STD(7,201),NCOLL,CTHD,CTH,STH,PHI,CPHI,SPHI,EGY,CPHI0,
15PHI0,ED,SFAC,AMBDA,N,IFLAG,MAXCOLL,LCHR,STAB(200),ETAB(200),TLIM,
2XO,YO,ZO,X,Y,Z,ECUT,CLAB,THICK(11),NMAX,ZMIN,ZMAX,IE,INRAN,XIRA
3,NPRINT,TLAST,CTMC,NBATCH,ETA,PATH,S,SIGTAB(200),ENTAB(200),XIRC,
4Z1,WF,NCOUNT(2),ICROSS(2),PCROSS(2),NTRUE
5DIMENSION TRANC(10,10),TRANF(10,10),BACKC(10,10),BACKF(10,10),
6APLUS(9,9,10),AMINUS(9,9,10),PLUS(9,9,10),MINUS(9,9,10),
7ALGO(10,10),LEG(10,10),ENERGY(10),YCOF(9,9),ANGLE(10),PL(10),
8POLM(9,9),CP(9),SP(9),SUM(10),TEMP(10),ISIG(10),ITP(9,9,10),
9ATM(9,9,10),ISGEX(10,10),IPR(10)
DO 1 L=1,9
FL=(2.*FLOAT(L)+1.)/6.283185307
DO 2 M=1,L
LM=L-M+1
LM=L+M
FI=1.
DO 3 J=LMI,LM
FI=FI+FLOAT(1)
2 YCOF(L,M)=SORT(FL/FI)
1 CONTINUE
REWIND 2
NIN=NMAX/10
FN=FLOAT(MIN)
DE=ED/10.
ENERGY(1)=EO-DE
ANGLE(1)=.1
DO 4 K=2,10
ANGLE(K)=ANGLE(K-1)+.1
4 ENERGY(K)=ENERGY(K-1)-DE
1 PLANE=2
100 REWIND 2
IB=NH-NHIST-0
IPLANE=IPLANE+1
PRINT 777,THICK(IPLANE)
777 FORMAT('10.40X,DETECTOR PLANE AT=.E12.5)

```

```

DO 40 I=1,10
  IW=0
  DO 40 J=1,10
    TRANC(I,J)=TRANF(I,J)+BACKC(I,J)+BACKF(I,J)*O.
  41 ALEG(I,J)=TLEG(I,J)*O.
  DO 40 J=1,9
    DO 40 K=1,9
      40 APLUS(J,K,I)=AMINUS(J,K,I)+PLUS(J,K,I)+TMINUS(J,K,I)*O.
5000 READ(2)((STO(K,L),K=1,7),L=1,201)
DO 90 L=1,201
  IF(STO(1,L).EQ.O.)GO TO 11
  IF(STO(1,L).LT.10.)GO TO 10
  NH=NH+1
  11 NHIST=NHIST+1
  GO TO 50
  10 IPL=FIX(STO(1,L)+1.E-6)
  IF(IPL.NE.IPLANE)GO TO 90
  W=STO(7,L)
  CLASSIFY EMERGENT ANGLE
  CA=ABS(STO(3,L))
  DO 80 J=1,10
    80 IF(CA.LT.ANGLE(J))GO TO 81
    J=10
  81 JC=J
  CLASSIFY EMERGENT ENERGY
  DO 82 K=1,10
    82 IF(STO(6,L).GT.ENERGY(K))GO TO 83
    K=10
  83 KC=K
  IPR(KC)=1
  CTN POSITIVE OR NEGATIVE?
  CTMET=STO(3,L)
  STMET=STO(4,L)
  PHI=STO(5,L)
  CALL LEP(PL,CTMET,9)
  DO 70 M=1,9
    CP(M)=COS(W*PHI)
    SP(M)=SIN(W*PHI)
    CALL ASSOC(PL,CTMET,STMET,PLM)
  70 SP(M)=SIN(W*PHI)
    IF(CTMET)84,85,85
  84 BACKC(JC,KC)=BACKC(JC,KC)+W/FN
    BACKF(JC,KC)=BACKF(JC,KC)-W/CTMET/FN
  DO 86 LL=1,10
    86 ALEG(LL,KC)=ALEG(LL,KC)+PL(LL)*W/FN
    DO 87 LL=1,9
      DO 88 M=1,LL
        FM=YCOF(LL,M)*PLM(LL,M)/FN
        APLUS(LL,M,KC)=APLUS(LL,M,KC)+FM*CP(M)*W
  88 AMINUS(LL,M,KC)=AMINUS(LL,M,KC)+FM*SP(M)*W
  87 CONTINUE
  GO TO 90
  85 TRANC(JC,KC)=TRANC(JC,KC)+W/FN
    TRANF(JC,KC)=TRANF(JC,KC)+W/FN/CTMET
  DO 89 LL=1,10
    89 TLEG(LL,KC)=TLEG(LL,KC)+PL(LL)*W/FN
    DO 99 LL=1,9
      DO 91 M=1,LL
        FM=YCOF(LL,M)*PLM(LL,M)/FN
        TPLUS(LL,M,KC)=TPLUS(LL,M,KC)+FM*CP(M)*W
  91 TMINUS(LL,M,KC)=TMINUS(LL,M,KC)+FM*SP(M)*W
  99 CONTINUE
  GO TO 90
  50 IF(NHIST.LE.NNH)GO TO 90
  NHIST=1
  KPUT=1
  18=18+1

```

```

DO 150 MM=1,4
DO 14 I=1,10
164 SU(I)=0.
IF(18.LT.10)GO TO 154
PRINT 220
GO TO (114,124,134,144),MM
114 PRINT 230
GO TO 153
124 PRINT 231
GO TO 153
134 PRINT 232
GO TO 153
144 PRINT 233
153 PRINT 234,(ANGLE(I),I=1,10)
230 FORMAT(///45X,'POSITIVE CURRENT-//')
231 FORMAT(///45X,'NEGATIVE CURRENT-//')
232 FORMAT(///45X,'POSITIVE FLUX-//')
233 FORMAT(///45X,'NEGATIVE FLUX-//')
234 FORMAT(45X,'EMERGENT POLAR COSINE-/9X,10E12.5/3X,'E(MEV)')
154 DO 164 K=1,10
DO 165 I=1,10
GO TO (115,125,135,145),MM
115 TEMP(I)=TRANC(I,K)
TRANC(I,K)=0.
GO TO 105
125 TEMP(I)=BACKC(I,K)
BACKC(I,K)=0.
GO TO 105
135 TEMP(I)=TRANF(I,K)
TRANF(I,K)=0.
GO TO 105
145 TEMP(I)=BACKF(I,K)
BACKF(I,K)=0.
105 CONTINUE
CALL STATS(TEMP,ISIG,10,KPUT,18)
IF(18.LT.10)GO TO 104
DO 156 I=1,10
156 SUM(I)=SUM(I)+TEMP(I)
IF(18.LT.10)GO TO 104
PRINT 221,ENERGY(K),(TEMP(I),ISIG(I),I=1,10)
104 CONTINUE
IF(18.EQ.10)PRINT 235,(SUM(I),I=1,10)
235 FORMAT(/2X,'TOTALS ',10(E12.5))
150 CONTINUE
DO 1700 I=1,9
DO 1700 J=1,9
DO 1701 K=1,10
TEMP(K)=TPLUS(I,J,K)
1701 TPLUS(I,J,K)=0.
CALL STATS(TEMP,ISIG,10,KPUT,18)
IF(18.LT.10)GO TO 1700
DO 1702 K=1,10
TPLUS(I,J,K)=TEMP(K)
1702 TPLUS(I,J,K)=ISIG(K)
1700 CONTINUE
DO 1710 I=1,9
DO 1710 J=1,9
DO 1711 K=1,10
TEMP(K)=TMINUS(I,J,K)
1711 TMINUS(I,J,K)=0.
CALL STATS(TEMP,ISIG,10,KPUT,18)
IF(18.LT.10)GO TO 1710
DO 1712 K=1,10
TMINUS(I,J,K)=TEMP(K)
1712 TMINUS(I,J,K)=ISIG(K)
1710 CONTINUE

```

```

DO 720 I=1,10
A
  FAL=SQRT((2.*AL-1.)/12.56637061)
  DO 1721 J=1,10
    TEMP(J)=TLEG(I,J)
1721 TLEG(I,J)=0.
    CALL STATS(TEMP,ISIG,10,KPUT,18)
    IF(18.LT.10)GO TO 1720
    IF(MA.NE.NMAX)STOP 7
    DO 1722 J=1,10
      TLEG(I,J)=FAL*TEMP(J)
1722 ISGEX(I,J)=ISIG(J)
1720 CONTINUE
    IF(18.EQ.10)PRINT 499,NH
    IF(18.LT.10)GO TO 1730
220 FORMAT(///)
221 FORMAT(1X,F7.3,1X,10(F9.5,13))
499 FORMAT(1X,*TOTAL NUMBER OF HISTORIES PROCESSED = *,I10)
PRINT 1523
1523 FORMAT(///45X,*LEGENDRE COEFFICIENTS OF TRANSMISSION*/
14X,*E(MEV) L= 4X,*0*,11X,*1*,11X,*2*,11X,*3*,11X,*4*,
21X,*5*,11X,*6*,11X,*7*,11X,*8*,11X,*9*)
DO 1626 K=1,10
  IF(IPR(K).EQ.0)GO TO 1626
  PRINT 526,ENERGY(K),(TLEG(LL,K),LL=1,10)
1626 CONTINUE
PRINT 651
DO 1525 K=1,10
  IF(IPR(K).EQ.0)GO TO 1525
  PRINT 524,ENERGY(K),(ISGEX(LL,K),LL=1,10)
1525 CONTINUE
DO 1345 KE=1,10
  IF(1PR(KE).EQ.0)GO TO 1345
  PRINT 1347,ENERGY(KE)
1347 FORMAT(///1X,*ENERGY = *,F10.5,* MEV*/1X,*SPHERICAL HARMONICS COEFF
ICENTS FOR TRANSMISSION, TPLUS*/
DO 1343 LL=1,9
  PRINT 349,LL,(TPLUS(LL,M,KE),M=1,LL)
1343 PRINT 651
DO 1652 LL=1,9
  PRINT 653,LL,(ITP(LL,M,KE),M=1,LL)
1652 PRINT 1348
1348 FORMAT(1X,*SPHERICAL HARMONICS COEFFICIENTS FOR TRANSMISSION, TMIN
1US*/
DO 1342 LL=1,9
  PRINT 349,LL,(TMINUS(LL,M,KE),M=1,LL)
1342 PRINT 651
DO 1654 LL=1,9
  PRINT 653,LL,(ITM(LL,M,KE),M=1,LL)
1654 PRINT 1345
1345 CONTINUE
1730 DO 700 I=1,9
  DO 700 J=1,9
    DO 701 K=1,10
      TEMP(K)=APLUS(I,J,K)
701 APLUS(I,J,K)=0.
      CALL STATS(TEMP,ISIG,10,KPUT,18)
      IF(18.LT.10)GO TO 700
      DO 702 K=1,10
        APLUS(I,J,K)=TEMP(K)
702 ITP(I,J,K)=TEMP(K)
700 CONTINUE
      DO 710 I=1,9
        DO 710 J=1,9
          DO 711 K=1,10
            TEMP(K)=AMINUS(I,J,K)
711 AMINUS(I,J,K)=0.

```

```

C 11  STATES(TEMP,ISIG,10,KPUT,18)
  IF (B.LT.10)GO TO 710
  DO 12 K=1,10
    AMINUS(I,J,K)=TEMP(K)
  712  ITM(I,J,K)=ISIG(K)
  710  CONTINUE
  DO 720 I=1,10
    DO 721 J=1,10
      TEMP(J)=ALEG(I,J)
  721  ALEG(I,J)=0.
  CALL STATES(TEMP,ISIG,10,KPUT,18)
  IF (B.LT.10)GO TO 720
  DO 722 J=1,10
    ALEG(I,J)=TEMP(J)
  722  ISGEX(I,J)=ISIG(J)
  720  CONTINUE
  IF (B.LT.10)GO TO 90
  PRINT LEGENDRE COEFFICIENTS
  523  FORMAT(/45X,'LEGENDRE COEFFICIENTS OF BACKSCATTER=4X,E(MEV) L=
    11X,80,11X,10,11X,20,11X,30,11X,40,11X,50,11X,60,11X,70,
    211X,80,11X,90')
  DO 625 LL=1,10
    AL=LL
    FAL=SQRT((2.*AL-1.)/12.56837061)
    DO 626 K=1,10
      ALEG(LL,K)=ALEG(LL,K)*FAL
  625  CONTINUE
  PRINT 523
  DO 626 K=1,10
    IF (IPR(K).EQ.0)GO TO 628
    PRINT 526,ENERGY(K),(ALEG(LL,K),LL=1,10)
  626  CONTINUE
  526  FORMAT(3X,F7.3,1X,10E12.5)
  PRINT 651
  DO 525 K=1,10
    IF (IPR(K).EQ.0)GO TO 525
    PRINT 524,ENERGY(K),(ISGEX(LL,K),LL=1,10)
  525  CONTINUE
  524  FORMAT(3X,F7.3,1X,10I4)
  DO 345 KE=1,10
    IF (IPR(KE).EQ.0)GO TO 345
  3461  PRINT 347,ENERGY(KE)
  347  FORMAT(/1X,'ENERGY = ',F10.5,' MEV',1X,'SPHERICAL HARMONICS COEFF
    11CIENTS - APLUS=')
  DO 343 LL=1,9
    343  PRINT 349,LL,(APLUS(LL,M,KE),M=1,LL)
  349  FORMAT(1X,L=,15,1X,9E12.5)
  PRINT 651
  651  FORMAT(1X,'PERCENT ERRORS IN ABOVE ITEMS=')
  DO 652 LL=1,9
    652  PRINT 653,LL,(ITP(LL,M,KE),M=1,LL)
  653  PRINT 348
  348  FORMAT(1X,'SPHERICAL HARMONICS COEFFICIENTS - AMINUS=')
  DO 342 LL=1,9
    342  PRINT 349,LL,(AMINUS(LL,M,KE),M=1,LL)
  PRINT 651
  DO 654 LL=1,9
    654  PRINT 653,LL,(ITM(LL,M,KE),M=1,LL)
  653  CONTINUE
  653  FORMAT(1X,L=,15,1X,9I4)
  GO TO 200
  90  CONTINUE
  GO TO 5000
  200  IF (IPLANE.LT.5)GO TO 100
  RETURN
  END

```

Program Listing

- CHDP -

(Reference: Vol. I, Section IV.B.5)

```

PROGRAM CHDP(INPUT,OUTPUT,TAPE1,TAPE2,TAPE3)

C THE PURPOSE OF THIS PROGRAM IS TO PROCESS AND ANALYZE DATA
C OBTAINED AS A RESULT OF IRRADIATION OF THE POLYMER CGH90
C BY ELECTRON BEAMS. THE MEASURED QUANTITIES ARE IN THE FORM OF
C CURRENTS. THE RESULTS OF THESE MEASUREMENTS ARE COMPARED WITH
C CALCULATIONS PERFORMED USING THE TIGER MONTE CARLO CODE AND ALSO
C WITH THE RESULTS PREDICTED USING TABATA'S ALGORITHM. THE
C EXPERIMENTAL CURRENT DATA WAS INTEGRATED USING THE ADAPTIVE SIMP-
C SON RULE INTEGRATION ROUTINE OF SHAMPINE AND ALLEN, TO OBTAIN
C CHARGE DEPOSITION PROFILES IN A FORM COMPATIBLE WITH THE TIGER
C CALCULATION. THE TIGER HISTOGRAMS WERE SMOOTHED USING A LEAST-
C SQUARES SPLINE SMOOTHING ALGORITHM "ICSSCU" OBTAINED FROM THE
C INSL PROGRAM LIBRARY. THE REFERENCES FOR THE SPLINE SMOOTHER,
C THE SIMPSON RULE INTEGRATOR AND THE TABATA ALGORITHM ARE GIVEN
C RESPECTIVELY BY REFS.20,21,22 GIVEN ON PAGE 265 OF VOL.1.

C DIMENSION X(100,3),NP(3),F(100),IF(100),ED(100,3),XMAX(3),
C 1XP(100),YP(100),S(100),DY(100,3),C(99,3),WY(100),WK(714)
C DIMENSION YI(51),ED(3),FRI(3),AA(3),ZA(3),CUR(100),ECUR(71,6)
C DIMENSION TTR(70,6),NMA(6),EGV(6),XD(100),YD(100),ERD(100)
C COMMON XP,C,W,XMAX
C EQUIVALENCE(XD(1),X(1)),(YD(1),X(101)),(ERD(1),X(201))
C EXTERNAL F1
C DATA NP/97,98,100/
C DATA FR,AA,ZA/74173,16482,09345,12,001,16,1,008,6,8,1,1./
C DATA XMAX/150,500,750./
C DATA ED/4,1,1,4/
C DATA EGV/4,6,6,1,1,2,1,4/
C DATA NMA/15,24,37,49,59,70/
C DATA ERROR/8,0004,002,0022,0014,0006,002,001,56,0,0,
C 18,0004,002,6,0005,0025,0038,0007,0005,0004,0002,0001,
C 2,0003,47,0,8,0003,0015,6,0007,0035,002,6,0004,002,004,
C 3,0007,0006,0005,0004,0003,0002,0006,5,0002,33,0,8,0003,
C 4,015,6,0007,0035,002,6,0004,002,002,6,0004,002,003,
C 52,0006,0005,0005,0004,0003,0013,0021,0003,0002,2,0001,
C 625,0,8,0003,0015,6,0006,003,0015,6,0003,0015,002,6,0004
C 7,002,0035,6,0007,0035,0015,6,0003,0015,001,7,0002,14,0,
C 88,0003,0015,6,0005,0025,002,6,0004,002,2,002,5,0004,002
C 9,0025,6,0005,0025,002,6,0004,002,0015,7,0003,0015,8,0003
C 1,0015,0002,0002,0001,3,0,0./
C ZE=ZDA=0.
C DO 17 I=1,3
C ZE=ZE+FR(I)*ZA(I)
C 17 ZDA=ZDA+FR(I)*ZA(I)/AA(I)
C AW=ZE/ZDA
C MODE=1
C PRINT 19,ZE,AW
C 19 FORMAT(1X,'EFFECTIVE Z =',F8.5,2X,'EFFECTIVE A =',F10.5)
C IC=99
C REMIND 1
C REMIND 2
C REMIND 3
C DO 100 L=1,3
C LL=L
C READ(LL,NX,(X(1),F(1),YP(1),I=1,NX)
C READ CHARGE DEPOSITION PROFILES FROM TIGER CALCULATIONS
C READ 45,((ED(1),I=1,NX)
C READ STANDARD PERCENT ERRORS FROM TIGER CALCULATIONS

```

```

      RE=0.46*(IF(I1,I=1,NX)
48 F( AT(4012)
C
C CONVERT PERCENT ERRORS TO ABSOLUTE ERROR ESTIMATES
C
      DO 47 I=1,NX
      DY(I,L)=.01*FLOAT(IF(I1)*ED(I,L)
47 IF(I1)=0
100 CONTINUE
      DO 101 I=1,100
101 F(I)=Y(I1)*0.
45 FORMAT(16F5.0)
      DO 200 L=1,3
      TO=EO(L)
      IMAX=NP(L)
      DO 160 I=1,IMAX
      XP(I)=X(I,L)
160 YP(I)=ED(I,L)
      PRINT 72,L
      PRINT 71,(XP(I),YP(I),I=1,IMAX)
      DN=FLOAT(IMAX+1)
      SM=FLOAT(IMAX+1)-DN
      DO 149 I=1,IMAX
149 S(I)=DY(I,L)
      DO 150 K=1,3
C
C CALL LEAST SQUARES SPLINE SMOOTHING ROUTINE
C
      CALL ICSSCU(XP,YP,S,IMAX,SM,NY,C,1C,MK,IER)
      OX=AMAX(L)/50.
      XX=0.5*DX
      DO 40 N=1,50
      DO 41 I=2,IMAX
41 IF(XX*LT.XP(I))GO TO 42
42 D=XX*XP(I-1)
      I=I-1
      Y=((C(I,3)*D+C(I,2))*D+C(I,1))*D*NY(I)
      X(N,L)=XX
      F(N)=Y
      IF(K.GT.1)GO TO 130
      XT=XX*1.E-3
C
C CALL TABATA PROGRAM TO CALCULATE CURRENT VALUES
C
      CALL DEPOSI(CU,VY,TO,ZE,AM,MODE,XT)
      YI(N)=VY
      CUR(N)=CU
130 XX=XX+DX
      IF(XX.GE.XP(IMAX))GO TO 48
40 CONTINUE
48 PRINT 73,SM
      PRINT 71,(X(N,L),FIN),N=1,51)
      IF(K.GT.1)GO TO 131
      PRINT 81
      PRINT 71,(X(N,L),YI(N),N=1,51)
      FORMAT(/1X,*TABATA ALGORITHM RESULTS*//)
      PRINT 82
      PRINT 71,(X(N,L),CUR(N),N=1,51)
      PRINT 71,(X(N,L),CUR(N),N=1,51)
82 FORMAT(1X,*TABATA CURRENT VALUES*)
131 XLO=XP(1)
      XHI=XX-DX
C
C INTEGRATE SMOOTHED CURRENTS TO GET TOTAL CHARGE DEPOSITED
C
      CALL SIMP(F1,XLO,XHI,1.E-5,AREA,ERR,AR,IFL)
      PRINT 9,AREA

```

```

9 FORMAT(1X,'INTEGRAL=',E12.5)
150 S( N=DN
200 C( INUE
72 FORMAT(1X,'DATA SET NO.=12/1X,'RAW DATA=)
73 FORMAT(1X,'1X,'INTERPOLATED VALUES',5X,'SM=',F10.5)
71 FORMAT(5I2F10.3,6X))
37 FORMAT(16F5.0)
DO 300 L=1,6
NM=NM+1(L)
300 READ 37,(ECUR(N,L),N=1,NM)
C
C EXPERIMENTAL CURRENT DEPOSITION DATA
C
DO 301 L=2,6
301 ECUR(1,L)=ECUR(1,L)
DO 302 L=4,6
302 ECUR(2,L)=ECUR(2,L)
XP(1)=10.7
XP(9)=95.6
XP(17)=175.
XP(25)=259.9
XP(33)=335.5
XP(41)=420.4
XP(49)=497.9
XP(57)=582.8
XP(64)=655.0
DO 303 N=2,8
XP(N)=XP(N-1)+10.7
XP(N+8)=XP(N+7)+10.7
XP(N+16)=XP(N+15)+10.7
XP(N+24)=XP(N+23)+10.7
XP(N+32)=XP(N+31)+10.7
XP(N+40)=XP(N+39)+10.7
XP(N+48)=XP(N+47)+10.7
XP(N+56)=XP(N+55)+10.7
XP(64)=655.
303 XP(N+63)=XP(N+62)+10.7
DO 350 L=1,6
NM=NM+1(L)
DO 351 N=1,NM
XD(N)=XP(N)
YD(N)=ECUR(N,L)
351 ERD(N)=ERROR(N,L)
DN=DN+1
SN=FLOAT(NM+1)-DN
C
C SMOOTH THE EXPERIMENTAL DATA
C
DO 352 K=1,3
CALL ICSSCU(XD,YD,ERD,NM,SM,WY,C,IC,WK,IER)
PRINT 360,EGY(L),SM
PRINT 71,({XD(N),WY(N),N=1,NM)
360 FORMAT(1X,'SMOOTHED EXPERIMENTAL DATA, SOURCE ENERGY=',F5.3,'ME
1V=3X,'SMOOTHING PARAMETER=',F8.3)
352 SM=SM+DN
TD=EGY(L)
MODE=1
C
C OBTAIN CURRENT VALUES FROM TABATA PROGRAM
C
DO 362 N=1,NM
XX=XD(N)+1.E-3
CALL DEPOSI(CU,YY,TO,ZE,AW,MODE,XX)
MODE=2
XX=(XD(N)-10.7)+1.E-3
CALL DEPOSI(CV,YY,TO,ZE,AW,MODE,XX)

```

362 WY(N)=CV-CU

PR : 81

PR : 71,(XDIN),WY(N),N=1,NM)

350 CONTINUE

STOP

END

IMSL ROUTINE NAME - ICSSCU

COMPUTER - CDC/SINGLE

LATEST REVISION - JUNE 1, 1980

PURPOSE - CUBIC SPLINE DATA SMOOTHER

USAGE - CALL ICSSCU (X,F,DF,NX,SM,Y,C,IC,WK,IER)

ARGUMENTS X - VECTOR OF LENGTH NX CONTAINING THE ABSCISSAE OF THE NX DATA POINTS (X(1),F(1)) I=1,...., NX. (INPUT) X MUST BE ORDERED SO THAT X(1) .LT. X(I+1).

F - VECTOR OF LENGTH NX CONTAINING THE ORDINATES (OR FUNCTION VALUES) OF THE NX DATA POINTS. (INPUT)

DF - VECTOR OF LENGTH NX. (INPUT) DF(1) IS THE RELATIVE WEIGHT OF DATA POINT 1 (SEE PARAMETER SM BELOW).

NX - NUMBER OF ELEMENTS IN X, F, DF, AND Y. (INPUT) NX MUST BE GE. 2.

SM - A NON-NEGATIVE NUMBER WHICH CONTROLS THE EXTENT OF SMOOTHING. (INPUT) THE SPLINE FUNCTION S IS DETERMINED SUCH THAT THE SUM FROM 1 TO NX OF ((S(X(I))-F(1))/DF(1))\*\*2 IS LESS THAN OR EQUAL TO SM, WHERE EQUALITY HOLDS UNLESS S DESCRIBES A STRAIGHT LINE.

Y,C - SPLINE COEFFICIENTS. (OUTPUT) Y IS A VECTOR OF LENGTH NX. C IS AN NX-1 BY 3 MATRIX. THE VALUE OF THE SPLINE APPROXIMATION AT T IS S(T) = ((C(1,3)+D+C(1,2))\*D+C(1,1))\*D+Y(1) WHERE X(1) .LE. T .LT. X(I+1) AND D = T-X(1).

IC - ROW DIMENSION OF MATRIX C EXACTLY AS SPECIFIED IN THE DIMENSION STATEMENT IN THE CALLING PROGRAM. (INPUT)

WK - WORK AREA VECTOR OF LENGTH 7\*NX+14.

IER - ERROR PARAMETER. (OUTPUT) TERMINAL ERROR IER = 129, IC IS LESS THAN NX-1 IER = 130, NX IS LESS THAN 2 IER = 131, INPUT ABSCISSAE ARE NOT ORDERED SO THAT X(1) .LT. X(2) ... .LT. X(NX)

PRECISION/HARDWARE - SINGLE AND DOUBLE/H32

- SINGLE/H36,H48,H60

REQD. IMSL ROUTINES - UERTST,UGETIO

NOTATION - INFORMATION ON SPECIAL NOTATION AND CONVENTIONS IS AVAILABLE IN THE MANUAL INTRODUCTION OR THROUGH IMSL ROUTINE UHELP

REMARKS 1. THE ROUTINE PRODUCES A NATURAL CUBIC SPLINE. HENCE,

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C      THE SECOND DERIVATIVE OF THE SPLINE FUNCTION S AT
C      X(1) AND X(NX) IS ZERO.
C      2. FOR EACH SET OF DATA POINTS THERE EXISTS A MAXIMUM
C      VALUE FOR THE SMOOTHING PARAMETER. LET US CALL THIS
C      MAXIMUM VALUE SM. IT IS DEFINED BY THE FOLLOWING
C      FORMULA:
C      SM = THE SUM FROM I EQUAL 1 TO NX OF
C      ((Y(I)-F(I))/DF(I))2
C      WHERE Y IS THE SET OF FUNCTION VALUES DEFINING THE
C      STRAIGHT LINE WHICH BEST APPROXIMATES THE DATA IN
C      THE LEAST SQUARES SENSE (WITH WEIGHTS DF).
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C      WARRANTY - IMSL WARRANTS ONLY THAT IMSL TESTING HAS BEEN
C      APPLIED TO THIS CODE. NO OTHER WARRANTY,
C      EXPRESSED OR IMPLIED, IS APPLICABLE.
C
C      SUBROUTINE ICSSCU (X,F,DF,NX,SM,Y,C,IC,WK,IER)
C      INTEGER
C      REAL
C      X(1),F(1),DF(1),Y(1),C(IC,1),WK(7,1)
C      I,J,M2,NP1,NP3
C      E,FF,F2,G,H,HMG,ONE,ONEDH,ONED3,P,SM,
C      TWOD3,ZERO
C      TWOD3=.666666666666667/
C      ONED3=.333333333333333/
C      ZERO/0.0,ONE/1.0/
C      FIRST EXECUTABLE STATEMENT
C      IER = 0
C      CHECK ERROR CONDITIONS
C      IF (IC .LT. NX-1) GO TO 65
C      IF (NX .LT. 2) GO TO 70
C      M2 = NX+2
C      NP1 = NX+1
C      WK(1,1) = ZERO
C      WK(1,2) = ZERO
C      WK(2,NP1) = ZERO
C      WK(3,M2) = ZERO
C      WK(3,NP1) = ZERO
C      WK(6,1) = ZERO
C      WK(6,2) = ZERO
C      WK(6,M2) = ZERO
C      WK(6,NP1) = ZERO
C      P = ZERO
C      H = X(2)-X(1)
C      IF (H .LE. ZERO) GO TO 75
C      F2 = -SM
C      FF = (F(2)-F(1))/H
C      IF (NX .LT. 3) GO TO 30
C      DO 5 I=3,NX
C      G = H
C      H = X(I)-X(I-1)
C      IF (H .LE. ZERO) GO TO 75
C      ONEDH = ONE/H
C      E = FF
C      FF = (F(I)-F(I-1))*ONEDH
C      Y(I) = FF-E
C      WK(4,I) = (G+H)*TWOD3

```

```

      WK(5,1) = H*Q*ED3
      ( (1,3,1) = DF(1-2)/G
        *X(1,1) = DF(1)-ONEDH
        WK(2,1) = -DF(1-1)/G-DF(1-1)*ONEDH
      5 CONTINUE
      DO 10 I=3,NX
        C(1-1,1) = WK(1,1)*WK(1,1)+WK(2,1)*WK(3,1)+WK(3,1)
        C(1-1,2) = WK(1,1)*WK(2,1)+WK(2,1)*WK(3,1)+1
        C(1-1,3) = WK(1,1)*WK(3,1)+2
      10 CONTINUE
      C
      15 IF (NX .LT. 3) GO TO 30
      DO 20 I=3,NX
        WK(2,1-1) = FF*WK(1,1-1)
        WK(3,1-2) = G*WK(1,1-2)
        WK(1,1) = ONE - (P-C(1-1,1)+WK(4,1)-FF*WK(2,1-1)-G*WK(3,1-2))
        WK(6,1) = Y(1)-WK(2,1-1)+WK(6,1-1)-WK(3,1-2)*WK(6,1-2)
        FF = P-C(1-1,2)+WK(5,1)-H*WK(2,1-1)
        G = H
        H = C(1-1,3)*P
      20 CONTINUE
      NP3 = NX+3
      DO 25 I=3,NX
        J = NP3-I
        WK(6,J) = WK(1,J)*WK(6,J)-WK(2,J)*WK(6,J+1)-WK(3,J)*WK(6,J+2)
      25 CONTINUE
      30 E = ZERO
      H = ZERO
      C
      DO 35 I=2,NX
        G = H
        H = (WK(6,1+1)-WK(6,1))/(X(1)-X(1-1))
        HMG = H-G
        WK(7,1) = HMG-DF(1-1)*DF(1-1)
        E = E+WK(7,1)*HMG
      35 CONTINUE
      G = -H*DF(NX)-DF(NX)
      WK(7,NP1) = G
      E = E-G*H
      G = F2
      F2 = E*P*P
      IF (F2 .GE. SM .OR. F2 .LE. G) GO TO 50
      FF = ZERO
      H = (WK(7,3)-WK(7,2))/(X(2)-X(1))
      IF (NX .LT. 3) GO TO 45
      DO 40 I=3,NX
        G = H
        H = (WK(7,1+1)-WK(7,1))/(X(1)-X(1-1))
        G = H-G-WK(2,1-1)*WK(1,1-1)-WK(3,1-2)*WK(1,1-2)
        FF = FF+G*WK(1,1)*G
        WK(1,1) = G
      40 CONTINUE
      45 H = E*P*FF
      IF (H .LE. ZERO) GO TO 50
      C
      C
      P = P+(SM-F2)/((SQRT(SM/E)+P)*H)
      GO TO 15
      C
      C
      50 NP1 = NX-1
      DO 55 I=1,NP1
        Y(I) = F(1)-P*WK(7,I+1)
        C(1,2) = WK(6,I+1)
        WK(1,1) = Y(1)
      55 CONTINUE

```

```

      1. NX) = F(NX) - P * WK(7, NX+1)
      DO 60 I=2, NX
      M = X(I) - X(I-1)
      C(I-1,3) = (WK(6, I+1) - C(I-1,2)) / (H+M+H)
      C(I-1,1) = (WK(1, I) - Y(I-1)) / H - H * C(I-1,3) + C(I-1,2) * H
60 CONTINUE
      GO TO 9005
65 IER = 129
      GO TO 9000
70 IER = 130
      GO TO 9000
75 IER = 131
9000 CONTINUE
      CALL UERTST(IER, SHICSSCU)
9005 RETURN
      END
C IMSL ROUTINE NAME - UERTST
C
C COMPUTER - CDC/SINGLE
C LATEST REVISION - JANUARY 1, 1978
C PURPOSE - PRINT A MESSAGE REFLECTING AN ERROR CONDITION
C USAGE - CALL UERTST (IER, NAME)
C ARGUMENTS IER - ERROR PARAMETER. (INPUT)
      IER = I+J WHERE
      I = 128 IMPLIES TERMINAL ERROR.
      I = 64 IMPLIES WARNING WITH FIX, AND
      J = 32 IMPLIES WARNING.
      J = ERROR CODE RELEVANT TO CALLING
      ROUTINE.
      NAME - A SIX CHARACTER LITERAL STRING GIVING THE
      NAME OF THE CALLING ROUTINE. (INPUT)
PRECISION/HARDWARE - SINGLE/ALL
REQD. IMSL ROUTINES - UGETIO
NOTATION - INFORMATION ON SPECIAL NOTATION AND
CONVENTIONS IS AVAILABLE IN THE MANUAL
INTRODUCTION OR THROUGH IMSL ROUTINE UHELP
REMARKS THE ERROR MESSAGE PRODUCED BY UERTST IS WRITTEN
ONTO THE STANDARD OUTPUT UNIT. THE OUTPUT UNIT
NUMBER CAN BE DETERMINED BY CALLING UGETIO AS
FOLLOWS... CALL UGETIO(1, NIN, NOUT).
THE OUTPUT UNIT NUMBER CAN BE CHANGED BY CALLING
UGETIO AS FOLLOWS...
      NIN = 0
      NOUT = NEW OUTPUT UNIT NUMBER
      CALL UGETIO(3, NIN, NOUT)
      SEE THE UGETIO DOCUMENT FOR MORE DETAILS.
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WARRANTY - IMSL WARRANTS ONLY THAT IMSL TESTING HAS BEEN
APPLIED TO THIS CODE. NO OTHER WARRANTY,
EXPRESSED OR IMPLIED, IS APPLICABLE.

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ICMU1930  
ICMU1940  
ICMU1950  
ICMU1960  
ICMU1970  
ICMU1980  
ICMU1990  
ICMU2000  
ICMU2010  
ICMU2020  
ICMU2030  
ICMU2040  
ICMU2050  
ICMU2060  
ICMU2070  
ICMU2080  
ICMU2090  
UERT0010  
UERT0020  
UERT0030  
UERT0040  
UERT0050  
UERT0060  
UERT0070  
UERT0080  
UERT0090  
UERT0100  
UERT0110  
UERT0120  
UERT0130  
UERT0140  
UERT0150  
UERT0160  
UERT0170  
UERT0180  
UERT0190  
UERT0200  
UERT0210  
UERT0220  
UERT0230  
UERT0240  
UERT0250  
UERT0260  
UERT0270  
UERT0280  
UERT0290  
UERT0300  
UERT0310  
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UERT0360  
UERT0370  
UERT0380  
UERT0390  
UERT0400  
UERT0410  
UERT0420  
UERT0430  
UERT0440  
UERT0450  
UERT0460  
UERT0470  
UERT0480  
UERT0490

```

C      SUBROUTINE UERTST (IER,NAME)
C      INTEGER IER,NAME
C      SPECIFICATIONS FOR ARGUMENTS (
C      SPECIFICATIONS FOR LOCAL VARIABLES
C      INTEGER NAMEQ,NAMEQ
C      DATA NAMEQ/6H
C      DATA NAMEQ/6H
C      DATA LEVEL/4, IEQDF/0, IEQ/1H
C      IF (IER.GT.999) GO TO 25
C      IF (IER.LT.-32) GO TO 55
C      IF (IER.LE.128) GO TO 5
C      IF (LEVEL.LT.1) GO TO 30
C      CALL UGETIO(1,NIN,IOUNIT)
C      IF (IEQDF.EQ.1) WRITE(IOUNIT,35) IER,NAMEQ,IEQ,NAME
C      IF (IEQDF.EQ.0) WRITE(IOUNIT,35) IER,NAME
C      GO TO 30
C      5 IF (IER.LE.64) GO TO 10
C      IF (LEVEL.LT.2) GO TO 30
C      CALL UGETIO(1,NIN,IOUNIT)
C      IF (IEQDF.EQ.1) WRITE(IOUNIT,40) IER,NAMEQ,IEQ,NAME
C      IF (IEQDF.EQ.0) WRITE(IOUNIT,40) IER,NAME
C      GO TO 30
C      10 IF (IER.LE.32) GO TO 15
C      IF (LEVEL.LT.3) GO TO 30
C      CALL UGETIO(1,NIN,IOUNIT)
C      IF (IEQDF.EQ.1) WRITE(IOUNIT,45) IER,NAMEQ,IEQ,NAME
C      IF (IEQDF.EQ.0) WRITE(IOUNIT,45) IER,NAME
C      GO TO 30
C      15 CONTINUE
C      IF (NAME.NE.NAMEQ) GO TO 25
C      LEVEL = LEVEL
C      LEVEL = IER
C      IER = LEVEL
C      IF (LEVEL.LT.0) LEVEL = 4
C      IF (LEVEL.GT.4) LEVEL = 4
C      GO TO 30
C      25 CONTINUE
C      IF (LEVEL.LT.4) GO TO 30
C      PRINT NON-DEFINED MESSAGE
C      CALL UGETIO(1,NIN,IOUNIT)
C      IF (IEQDF.EQ.1) WRITE(IOUNIT,50) IER,NAMEQ,IEQ,NAME
C      IF (IEQDF.EQ.0) WRITE(IOUNIT,50) IER,NAME
C      RETURN
C      30 IEQDF = 0
C      35 FORMAT(19H *** TERMINAL ERROR,10X,7H( IER = ,13,
C      20H) FROM IMSL ROUTINE ,AG,A1,AG)
C      40 FORMAT(36H *** WARNING WITH FIX ERROR ( IER = ,13,
C      20H) FROM IMSL ROUTINE ,AG,A1,AG)
C      45 FORMAT(18H *** WARNING ERROR,11X,7H( IER = ,13,
C      20H) FROM IMSL ROUTINE ,AG,A1,AG)
C      50 FORMAT(20H *** UNDEFINED ERROR,9X,7H( IER = ,15,
C      20H) FROM IMSL ROUTINE ,AG,A1,AG)
C      SAVE P FOR P = R CASE
C      P IS THE PAGE NAME
C      R IS THE ROUTINE NAME
C      55 IEQDF = 1
C      NAMEQ = NAME
C      65 RETURN
C      END
C      IMSL ROUTINE NAME - UGETIO
C

```

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UERT0500
UERT0510
UERT0520
UERT0530
UERT0540
UERT0550
UERT0560
UERT0570
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UERT0590
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UERT0690
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UERT0970
UERT0980
UERT0990
UERT1000
UERT1010
UERT1020
UERT1030
UERT1040
UERT1050
UERT1060
UERT1070
UERT1080
UERT1090
UERT1100
UERT1110
UERT1120
UERT1130
UGET0010
UGET0020

```



IT IS DESIGNED TO EVALUATE THE DEFINITE INTEGRAL OF A CONTINUOUS FUNCTION WITH FINITE LIMITS OF INTEGRATION. ( REF.: L.F. SHAMPINE AND R.C. ALLEN, JR., NUMERICAL COMPUTING: AN INTRODUCTION, W.B. SAUNDERS CO., PP. 238-242 (1974)

F - NAME OF FUNCTION WHOSE INTEGRAL IS DESIRED. THE FUNCTION NAME F MUST APPEAR IN AN EXTERNAL STATEMENT IN THE CALLING PROGRAM.

A,B - LOWER AND UPPER LIMITS OF INTEGRATION.

ANS - APPROXIMATE VALUE OF THE INTEGRAL OF F(X) FROM A TO B.

AREA - APPROXIMATE VALUE OF THE INTEGRAL OF ABS(F(X)) FROM A TO B.

ERROR - ESTIMATED ERROR OF ANS. USER MAY WISH TO EXTRAPOLATE BY FORMING ANS+ERROR TO GET WHAT IS OFTEN A MORE ACCURATE RESULT, BUT NOT ALWAYS.

ACC - DESIRED ACCURACY OF ANS. CODE TRIES TO MAKE ABS(ERROR).LE.ACC\*ABS(AREA).

IFLAG = 1 FOR NORMAL RETURN.

2 IF IT IS NECESSARY TO GO TO 30 LEVELS OR USE A SUBINTERVAL TOO SMALL FOR MACHINE WORD LENGTH, ERROR MAY BE UNRELIABLE IN THIS CASE.

3 IF MORE THAN 2000 FUNCTION EVALUATIONS ARE USED. ROUGH APPROXIMATIONS ARE USED TO COMPLETE THE COMPUTATIONS AND ERROR IS USUALLY UNRELIABLE.

DIMENSION FV(5),LORR(30),FIT(30),F2T(30),F3T(30),DAT(30),AREST(30),ESTT(30),EPST(30),PSUM(30)

SET U TO APPROXIMATELY THE UNIT ROUND-OFF OF SPECIFIC MACHINE (HERE IBM 360/67)

U=1.E-12

INITIALIZE

FOURU=4.0\*U

IFLAG=1

EPS=ACC

ERROR=0.0

LVLS=1

LORR(LVLS)=1

PSUM(LVLS)=0.0

ALPHA=A

DA=B-A

AREA=0.0

AREST=0.0

FV(1)=F(ALPHA)

FV(3)=F(ALPHA+0.5\*DA)

FV(5)=F(ALPHA+DA)

KOUNT=3

WT=DA/6.0

EST=WT\*(FV(1)+4.0\*FV(3)+FV(5))

'BASIC STEP'. HAVE ESTIMATE EST OF INTEGRAL ON (ALPHA,ALPHA+DA). BISECT AND COMPUTE ESTIMATES ON LEFT AND RIGHT HALF INTERVALS. SIMILARLY TREAT INTEGRAL OF ABS(F(X)). SUM IS BETTER VALUE FOR INTEGRAL AND DIFF/15.0 IS APPROXIMATELY ITS ERROR.

DX=0.5\*DA

FV(2)=F(ALPHA+0.5\*DX)

FV(4)=F(ALPHA+1.5\*DX)

KOUNT=KOUNT+2

```

W1=6.0
E1=WT*(FV(1)+4.0*FV(2)+FV(3))
E2=WT*(FV(3)+4.0*FV(4)+FV(5))
SUM=ESTL+ESTR
AREST=WT*(ABS(FV(1))+ABS(4.0*FV(2))+ABS(FV(3)))
ARESTR=WT*(ABS(FV(3))+ABS(4.0*FV(4))+ABS(FV(5)))
AREA=AREA1+(ARESTL+ARESTR)-AREST
DIFF=EST-SUM
C
C IF ERROR IS ACCEPTABLE. GO TO 2. IF INTERVAL IS TOO SMALL OR
C TOO MANY LEVELS OR TOO MANY FUNCTION EVALUATIONS, SET A FLAG
C AND GO TO 2 ANYWAY.
C
IF(ABS(DIFF).LE.EPS*ABS(AREA)) GO TO 2
IF(ABS(DX).LE.FOUR*ABS(ALPHA)) GO TO 5
IF(LVL.GE.30) GO TO 5
IF(MOUNT.GE.2000) GO TO 6
C
C HERE TO RAISE LEVEL. STORE INFORMATION TO PROCESS RIGHT HALF
C INTERVAL LATER. INITIALIZE FOR 'BASIC STEP' SO AS TO TREAT
C LEFT HALF INTERVAL.
C
LVL=LVL+1
LORR(LVL)=0
FIT(LVL)=FV(3)
F2T(LVL)=FV(4)
F3T(LVL)=FV(5)
DA=DX
DAT(LVL)=DX
AREST=ARESTL
ARESTT(LVL)=ARESTR
EST=ESTL
ESTT(LVL)=ESTR
EPS=EPS/1.4
EPST(LVL)=EPS
FV(5)=FV(3)
FV(3)=FV(2)
GO TO 1
C
C ACCEPT APPROXIMATE INTEGRAL SUM. IF IT WAS ON A LEFT INTERVAL.
C GO TO 'MOVE RIGHT'. IF A RIGHT INTERVAL, ADD RESULTS TO FIN-
C ISH AT THIS LEVEL. ARRAY LORR (MNEMONIC FOR LEFT OR RIGHT)
C TELLS WHETHER LEFT OR RIGHT INTERVAL AT EACH LEVEL.
C
2
3
ERROR=ERROR+DIFF/15.0
IF(LORR(LVL).EQ.0) GO TO 4
SUM=PSUM(LVL)+SUM
LVL=LVL+1
IF(LVL.GT.1) GO TO 3
ANS=SUM
RETURN
C
C 'MOVE RIGHT'. RESTORE SAVED INFORMATION TO PROCESS RIGHT HALF
C INTERVAL.
C
4
PSUM(LVL)=SUM
LORR(LVL)=1
ALPHA=ALPHA+DA
DA=DAT(LVL)
FV(1)=FIT(LVL)
FV(3)=F2T(LVL)
FV(5)=F3T(LVL)
AREST=ARESTT(LVL)
EST=ESTT(LVL)
EPS=EPST(LVL)
GO TO 1

```

```

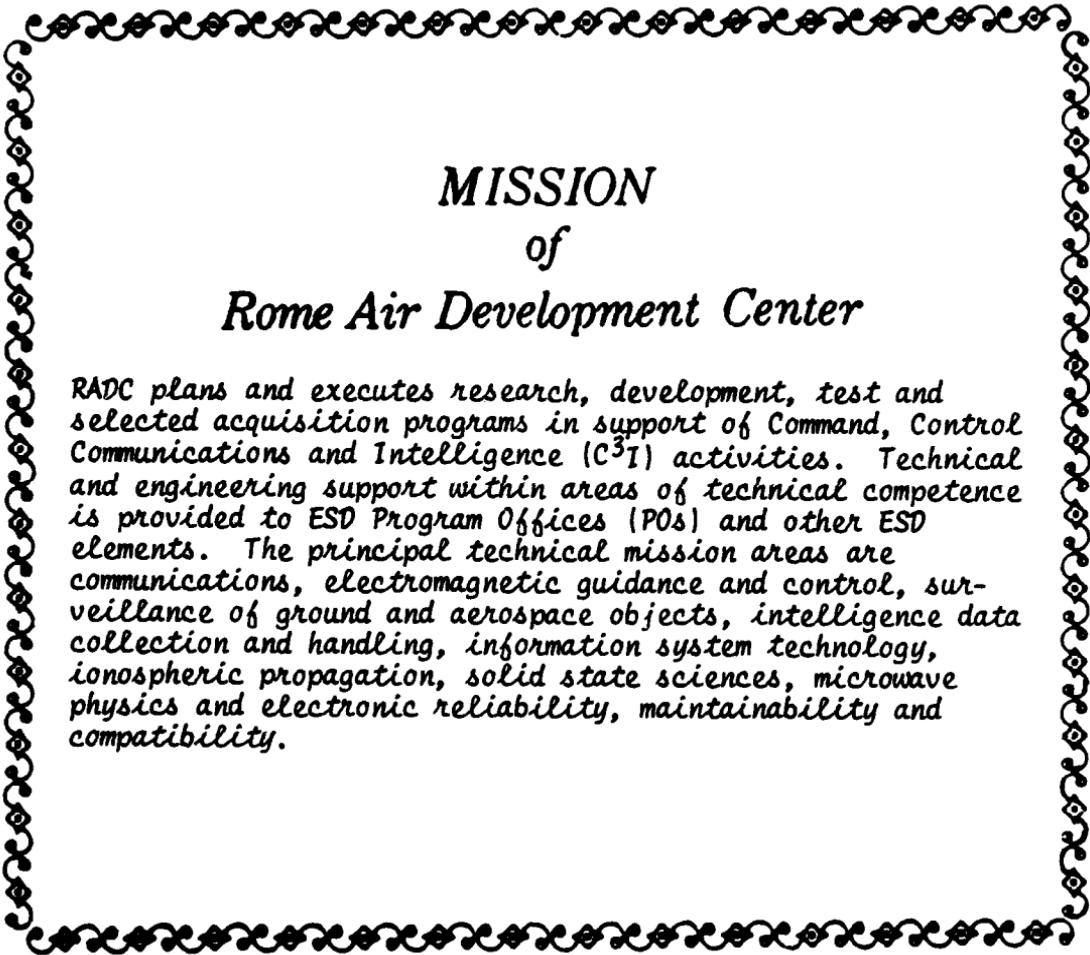
C C A PT 'POOR' VALUE. SET APPROPRIATE FLAGS.
C C
C C 5 IFLAG=2
C C GO TO 2
C C 6 IFLAG=3
C C GO TO 2
C C END.
C C FUNCTION F1(X)
C C COMMON X,C,Y,NX
C C DIMENSION X(100),Y(100),C(99,3)
C C DO 1 I=2,NX
C C 1 IF(XX.LT.X(I))GO TO 2
C C 2 D=XX-X(I-1)
C C I=I-1
C C F1=((C(I,3)+D*(C(I,2))-D*(C(I,1)))*D*(Y(I)
C C RETURN
C C END.
C C SUBROUTINE DEPOS(CURR,EDEPOS,TO,Z,AM,MODE,DEPTH)
C C DEPOS IS TABATA'S ALGORITHM MODIFIED TO CALCULATE FAST ELECTRON CURRENTS AS
C C WELL AS DOSE RATES UNDER ELECTRON IRRADIATIONS. SEE TEXT.
C C
C C MODIFICATIONS MADE BY A.R. FREDERICKSON OF RADC. FOR ORIGINAL
C C ALGORITHM, SEE REF.22, VOL.1, PAGE 265.
C C
C C PURPOSE
C C CALCULATE THE ENERGY DEPOSITION OF ELECTRONS NORMALLY INCIDENT
C C ON THE SURFACE
C C
C C USAGE
C C EDEP = EDEPOS(DEPTH,TO,Z,AM,MODE)
C C
C C DESCRIPTION OF PARAMETERS
C C DEPTH - DEPTH FROM THE INCIDENT SURFACE EXPRESSED IN G/CM2
C C TO - INCIDENT KINETIC ENERGY OF THE ELECTRON IN MEV
C C Z - ATOMIC NUMBER OF THE ABSORBER
C C AM - ATOMIC WEIGHT OF THE ABSORBER
C C MODE - SHOULD BE SET AS FOLLOWS
C C =1 IN THE 1ST USAGE FOR A GIVEN SET OF
C C VALUES OF TO, Z, AND AM
C C GREATER THAN 2 IN THE 2ND AND LATER USAGES FOR THE
C C GIVEN SET OF VALUES OF TO, Z AND AM
C C EDEPOS - ENERGY DEPOSITION IN MEV/(G/CM2)
C C
C C STATEMENT FUNCTIONS FOR THE INVERSE EXTRAPOLATED RANGE-ENERGY
C C RELATION AND ITS DERIVATIVE
C C
C C D1(L,Z)=1470./Z**0.691*L**C3
C C D2(L,Z)=1.+D1(L,Z)
C C D3(L,Z)=(257.-0.34*Z)/D2(L,Z)
C C D4(L,Z,AM)=EXP(L*(C2+D3(L,Z)))/C1)
C C T(L,Z,AM)=C1*(D4(L,Z,AM)-1.)
C C DIDX(L,Z,AM)=D4(L,Z,AM)*(C2+D3(L,Z))*(1.-C3*D1(L,Z)/D2(L,Z))
C C
C C CALCULATE DEPTH-INDEPENDENT TERMS
C C
C C IF(MODE=1) 7,7,27
C C 7 C1=2980./Z
C C C2=6.14*Z**1.026/AM
C C C3=0.505/Z**0.1874
C C C2=0.000178*Z
C C FK1=1.277/Z**0.134
C C TAU=TO/0.511004
C C BAK=1.15*EXP(-8.35/Z**0.925)/(1.+(.0185+15.7/Z**1.59)*TAU**0.156

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1      -4.42/Z))
1      F      -1.33/Z+0.385*TAU+((0.086+Z+0.33)-0.048/Z+0.35/1) (0.47
1      -Z+0.12)
1      IF (FEB) 14,14,16
14      FB=0.
15      GO TO 17
16      FB=BAK*FEB
17      FRZTAU=C.000236*Z*TAU
18      REX=0.2335*AW/Z+1.209*(ALOG(1.+CZ*TAU)/CZ-(0.9891-0.000301*Z)*TAU
19      /11.+11.568-0.0118*Z)*TAU+((1.232/Z+0.109)))
20      FACTOR=(1.-FB-FRZTAU/(1.+FRZTAU))*70/1 (REX,Z,AW)
21      BETA=4.09/Z+0.28*TAU+((0.106+Z+0.103)
22      ALPHA=(1.-1./BETA)*((1.-BETA)
23      XRLIM=0.1*REX
24      W1=T(XRLIM,Z,AW)
25      A1=DTDX(XRLIM,Z,AW)/W1
26      C
27      C
27      15 (DEPTH) 28.30.30
28      EDEPOS=0.
29      CURR=0.
30      RETURN
31      XRESID=REX-FX1*DEPTH
32      ADEP=ARLIM-XRESID
33      IF (ADEP) 33,33,37
34      DMDX=DTDX(XRESID,Z,AW)*FX1
35      IF (DEPTH.EQ.0.) GO TO 43
36      A1=(ARESID,Z,AW)
37      GO TO 39
38      W1=EXP(-A1*XDEEP)
39      DMDX=A1*FX1*W
40      S=DEPTH/REX
41      ASB=ALPHA*S+0.5*BETA
42      EDEPOS=FACTOR*EXP(-ASB)*(ASB*BETA/S/REX+W+DMDX)
43      CURR=(1.-BAK)*EXP(-ASB)
44      RETURN
45      EDEPOS=FACTOR*DMDX
46      CURR=1.-BAK
47      RETURN
48      END

```



*MISSION  
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Rome Air Development Center*

*RADC plans and executes research, development, test and selected acquisition programs in support of Command, Control Communications and Intelligence (C<sup>3</sup>I) activities. Technical and engineering support within areas of technical competence is provided to ESD Program Offices (POs) and other ESD elements. The principal technical mission areas are communications, electromagnetic guidance and control, surveillance of ground and aerospace objects, intelligence data collection and handling, information system technology, ionospheric propagation, solid state sciences, microwave physics and electronic reliability, maintainability and compatibility.*

END

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